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Mathematical Modelling in Industry: A Review

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ABSTRACT

Modelling is an important part of any large scale decision making process in a system. There are large numbers of factors that affect the performance of the system. It is not possible for the human brain to keep track of the all the players in the system, their interactions and interrelationships at a time. Therefore, to get the solution of the complex theoretical problems, one resort to the techniques, which are simple to use. Also, these techniques should be capable to reproduce all the essential and important relationships of the reality. Mathematical modelling is one such basic technique. This paper presents a literature review of the work done by several researchers to study the problems faced by industries, using mathematical modelling; particularly in the field of Diffusion innovation, Inventory Control and Production function.

Introduction

To analyze a real world problem or situation by representing it in a mathematical form, solving, interpreting the result and predicting the future implications is called *mathematical modelling*. It can be represented as follows:

Real world problem

↓

Mathematical formulation

↓

Using existing technique or developing a new tool for its solution

↓

Interpretation of result in terms of real world problem

↓

Prediction

To fit an existing technique or developing a new tool requires detailed information about the situation. For developing a model, it is advisable to break the complex situation into smaller segments and develop them independently. After getting the satisfactory results for these sub modules, combine them to get a model, which describes the real world situation to its optimal possible level. Because, most of the times a single model is not able to represent the real world situation to its fullest due to inherent constraints. However according to *M. S. Bartlett* a mathematical model can be considered successful if

✓ Known facts are accounted for

✓ Greater insight has been achieved of the situation under study.

✓ The theory of model can correctly predict the future patterns, even under different conditions than those pertaining to the current observed data.

Thus, mathematical modelling is actually the pure mathematics, applied to the real world problems to give solutions. Mathematical modelling and Mathematical technique are two sides of the same coin namely applied mathematics. Thus, applied mathematics is the connecting link between pure mathematics and other branches of science and technology.

Emergence of Mathematical Modelling

Though the roots of mathematical modelling extend to early 1800's, it was 1885, when Frederick W. Taylor [1] emphasized the application of mathematical analysis to the methods of production, that the real start took place.

Another man of early era was Henry L. Gnatt, who applied it to job scheduling methods [2]. A.K. Erlang was the first person to study the problem of telephone networks. By studying a village telephone exchange he worked out a formula, now known as Erlang's formula, to calculate the fraction of callers attempting to call someone outside the village that must wait because all of the lines are in use. He published this work. A few years after its appearance, his work was accepted by the British post office as the basis for calculating circuit facilities. Although Erlang's model is a simple one, the mathematics underlying today's complex telephone networks is still based on his work. [3]

During 1930's, H .C. Lavinson, an American astronomer, applied it to the problems of merchandising which included buying habits of customers, response to the advertising and relation of environment to the type of article sold (School of Science and Technology, 2011).

However, it was the first industrial revolution, which contributed mainly towards the development of *Operations Research* [4-5]. The industrial development brought with it new types of problems called executive type of problems. Various types of problems, which industry managers often come across, can be summarized as follows:

- Production house wants to maximize the production and minimize the cost.
- Marketing department wants a large but diverse inventory so that customer may be provided immediate delivery. It also likes to have a flexible production policy, so as to meet special demands on short notice.
- Finance department wants to minimize inventory so as to minimize the unproductive capital investment 'tied up' with it.
- Personal department wants to hire good labour and retain it.

Formulation of a policy which serves the interest of the whole organization and rather than of an individual department

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can be satisfactorily achieved by mathematical modelling, and such a decision is called *optimal decision* [6].

Thus, the mathematical modelling existed even in pre World War II era [7], but it was the work of Operations Research team, during World War II which attracted the attention of industry managers towards mathematical modelling. The successful jobs conducted by Operations research team during World War II included:

- ✓Developing strategies for mining operations.
- ✓Allocation of British Air force.
- ✓Designing new flight patterns.

Managing Industry

Industry is defined as the manufacturing or technically productive enterprises in a particular field, country, region, or economy collectively, or one of these individually. Apart from manufacturing it includes transport, energy supply and demand, mining, construction, and related informal production activities. Other sectors such as wholesale/retail trade, communications and real estate business activities are also included in the industrial classifications, in the categories of services and infrastructure.

Need of Mathematics in Industry

“Mathematics offers business a formula for success’’

“Mathematicians” have come up with an impressive multiplication formula for British commerce and industry: spend a few million pounds promoting the use of maths as a strategic tool, and add billions of pounds of value to businesses. That is the thinking about a new government-industry consortium, the Mathematics Knowledge Transfer Network. The network aims to boost the use of maths throughout the economy from grocery distribution to banking, telecoms to manufacturing.

Financial Times, February 2006 (Report on Mathematics in Industry, 2008)

Managing an industry is not an easy task. Especially the managers of large industrial establishment face very complex situation many a times. These problems include the inventory and production planning, capital limitations, transportation challenges; inflation, competition from other players on the ground, etc.

While dealing with all these problems, management has to be careful of their time management. If solutions are not obtained timely, then there is no use of any service or production as there will be no takers available in the market. To solve these problems industry managers generally undertake following measures:

- ✓Direct experiment
- ✓Trial and error methods.
- ✓Insights and intuition of industrial managers.

However, there arises a situation when direct experiments are not possible as either they are too costly or involve destructive testing. (Eg. designing the nuclear reactors). Also, if problem involves too many facts and figures, it is not possible to arrive at an efficient solution just by intuition and insight of the managers. Since in such situations, the number of possible solutions are too large to work out all of them but optimization is still desired. This is where the mathematics comes in.

The rapid industrial growth and innovation is the result of scientific research and techniques and is driven by mathematics. This industry–mathematics connection is very strong, not only in the field, which is beneficial for the industry, but it is also contributing in the areas such as health, security, communications, and environment which are concerned with the well-being of society.

The search for new life-saving drugs, the development of highly efficient machines and materials, the protection of sensitive ecosystems – all of these application-oriented activities, and many others, are strongly dependent on fundamental research, and that research is inextricably linked to mathematics.

In the 21st century there are major opportunities in industry as a result of mathematical interface. In the days of recession there is strong pressure on the industries to reduce their funds on research and development activities. The companies which will properly and efficiently utilize mathematics can rapidly gain a commercial edge over their competitors as over the years it has been a well established fact that mathematics is capable of enhancing the organisational capabilities without being expansive.

Mathematical Areas and their Applications

Various areas of mathematics and their industrial applications can be summarized as below [8-10]:

Table 1: Summarizing Mathematical Areas and their Applications

S. No.	Mathematical Area	Application
1	Algebra and number theory	Cryptography
2	Computational fluid dynamics	Aircraft and automobile design
3	Differential equations	Aerodynamics, porous media, finance
4	Discrete mathematics	Communication and information security
5	Formal systems and logic	Computer security, verification
6	Geometry	Computer-aided engineering and design
7	Nonlinear control	Operation of mechanical and electrical systems
8	Numerical analysis	Essentially all applications
9	Optimization	Asset allocation, shape and system design
10	Parallel algorithms	Wealth modelling and prediction, crash simulation
11	Stochastic processes	Signal analysis

Mathematical Modelling in Industry; A Literature Review

The mathematical sciences form an essential part of science, engineering, medicine, industry and technology. It serves as one of the pillars of education at all levels. Since beginning the mathematics has been a great tool to establish the natural laws. In its early days, advances in mathematics were guided purely by the interests of individuals like Aryabhatta, Srinivas Ramanujan, Bhāskara, Leelavathi(India) [11], Aristotle, Archimedes, Euclid, Pythagoras [12] (Greek), Guo Shoujing, Liu Hui, Liu Xin , Li Zhi (China) [13] etc. For several years industrial applications of mathematics were almost nill. No visible real life applications and its abstractness were the reasons that mathematics, for a long time, remained a subject of the chosen ones.

At the turn of the twentieth century, industry and military concern with technology sought a new class of mathematicians that required a new mathematical discipline. Now came the time when mathematics should come out the aesthetics and abstractness of the curriculum and start playing an important role in the development and planning of defence and industry mechanism.

Concerned with real and concrete environments, engineers were a growing class of scientific professionals to solve the problems demanding technical expertise. They were technological pioneers in the field of construction of bridges, canals, and railroads, but their occupation now demanded better mathematical skills within technological-savvy world [14]. For example, during World War I production engineers and entrepreneurs had worked to improve airplane production and aerodynamics. Most of these designs based on very little analytical or mathematical understanding of the theory of lift, drag, or airfoil. These aspects of plane construction were very complex in nature. These designs were critical to building safe and efficient aircraft. When mathematicians applied their skills in understanding and manipulating the complexities of numbers and theories related with the design, solutions obtained were better.

In 1930 government and businesses such as the automobile and air traffic industries started thinking for developing their resources. They wanted to find alternative ways of increasing production, and the military wanted to develop a mechanism for planning large-scale military action. Hence need for quantitative models arouse. [15].

These objectives required advanced knowledge of mathematics that engineers lacked. This was the beginning of an era where mathematician had to come out from the exile and play a very important and active role in building the society [16].

In 1940, during the World War II, the National Resources Planning Board submitted a review of industry's conditions entitled, "Research—A National Resource." Among the many reports within this compendium was Fry's 38-page report. "Industrial Mathematics," Fry (1941) reported that *"Throughout the whole of industry, research is becoming more complex and theoretical, and hence the value of consultants in general and mathematical consultants in particular, must increase. Theories of supply and demand required a new class of mathematicians. They could help in reducing the labour costs and avoid unnecessary experimentation."* [17]

Thus, slowly but steadily mathematics became an essential research and development tool in managing industry. Since then many models have been proposed, modified and used to strengthen the economic growth and industrialisation.

Since large scale industries have not one but many problems and solving them collectively involved several variables leading to the complex structures. So, for the sake of simplicity and effective use of mathematics as per the particular query, many sub-areas developed and have been enhanced with the passage of time. Some of these sub-areas are

✓ Diffusion innovation
✓ Inventory Control
✓ Production function
✓ Cost management
✓ Revenue management
✓ Resource Allocation
✓ Transportations and scheduling
✓ Quality Control
✓ Network analysis etc.

A detailed review of some of these topics is given as follows:

Diffusion innovation

The spread of an innovation in a market is termed as "diffusion". Diffusion research seeks to understand the spread of innovations by modelling their entire life cycle from the perspective of communications and consumer interactions. The original attempt in diffusion research has been made by the French sociologist Gabriel Tarde [18]. He plotted the original S-shaped diffusion curve and observed that the rate of adoption of a new idea usually follows an S-shaped curve over time. Nothing fruitful happened for forty years and then in 1940's, two sociologists, Bryce Ryan and Neal Gross presented their study on diffusion of hybrid seed among Iowa farmers and proved that Tarde [18] was right in his prediction of S-shaped diffusion curve [19].

This study aroused the interest of academician in the field and then Mansfield [20] investigated the factors determining how rapidly the use of a new technique spreads from one firm to another. He presented a simple model which can explain differences among innovations in the rate of imitation. Both the deterministic and stochastic versions of this model were presented and are tested against data showing how rapidly firms in four industries came to use twelve important innovations. The empirical results seem quite consistent with both versions of the model. Fourt and Woodlock [21], Floyd [22], Rogers [23], Chow [24] presented their studies and further enriched the topic.

First Tarde [18] and then Roger [23] laid the foundation of mathematical modelling of diffusion innovation. Their work was more theoretical in nature but that doesn't make it unimportant. As on this theoretical framework is based the Bass model [25]. This model is a breakthrough in the history of mathematical modelling of diffusion innovation and is the most sought after model in this field.

The Bass model states that the probability that an individual will adopt the innovation — given that the individual has not yet adopted it — is linear with respect to the number of previous adopters. The model parameters p, q, and m can be estimated from the actual adoption data. Parameter estimation issues are discussed in Jiang, Bass and Bass [26]; Boswijk and Franses [27], Van den Bulte and Stremersch [28]; Venkatesan, Krishnan and Kumar [29], Lilien *et al*., [30]. Sultan, Farley and Lehmann [31].

The Bass model [25] is a very useful tool for forecasting the adoption (first purchase) of an innovation (more generally, a new product) for which no closely competing alternatives exist in the marketplace. A key feature of the model is that it embeds a "contagion process" to characterize the spread of word-of - mouth between those who have adopted the innovation and those who have not yet adopted the innovation. The model, however, is not without its imperfections. Parameter estimates of the model may not be stable for relatively new products because adoption data are limited. Studies by Heeler and Hustad [32] Srinivasan and Mason [33] Bemmaor and Lee [34] Suggested that stable and robust parameter estimates for the Bass model are obtained only if the data under consideration include the peak of the non-cumulative adoption curve

The extent of research into modelling and forecasting the diffusion of innovations is impressive and reflects its continuing importance as a research topic. Modelling developments in the period 1970 onwards have been in modifying the existing models by adding greater flexibility in various ways.

Bass [35] himself commented on Bass [25] model and discussed the background and history of the development of the paper, the reasons why the model has been influential, some important extensions of the model, some examples of applications, and some examples of the frontiers of research involving the Bass Model.

Over the years, several diffusion models incorporating both social and marketing mix variables have been proposed. Such models are used not only to describe and predict new product sales but also to provide normative insights. The Generalized Bass Model, or GBM, proposed by Bass, Krishnan, and Jain [36] includes decision variable like price, advertisement etc in the Bass model. GBM has been very popular in both descriptive and normative applications. Fruchter and Van den Bulte [37] studied optimal advertising policy under the GBM structure and concluded that this strategy remains optimal in the presence of decreasing prices that affect both margins and diffusion speed. The optimal GBM policy is to spend extremely little at first (the lower the better) and then increase spending throughout the planning period.

Krishnan *et al*. [38] considered optimal pricing policies under the GBM, considering both initial price $Pr(0)$ and subsequent price evolution $R(t)$. But their work was different from the work of Fruchter and Van den Bulte (2011) [37] in many ways. Krishnan *et al.* did not work with the traditional, empirically validated GBM specification but rather with a variation of the model that has not been empirically supported. They used numerical analyses to arrive at the optimal pair $\{Pr(0), R(t)\}$". Also, they did not provide analytical expressions to identify the switching times at which the optimal pricing path changes direction (e.g., from increase to decrease).

Chun-Yao Huang *et. al.* [39] presented a stochastic model for the sales of a new CPG product that integrates all of an individual's purchases of the new product (as opposed to developing separate models for trial, first repeat, etc.) and simultaneously captures the effects of marketing activities in initial repeat buying behaviour at the individual consumer level.

Fader, Hardie and Chun-Yao Huang [40] developed a dynamic change-point model that captures the underlying evolution of the buying behavior associated with the new product. This extends the basic changepoint framework, as used by a number of statisticians, by allowing the changepoint process itself to evolve over time. Additionally, this model nested a number of the standard multiple-event timing models considered in the marketing literature. In their empirical analysis, they showed that the dynamic change-point model accurately tracks (and forecasts) the total sales curve as well as its trial and repeat components and other managerial diagnostics.

In another study Fader, Peter S. and Bruce G.S. Hardie [41] reviewed the E/KS model; they first examined its ability to provide insights into the structure of the repeat buying process. In a series of simulations for a simple stationary market, we show that the model fares very poorly in this regard. They showed that the forecasting performance of the model, even under data conditions that include different types of non-stationarity, is quite impressive. They systematically varied three factors (length of calibration period, purchase cycle (fast vs. slow), and degree of consumer heterogeneity), and find that the week 52 E/KS forecasts are remarkably robust. As expected, forecast accuracy improved significantly as more data are available to fit the model, but variations in the other two factors lead to relatively modest differences, as measured by the absolute percentage error in the year-end sales estimates as well as a measure of forecast bias.

The diffusion of an innovation rarely takes place in a stable, homogeneous and unchanging environment. The first mathematician to address the income heterogeneity hypothesis was Duesenberry [42]. The heterogeneity of income distribution has been cited by other authors like Bonus, [43] as a driver for the S shaped diffusion curve. Their view is that the diffusion curve reflects the nature of income distribution. As the price of an innovation falls, more consumers can afford it, provided the income distribution is bell-shaped, and the price falls monotonically, an S curve will result. Liebermann and Paroush [44] argued that income heterogeneity, price and advertising are important drivers of the diffusion process.

Rogers [45] in his frame-work of heterogeneous innovativeness revealed that the diffusion of an innovation will not accelerate if critical mass is not reached. In this context, he makes the distinction between interactive and non-interactive innovations. Wareham, Levy and Shi [46] investigate socio-economic factors underlying the diffusion of the internet and 2G mobiles in the US. Mobile adoption is positively correlated with income, occupation and living in a metropolitan area.

Other than income, heterogeneity can be geographical also. Culture which changes as the location changes, drives the adopters need and behaviour. Goldenberg, Libai, Solomon, Jan and Stauffer [47] theoretically examine innovation diffusion in a spatial context. They used simulation. The S- curve produced by this model has a very late point of inflexion, very close to the saturation level. In an empirical study, Baptista [48] examined the diffusion of numerically controlled machines and microprocessors in various areas of UK. He found that there were significant regional effects on the rate of diffusion.

Tanner [49] used GDP/capita and the cost of car usage as additional variables to forecast the growth of car ownership in the UK

Innovativeness is an individual's characteristics and can be described as risk taking ability and quest for trying new things before the majority does. Leavitt and Walton [50-51] describe innovators as individuals open to new experiences. Individuals who may be seen as innovators in one product domain, however, may not necessarily be innovators in other domains. The measurement of individuals into adopter categories such as innovators, early adopters, early majority, late majority and laggards should be done by keeping this in mind. The categorization scheme proposed by Roger [52], however, has potential limitations.

Several researchers have argued that Roger's assumptions that all new products follow a normal-distribution diffusion pattern, products is not reliable and should be further investigated.[53]. Definition of innovativeness is solely time-dependent and requires a product launch if it is to be observed and measured. Studies by Gatignon and Robertson [54], Citrin *et al*. [55], Goldsmith [56] and Blake *et al*. [57] agrees that innovativeness can differ and hence must be identified for a specific product category or region. Measurement scales developed by Goldsmith and Hofacker [58], therefore, are domain-specific and reflect an individual's tendency to learn about and adopt innovations within a particular area of interest. Another approach to measuring innovativeness is to determine the individual's adopter category on the basis of the relative time of adoption of the innovation. Rogers [59], defines a person's innovativeness as 'the degree to which an individual is relatively earlier in adopting new ideas than other members of a social system'.

Critical mass is defined as "the point after which further diffusion becomes self-sustaining. Grajek and Kretschmer [60] developed a structural model of demand for a network good to provide a rigorous definition of critical mass. Using simulations, they demonstrate that their model of critical mass can be used easily and can generate theoretically grounded insights about critical mass phenomena identified in empirical settings

In a world where many of the Internet-based communication applications have proven to be very popular amongst consumer groups; David H. Wong *et.al.* (2011) [61] studied 10 years of diffusion data to examine the diffusion pattern of a mature internet application –email. The Bass Model approach was utilised to classify individuals into adopter categories. The results revealed a q/p ratio of 50.7 for the adoption of email; indicating that the imitation effect is greater than the innovation effect in the diffusion of similar Internet-based communication technologies. It was also found that for such technologies, the peak of the non-cumulative adoption curve can be expected in 5.4 years after launch.

Inventory Control

The mathematical analysis of inventory systems and the consequent theory of inventory control are neither new nor complete. The earliest attempts to determine a mathematical basis for the control of inventories originated from the manufacturing industries, and constituted the problem of economic lot sizes. The earliest known work published in this field was by George D. babcock [63], based on work done in installing the Taylor system in the Franklin Manufacturing Company in 1912. This paper could not develop the model, but it attempted to equate set-up costs and "capital costs". This approach gave rise to a cubic equation.

The next major development occurred in 1918, when E. W. Taft [63] dropped the previous assumptions that production of the finished product occurred at an instantaneous rate. J. A. Bennie [64] attempted to include in his stock model the cost of stock held to protect against unexpectedly large demand -- safety stock. R. C. Davis [65], introduced this concept, and explicitly included the costs of this reserve stock in determining the optimum lot size.

Fry [66] attacked an inventory problem wherein demand for the product was allowed to be a random variable (Poisson distribution). Using a criterion of fixed run-out probabilities, he was able to find the probability of running out of stock as a function of the maximum quantity stocked. From this the quantity to be stocked for fixed run-out probabilities was immediately found.

Masse seems to have been the first person to successfully introduce risk concepts into the problem of dynamic inventory systems [67].

Rosenblatt added a new problem to the inventory field. He posed the question of randomly-arriving quantities of supply. Viewed in an agricultural setting, his problem is to find the optimum quantity of a good -- say, wheat -- to plant, given that the quantity produced (arriving) is a random function of the quantity planted. [68]

Harris [69] developed *EOQ model*. Many researchers have extended the above model with different types of demands and replenishment. A detailed literature is available in the text book such as Hadley and Whitin [70] , Tersine [71], Silver and Peterson [72], etc. In classical inventory models, demand is normally assumed to be constant.

Many researchers have examined the classical inventory policy with *infinite and finite production rate*. Donaldson [73], Silver [74], Ritchie [75], Dave [76], Urban [77] and others who formulated and solved inventory models with infinite production rate taking various types of demand, namely constant or time dependent or stock dependent.

There is some literature on the inventory models with *stock-dependent demand*. Several authors like Mandal and Phaujdar [78], Urban [79] worked in this direction.

A lot of research literature available on diffusion innovation has used simple *ABC analysis* to resolve the inventory control problem. It can be found in the work of Canen and Galvao [80] and Cohen and Dunford [81]. In 1986, Dias [82] performed principal component analysis is used in Sri Lanka and inventory control in the copper mine of Zambia was studied by Magson [83]. An indigenous application of inventory control in the beedi industry can be found in the study made by Lingraj [84].

A computer based model by Gupta, Shukla and Tripathy [85] is used for fertilizer distribution in India. Bhunia and Maiti [86-87] proposed two deterministic inventory models for a single item, where for the first model, the production rate at any instant depends on the on-hand inventory and for the second one, it is demand dependent. However, in both cases, the demand rate at any moment of time is a linear function of time for the scheduling period. Both the models are formulated and solved without allowing shortages.

For the solution of inventory models with *finite rate of replenishment*, Deb and Chaudhuri [88] considered the constant rate of demand whereas Goswami & Chaudhuri [89] and Hong, Cavalier & Hayya [90] took the time dependent demand and Mondal and Phaujdar [91] the stock dependent demands.

In 2^{nd} International Conference on Economics, Trade and Development Napaporn Rianthong and Aussadavut Dumrongsiri [92] presented a mixed integer linear programming model for an integrated decision of production, inventory and transportation planning problem. This model combines the direct shipment into distribution decisions.

In *direct shipment*, a manufacturer directly delivers the products to retailers by bypassing warehouse, thereby saving transportation cost from a plant to warehouse, and inventory holding cost at warehouse. The objective is to minimize overall cost comprising of production setup cost, inventory holding cost, transportation cost and reorder cost. A deterministic, multi-item inventory model with supplier selection and imperfect quality was proposed by Rezaei and Mansoor [93]. This model considers a supply chain with multiple products and multiple suppliers, with limited capacity. It is assumed that received items from suppliers are not of perfect quality. These imperfect items could be used in another inventory situation. Imperfect items are sold as a single batch, prior to receiving the next shipment, at a discounted price.

Mehar [94] presented a computational model to determine the factors of the optimal level of merchandizing inventory. The study is based on a mathematical model. The results revealed that the 'Usage of Material' or the Sales Volume is not the real determinate of the inventory volume. It is concluded in the model that the volume of inventories depends on the difference between the return on investment in the inventories and the rate of interest on short-term deposits.

Deteriorating items are common in our daily life. The inventory problem of deteriorating items was first studied by Whitin [95], he studied fashion items deteriorating at the end of the storage period. Then Goyal and Giri [96] made comprehensive literature reviews on deteriorating inventory items. Hill [97] was the first to introduce the ramp type demand to the inventory study. Then Mandal and Pal [98] introduced the ramp type demand to the inventory study of the deteriorating items.

Hartely [99] and Sarma [100] were the pioneer in discussing the *two warehouse inventory models* with infinite production rate, but without shortages. Dave [101] discussed the inventory models for finite and infinite rate of replenishment, rectifying the errors for the model given by Sarma [100] and gave a complete solution. Further, Goswami and Chaudhuri [102] considered the models with or without shortages taking linearly increasing time dependent demand. Correcting and modifying the assumptions of this model, Bhunia and Maiti [103] analysed the same inventory model and graphically presented a sensitivity analysis on the optimal average cost and the cycle length for the variations of the demand parameters. In all these models, only the cases of non-deteriorating items were discussed.

Bhunia and Maiti [104] Proposed deterministic inventory model with two warehouses. The model allows different levels of item deterioration in both warehouses. The demand rate is supposed to be a linear (increasing) function of time and the replenishment rate is infinite. The stock is transferred from rented warehouse to own warehouse in continuous release pattern and the associated transportation cost is taken into account. Shortages in own warehouse are allowed and excess demand is backlogged.

The effects of *inflation* and time-value of money were ignored in the classical inventory models. It was believed that inflation would not influence the cost and price components. The economic situation of most of the countries has changed considerably due to large-scale inflation and decline in the purchasing power of money. Buzacott [105] and Misra [106] were the first to develop EOQ models taking inflation into account. Both of them considered a constant inflation rate for all the associated costs and minimized the average annual cost to derive an expression for the economic lot size.

Their work was extended by researchers like Chandra and Bahner [107], Aggarwal [108], Misra [109], Bierman and Thomas [110], Sarker and Pan [111], etc. to cover considerations of time value of money, inflation rate. Manna and K. S. Chaudhuri [112] incorporated inflation in EOQ model. This model is based on discounted cash flows (DCF) approach where demand rate is assumed to be non-linear over time. An infinite time-horizon deterministic economic order quantity (EOQ) inventory model with deterioration is developed. The effects of inflation and time-value of money are also taken into account under a trade-credit policy.

Production Management

Production is one of the main focuses in industry. All the efforts that an industry undertakes directly or indirectly are to increase the production and decrease the cost. A production function deals with the problem of optimizing the production under available resources and hence helps in increasing the efficiency and profit of the industry. It gives a mathematical estimate of the maximum quantity of an output that can be produced using various combinations of inputs.

It is well established fact that that the concept of diminishing marginal productivity of production factors (land, labour, or capital) is credited to Anne Robert Jacques Turgot [113] and can be found in the works of Smith and Malthus who introduced the logarithmic production function. Stigler [114].

But an extensive and quantitative study of the marginal productivity analysis was first given by von Thünen [115-116]. He had examined separate expressions for both the marginal products of labour and capital. Then he first tentatively assumed and later explained that the marginal product of capital $(MP)_k$ is numerically equal to the rate of increase in average labor productivity $(AP)_L$ with respect to capital. With this crucial economic insight and mathematical synthesis, the modern production function concept was born. In 1969 Lloyd [117] provided a complete account of von Thünen's exponential production functions and their derivation.

Mathematically, a production function relates the amount of output (Y) as a function of the amount of input (X) used to generate that output. $Y = f(X)$

In The Isolated State, vol-II, von Thünen a farmer and noted economists of his times wrote down the first algebraic production function $P = hQ^n$, where P is output per worker, Q is capital per worker and h is the parameter that represents fertility of soil and efficiency of labour.

Philip Wicksteed [118] demonstrated that if production was experiences constant returns to scale then with each input receiving its marginal product, the total product would then be absorbed in factor payments without any deficit or surplus.

In the 1920s the economist Paul Douglas was working on the problem of relating inputs and output at the national aggregate level. A survey by the National Bureau of Economic Research found that during the decade 1909–1918, the share of output paid to labour was fairly constant at about 74% despite the fact the capital/labour ratio was not constant. He enquired of his friend Charles Cobb, a mathematician, if any particular production function might account for this [119]. This gave birth to the original Cobb–Douglas [120] production function.

Since production doesn't depend just on capital so it required a mathematical growth model with labour and capital as state variables. Such mathematical model was the Solow growth model [121]. Solow along with Kenneth Arrow, H.B. Chenery and B.S. Minhas presented ACMS [122] production function. In literature this is popularly known as constant elasticity of substitution or CES production function. This function allowed the elasticity of substitution to vary between zero and infinity. But once established it remained fixed along and across the isoquants irrespective of the size of output or inputs.

Y. Lu and L.B. Fletcher [123] developed a generalized CES production function. They allowed the elasticity of substitution to vary along different levels of output under certain circumstances. The Cobb-Douglas production function [120] is a special case of CES model. Cobb-Douglas kept the elasticity of substitution of capital for labour fixed to unity. Other attempts to generalize CES were by Mukerji [124] and Bruno [125]

Diewert [126] made two very important generalizations of production functions. First, he obtained a functional form that can incorporate many inputs in it and the second that such a functional form permitted variable elasticities of substitution.

Aschauer [127] stimulated an extensive discussion of the nature and magnitude of the impact of infrastructure capital on output and productivity growth. He estimated a Cobb-Douglas function and established that the decline in US productivity growth in the 1980s was associated with the neglect for public infrastructure over the same period.

Alicia Munnell [128] used a similar procedure, but different data, on aggregate private capital stock for a period ranging from 1948-87. Both Aschauer and Munnell employ aggregate time-series data of the United States to estimate the relationship between private output and the stock of non-military public capital. Several production function studies address infrastructure and productivity relationships at the state level using time-series cross-section data for the 48 contiguous states. The cross-sectional aspect of these data has certain advantages that minimize the possibility of deceiving correlation over time. As a whole, studies based on state-level data support a relatively lower but still positive relationship between public infrastructure and productivity. Munnell's elasticity estimates show that, while public capital has a positive effect on output , it is only half the size of the effect of private capital.

Using Munnell's data, Eisner [129] suggests that for all functions considered, the coefficient of public capital in the estimated equation remains significant when the data are arranged to reflect cross-sectional variation, but becomes insignificant when the data are arranged to allow for time-series variation. This shows that states with more public capital per capita have more output per capita. However, a state that increases its public capital in one year does not produce more output in that same year as a result. Therefore, Eisner regards the direction of causation between output and public capital as undetermined, and postulates that a lag structure is required to obtain a true time-series relationship between output and public capital.

The endogeneity problem is internal to the firm's optimal choice of inputs. According to Marschak and Andrews [130], using the inputs and outputs of profit maximizing firms to estimate production functions gives rise to an endogeneity problem. Some of the methods of controlling for the endogeneity problem in the production function are instrumental variables and fixed effect estimation. But these solutions have proved out to be unsatisfactory on both theoretical and empirical grounds. Olley and Pakes [131] developed a new solution to the endogeneity problem. This work was further extended by Levinsohn and Petrin [132], and Ackerberg *et al.* [133].

The replacement function cannot be used to identify the production function in the presence of a productive input which is both variable and static as suggested by Bond and Soderbom [134]. This is an inherent limitation of scalar unobservability: It does not allow variation in such inputs which arise from outside of the production function. As a consequence, the replacement function approach cannot identify gross output production functions when some inputs are variable and static.

Value-added production functions are problematic for applied work as they are based on many strong restrictions e.g., Basu and Fernald (1997). In general, when the assumptions of constant return to scale and perfect competition are violated, using the value-added production function to recover productivity is no longer a valid option. Perhaps more importantly, gross output production functions are required to study a number of important empirical problems, such as the problem of revenue production functions, or analyzing productivity among exporting firms (Rivers, [136]).

Future Implication and Direction for research

Today the impact of mathematical modelling can be felt in many areas such as health industry, library, city planning, transportation system, crime investigation, educational psychology, Behavioral science, Social sciences and environmental science and industries.

Although work is being done in this field, mathematical modelling is still in its infancy and a great deal of work is still necessary to develop more powerful models, so as to predict the future and plan the entire course of action, even before the problem arises, without much loss of money or/and manpower and natural resources.

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