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Modelling diameter distributions of a tropical natural forest in south west Nigeria with the beta distribution function

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ABSTRACT

The diameter distribution models of a natural stand in Shasha forest reserve (1.44ha permanent sample plots), South West Nigeria, were modeled with Beta distribution function. The data consisted of dbh measurement of all the trees with dbh ≥ 10cm. The data set for 1960 was used to model the diameter distribution while 1976 data set was used for model validation which should not be significance to the observed values. Simple linear regression equation was used to fit the Beta function/models. The best model from all the parameters was selected based on least values of standard deviation error, significance and high coefficient of determination. The result revealed that 68 tree species were currently available in the study area. The maximum dbh was observed in 1960 data set (164.91cm) while the least was observed in 2011 data set (81cm). The best model fit was $(a+b) = 0.80 lnDq – 200.79$ (R^2 = 61.31%, SE = 24.78 and p = 0.0003). The results of the models validation revealed that there were no significant differences between the observed and the predicted value of the parameters. Models developed are therefore recommended for application in the projection of diameter distribution for proper management.

Introduction

The tropical rain forest is one of the major vegetation types of the globe [1][2]. It occupies a total area of 1818.43 million hectares, representing 47% of the total land area occupied by all forest types of the world [3]. The tropical rain forest is the most diverse of all terrestrial ecosystems, containing more plant and animal species than any other biome [4]. In spite of this diversity, most species are locally endemic or rare and patchily distributed [1]. It is worthy to note that in recent times, the concern has been to concentrate conservation effort in the tropical rainforest because of its richness in biodiversity. This rainforest is one of the major vegetation belts in Nigeria. It consists of the moist tropical, and the lowland semi-deciduous forests, which form a narrow strip of green belt, a few kilometers inland along the coast and covers a total area of 13,300,000ha [5]. Although several studies describing the Nigerian rainforest abounds, information is partially or completely lacking on the changes in stand structure, differences in microclimate, soil and other biotic factors [6]. Such information if made available will enhance effective planning and management of the forest reserves.

Development of growth models for tropical species enables promotion of the productive and protective aspects of diverse species present [7]. Diameter class models allow planning of various uses and provide data about stand structure. These models are used to estimate stand variables and their structure with a density or distribution function, which is fitted to diameter distributions at breast height (dbh) or individual tree volume.

Forest managers are interested, for example, in being able to estimate the number of trees in different diameter classes in a stand, because the size of the diameter determines the industrial use of the wood and thus the price of the different products. Diameter distributions also provide information about stand structure, age structure, stand stability, etc. and enable the planning of silvicultural treatments. Furthermore, tree diameter is an important factor in harvesting because it determines the type of machines used and how they perform during felling and transportation of the wood.

The first mathematical description of a specific form of the diameter function in all-aged forest was provided by [8]. He found that plotting the number of stems against equal-diameter classes as a frequency histogram results in a reverse J-shaped curve. Field studies in virgin and old-growth forest have confirmed the utility of the negative exponential model [9][10][11][12], although there are occasionally small changes in the decreasing curve [13].

Diameter class models also have been applied to even-aged stands. In this case the frequency histogram in a diametric range is similar to the Gauss distribution but with a different shape because of the skewness and the kurtosis of the corresponding curve. Various distribution functions have been used to describe and predict stem frequency in even-aged and uneven- aged stands, such as the normal [14], gamma [15], Gram-Charlier [16], Johnson's SB [17][18][19] and Weibull function [20][21]. The popularity of the latter is based on its relative simplicity and flexibility [22].

The beta density function was used by [23][24] because it had the considerable advantage of being simple, highly

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adaptable and practically any form of diameter distribution can be described. Furthermore, this density function was used in the first models in which the function parameters were related to stand variables, in *Pinus elliottii* Engelm stands in Georgia [25].

Stand models that provide accurate estimates of stand growth and yield have become essential tool for evaluating the numerous management and utilization decision. No single type of stand model can be expected to provide information efficiently for all levels of decision making [26]. Therefore, there is need for wide variety of models of varying degree of complexity for the management of natural forest and plantation. Dbh of forest trees is an essential variable in determining the basal area and more importantly the volume of forest stands. It is the easiest determined/measured variable which can be use to predict or project the growth and yield of the forest estate. Therefore, the main objective of this study is to determine diameter distributions using appropriate distribution function such as Beta. This is of major importance to the forest managers in order to select system that emphasized the importance of recreating a specified diameter distribution or stand structure at the end of each cutting cycle [27][28][29].

Materials and methods

The study area

This study was carried out in Permanent Sample Plot (PSP) 133 Area 5 in Shasha Forest Reserve (Fig 1). The PSP is 1.44ha divided into fifteen sub-plots of sizes 30m x 30m each. The Forest Reserve is located in Ife South Local Government Area in Osun state, Nigeria which lies on Latitude 09^0 4' and 9^0 50'N and Longitude 3^0 54' and 4^0 6'E at altitude 122m above sea level with a mean annual rainfall of 1421mm. Soil type is ferruginous tropical soils on crystalline acid rock. The topography is gently undulating to undulating plain. The vegetation is mainly of the high forest type [30].

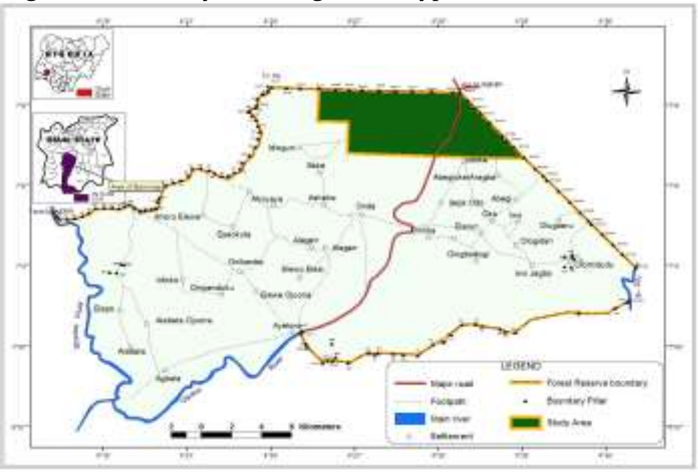


Fig 1: Map of Shasha forest reserve

Data set and processing

The data used to develop diameter distribution models were obtained from six sets of data on the PSP. These were the 1960, 1962, 1976, 1982, 2006 and 2011 data sets. This involved the dbh measurement data of all trees (dbh≥10cm) in each plots. The 2011 data collected was primary while others are secondary obtained from the Forestry Research Institute of Nigeria (FRIN). All living trees within the range of specified dbh (≥ 10 cm) were identified by their botanical name. In few cases where a tree's name is unknown, twig samples of such tree were collected and identified later at the Forestry Herbarium Ibadan. The data collected were sorted into species, families and dbh size class frequency using descriptive statistics. The results were presented in form of tables and chart. The following stand variables were calculated from the inventory data: quadratic mean diameter, mean diameter, minimum diameter, maximum diameter, number of trees per hectare, basal area and current stand volume.

Fitting of the Models

The data set for 1960 was used to model the diameter distribution base on the fact that the stand diameter distribution had fewer disturbances as at the time of establishment and to avoid serial correlation of observations as noted by [31]. The Beta distribution function is as stated by [32]. The general formula for the probability density function of the beta distribution is

$$f(x) = \frac{(x-a)^{p-1}(b-x)^{q-1}}{B(p,q)(b-a)^{p+q-1}} \quad a \le x \le b; p,q > 0 \text{…………….1}$$

Where:

p and q are the shape parameters

a' and b' are the lower and upper bounds, respectively, of the distribution, and

$B(p,q)$ is the beta function. The beta function has the formula

$$B(\alpha,\beta) = \int_0^1 t^{x-1}(1-t)^{\beta-1}\,dt \text{……………………………2}$$

The case where $a = 0$ and $b = 1$ is called the standard beta distribution.

$$f(x) = \frac{x^{p-1}(1-x)^{q-1}}{B(p,q)} \qquad 0 \le x \le 1; p,q > 0 \text{……………3}$$

Typically the distribution is defined in terms of location and scale parameters. The beta is different in that it defines the general distribution in terms of the lower and upper bounds. However, the location and scale parameters can be defined in terms of the lower and upper limits as follows:

Location (lower bound) = a Scale (Upper bound) = $b - a$

The formula for the cumulative distribution function of the beta distribution is also called the incomplete beta function ratio (commonly denoted by I_x) and is defined as

$$F(x) = I_x(p,q) = \frac{\int_0^x t^{p-1}(1-t)^{q-1}dt}{B(p,q)} \qquad 0 \le x \le 1; p,q > 0 \text{……4}$$

Where B is the beta function defined above.

The parameters are estimated as follows

$$p = \bar{x}\left(\frac{\bar{x}(1-\bar{x})}{s^2} - 1\right) \text{………………………………….. 5}$$

$$q = (1-\bar{x})\left(\frac{\bar{x}(1-\bar{x})}{s^2} - 1\right) \text{…………………… 6}$$

Where: p and q are shape 1 and 2 parameters respectively. $\bar{x}$ is the sample mean and s^2 is the sample variance. If a and b are not 0 and 1 respectively, then replace $\bar{x}$ with $\frac{\bar{x} - a}{b - a}$ and s^2 with $\frac{s^2}{(b-a)^2}$ in the above equations.

Model validation

The data set for 1976 was used as validation set since this set can still be used to describe the dynamics in the plot because little disturbances was observed here. It is important to subject the models formulated to a process of validation before inferences about the real world obtained from them can be used with confidence. Validation involves the testing and comparing of the model output with what is observed in the real world [33]. The constructed models were used to predict the values of the Beta parameters known as expected values. These values were compared with the observed values estimated from the validation data sets using the student's t-test. For a valid model, this comparison should show no significant difference at 5% level of significance.

$$t = \frac{\bar{X}_1 - \bar{X}_2}{S_{X_1 X_2} \cdot \sqrt{\frac{2}{n}}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ 7$$

Where

$$S_{X_1 X_2} = \sqrt{\frac{1}{2}(S^2_{X_1} + S^2_{X_2})} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ 8$$

$\overline{X}$ = Means for prediction model and real data respectively

$S_{X1\&2}$ = Pooled standard deviation

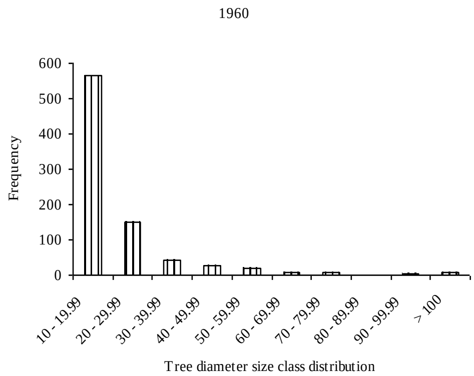
Results

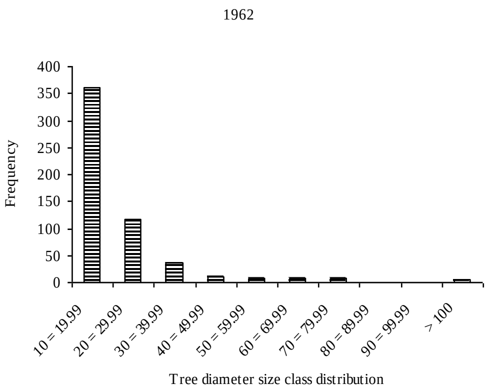
Stand density

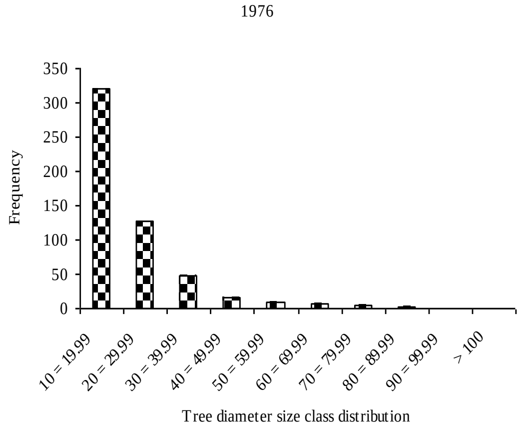
Table 1 showed that the number of trees per hectare followed a downward trend between 1960 and 2011 having decreased from 831 stems/ha to 499 stems/ha. Similar trend was also observed in the basal area as it decreased from 48.69m^{2}ha^{-1} in 1960 to 27.88m^{2}ha^{-1} in 2006 and later increased to 30.87m^{2}ha^{-1} in 2011. Meanwhile, maximum dbh encountered was 164.91cm in 1960 while the value for 2011 was 81.00cm. There was a steady decrease in the total number of species and families within the period of assessment in the study area. Ninety (90) species were encountered in 1960, 87 species in 1962, 82 species in 1976, 78 species in 1982, 72 species in 2006 and in 2011, it was 68 species. In case of families, there was reduction from total number of 38 in 1982 to 32 in 2011.

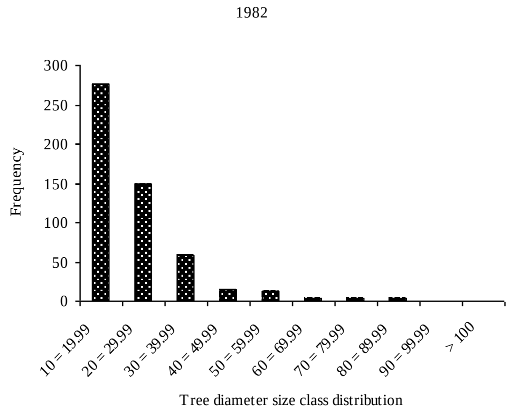
Diameter distribution

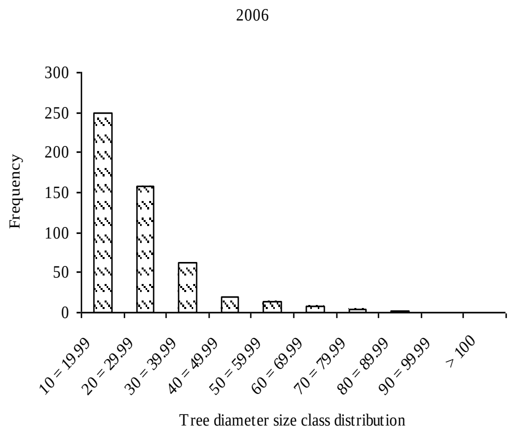
Figure 2 shows the diameter size class distribution in the plot between 1960 and 2011. Higher proportions of the trees were in the smaller diameter classes in all the years. In other words, there was a decline in the number of stems with increasing size classes.











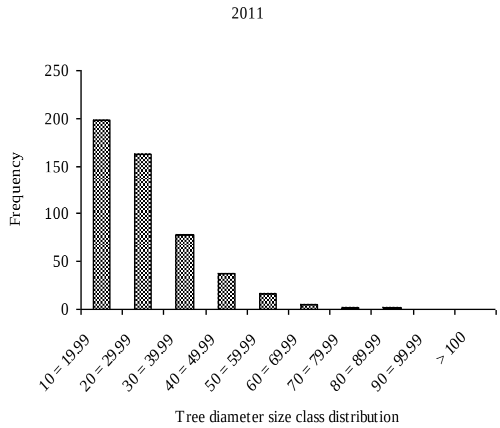


Table 1: Assessed Data on PSP 133 in Shasha Forest Reserve, Nigeria

Years of Assessment	No of Families	No of Species	No of Stems/ha	Mean dbh (cm)	Max. dbh (cm)	BA (m^2/ha)
1960	36	90	831	21.20	164.91	48.69
1962	34	87	551	21.20	145.51	28.55
1976	35	82	533	21.40	83.90	25.36
1982	38	78	521	22.43	88.80	26.68
2006	33	72	514	23.30	89.50	27.88
2011	32	68	499	25.35	81.00	30.87

Table 2: Plot summary statistics

Plot	A-DBH (cm)	dbh Std.Dev	Min. dbh	Max dbh	Q-DBH (cm)	No of stem/ha	BA (m^2/ha)
1	34.82	41.15	10.00	164.90	53.52	451	101.44
2	26.75	14.49	10.10	58.20	30.35	462	33.41
3	20.02	14.63	10.00	74.00	24.72	627	30.10
4	23.34	18.85	10.10	87.50	29.91	715	50.25
5	14.99	5.69	10.10	38.00	16.01	396	7.97
6	14.50	4.38	10.00	28.00	15.14	726	13.07
7	18.87	10.14	10.00	63.10	21.39	814	29.26
8	15.73	10.95	10.00	98.60	19.12	847	24.32
9	25.08	20.07	10.00	104.50	31.97	451	36.20
10	15.84	5.45	10.10	37.80	16.73	627	13.79
11	26.16	21.73	10.10	126.10	33.90	737	66.52
12	24.41	16.24	10.10	106.70	29.23	539	36.16
13	18.07	17.16	10.00	106.70	24.74	363	17.44
14	19.22	11.03	10.10	71.90	22.09	418	16.01
15	23.00	17.08	10.50	97.00	28.53	484	30.94

A-DBH = Arithmetic mean dbh, Q-DBH = Quadratic mean dbh and BA = Basal area

Table 3: Values of the model Parameters

PLOT	BETA PARAMETERS			
	p	q	a	b
1	0.19	0.37	10.00	164.90
2	0.52	0.98	10.10	58.20
3	0.26	1.29	10.00	81.40
4	0.26	1.15	10.10	96.25
5	0.43	2.04	10.10	45.60
6	0.54	1.62	10.00	33.60
7	0.47	2.35	10.00	75.72
8	0.19	2.77	10.00	118.32
9	0.32	1.66	10.00	125.40
10	0.67	2.57	10.10	45.36
11	0.33	2.07	10.10	151.32
12	0.51	2.95	10.10	128.04
13	0.13	1.31	10.00	117.39
14	0.44	2.52	10.10	86.28
15	0.31	1.86	10.50	116.40

p = shape 1; q = shape 2; a = lower bound; b = upper bound

Table 4: Statistical summary of the fitted Beta models

S/N	Regression equation	R^2 (%)	P-level	Std.Error	Mean residual	Std.dev. of residual	Sum of square of residual	Coeff of variation of residual
	(a+b) = 0.80lnDq – 200.79	61.31	0.0003*	24.7810	-7.63×10^{-07}	23.8797	7983.3529	-31299583.92
	(a+b) = 0.70lnD – 227.74	48.74	0.0038*	29.6020	-2.54×10^{-07}	28.5257	11391.9972	-112167509.00
	B = 0.70lnD – 237.61	48.69	0.0038*	29.6100	-1.27×10^{-06}	28.5327	11397.6382	-22439055.85
	b = 0.80lnDq – 210.78	64.06	0.0003*	24.7830	7.63×10^{-07}	23.8817	7984.7263	31302276.24
	(p+q) = 6.78 – 0.59lnDq	34.37	0.0216*	0.6806	3.58×10^{-08}	0.6558	6.0219	18338765.83
	(p+q) = 2.83 – 0.55BA	30.21	0.0338*	0.7018	7.95×10^{-09}	0.6763	6.4035	85099148.00
	q = 5.70 – 0.55lnDq	24.76	0.0041*	0.6343	-4.80×10^{-08}	0.6112	5.2300	-12817815.26
	q = 2.37 – 0.52BA	27.11	0.0466*	0.6479	-2.78×10^{-08}	0.6243	5.4568	-22444876.28

Dq = Quadratic mean dbh, D = Arithmetic mean dbh, lnD = Natural log of D, lnDq = Natural log of Dq, BA = Basal area/ha and
* = Significant at 5% level of probability

Table 5: Validation of model predicting the location parameter

Plot	Variable	Fitted equations	Observed	Predicted
1	lnDq	*(a+b) = 0.80lnDq – 200.79*	174.9	177.99
2			68.30	124.48
3			91.40	104.95
4			106.35	123.12
5			55.70	63.51
6			43.60	58.20
7			85.72	91.16
8			128.32	80.46
9			135.40	129.45
10			55.46	67.74
11			161.42	135.04
12			138.14	120.91
13			127.39	105.00
14			96.38	94.20
15			126.9	118.60

t-stat = 3.07×10^{-6}; df = 14 p = 0.5000 (not significant at $p > 0.05$)

Table 6: Validation of model predicting the scale parameter

Plot	Variable	Fitted equations	Observed	Predicted
1	lnDq	B = 0.80lnDq – 210.78	164.9	170.11
2			58.2	65.40
3			81.4	100.87
4			96.25	101.23
5			45.6	51.24
6			33.6	58.10
7			75.72	89.44
8			118.32	71.39
9			125.4	119.36
10			45.36	50.27
11			151.32	125.15
12			128.04	121.29
13			117.39	98.21
14			86.28	84.13
15			116.4	118.36

t-stat = 0.2782; df = 14 p = 0.3925 (not significant at $p > 0.05$)

Table 7: Validation of model predicting the shape 1 parameter

Plot	Variable	Fitted equations	Observed	Predicted
1	lnDq	*(p+q) = 6.78 – 0.59lnDq*	0.56	0.83
2			1.50	1.34
3			1.55	2.13
4			1.41	1.89
5			2.47	2.56
6			2.16	2.92
7			2.82	2.44
8			2.96	2.99
9			1.98	1.36
10			3.24	2.52
11			2.4	1.90
12			3.46	2.09
13			1.44	1.73
14			2.96	3.00
15			2.17	2.11

t-stat = 0.5844; df = 14; p = 0.2841 (not significant at $p > 0.05$)

Table 8: Validation of model predicting the shape 2 parameter

Plot	Variable	Fitted equations	Observed	Predicted
1	*lnDq*	*q = 5.70 –0.55lnDq*	3.98	3.27
2			3.41	3.36
3			3.21	3.98
4			3.40	2.56
5			2.77	2.36
6			2.72	2.84
7			3.06	3.15
8			2.95	2.53
9			3.46	2.84
10			2.82	2.27
11			3.52	2.69
12			3.38	2.45
13			3.21	3.71
14			3.09	3.89
15			3.35	3.56

t-stat = 1.2658;df = 14; p = 0.113114 (not significant at $p > 0.05$)

Fitting of the models

It was observed that plot 8 had the highest number of stems/ha (847) while the least was observed on plot 13 with 363 stems/ha (Table 2). Meanwhile, plot 1 had the highest arithmetic mean and quadratic mean dbh of 34.82cm and 53.52cm respectively while the least values were observed on plot 6 with respective values of arithmetic and quadratic mean of 14.50cm and 15.14cm. The stand basal area ranges from 7.97m^2/ha on plot 5 to 101.44 m^2/ha on plot 1

The fitted models

Having fitted the beta function to the diameter distribution data, parameter values obtained are shown in table 3. The values for shape 1 (p) ranges between 0.13 to 0.67 while that of shape 2 (q) varies from 0.37 to 2.95. The values for lower bound (a) varies from 10.00 to 10.50 while the values for upper bound (b) ranges between 33.60 to 164.900. The results of the fitted models revealed that all the models are significant at 5% level of probability (Table 4). All the independent variables have positive relationship with the dependent variables which are the estimated parameters. With the least values of mean residuals, standard deviation of residuals, sum of squares of residuals, coefficient of variation of residuals; significance and high coefficient of determination, the best models for each parameters were selected as stated below:

$(a+b) = 0.80 \ln Dq - 200.79$ ………………………….. 9

$b = 0.80 \ln Dq - 210.78$ ……………………………….. 10

$(p+q) = 6.78 - 0.59 \ln Dq$ ……………………..………… 11

$q = 5.70 - 0.55 \ln Dq$ ………………..………………... 12

Model Validation

To demonstrate the consistency, accuracy and efficiency of the regression models developed, validation tests were carried out using 1976 data set as predicted value. The validations of the models for predicting the Beta parameters are presented in Tables 5 – 8. The results revealed that there were no significant differences between the observed and the predicted value of all the parameters models fitted.

Discussions

An observation of downward trend in the number of stems per hectare in the plot between 1960 and 2011 in this study was similar to the observation of [34][5]. [34] observation of tree population in a permanent sample plot in Gambari forest reserve, Nigeria, revealed that there was consistent decline in the number of trees per hectare for a period of 22 years. This decline in the number of stems per hectares may have been through natural mortality and indiscriminate exploitation by illegal fellers. Weak forest management practices in the study area were identified as the major reason for some of the indiscriminate exploitation of tree species. This also confirms the [35] report on the rate of deforestation in Nigeria that between 1990 and 2010, Nigeria lost an average of 409,650ha or 2.38% per year. This also confirms decline in species composition observed in the study area. The reductions in the number of stems per hectare as the dbh size class increased reflect the characteristics of a natural forest. This confirms the report of [36] that trees in an uneven-aged forest grow continuously and have different reproductive periods. This continuous reproduction of new trees has been noted to bring about variation in ages especially in an undisturbed stand. Furthermore, diameter distribution in an uneven-aged stand is irregular. [37] stressed that as the area of the stand increases, the irregularities tend to even out and the inverse J-shaped diameter distribution becomes apparent. The maximum dbh was observed the highest in 1960 (164.91cm) as against that of 2011 which recorded the least (81cm). This was believed to be as a result of fewer disturbances on the plot in 1960.

Diameter distribution of the stand can further be described using the values of all the parameters estimated for the Beta distribution. In this study, quadratic mean dbh and natural log of quadratic mean dbh performed good in development of the models. All the parameters used in testing the fitness and the functional relationship between the estimated growth variables and the Beta parameters were satisfactory. The shape 2 parameter model was observed the best for Beta distribution prediction. The coefficient of determination (R^2) for shape 2 equation was high with low standard error; the equation was significant at 5% level of significance; least values of mean residuals, standard deviation of residuals, sum of squares of residuals, coefficient of variation of residuals and high coefficient of determination. All these criteria formed the basis for selecting the models for the Beta functions ($q = 5.70 - 0.55 \ln Dq$).

To further test the validity, accuracy and consistency of the models, validation tests carried out revealed that there were no significant differences between the observed and the predicted values of all the Beta parameter models fitted.

Conclusion

The changes in stand structure can be assessed by present measurements and the past knowledge of a stand. These are of great importance in detecting the changes that have taken place over a period of time and possible causes of those changes. This is the attempt made in this study. Illegal logging / deforestation was identified as the major threat to species diversity in the study area. The models developed for predicting the Beta parameters from quadratic mean dbh were consistent and good prediction models since there were no significant differences in the observed and the expected values of the parameters. Therefore, the stand structure in the tropical rainforest condition can be projected given the relevant stand growth variables.

Recommendation

Models developed in this study are recommended for application in the projection of diameter distribution in Shasha forest reserve using Beta distribution.

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