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#### ABSTRACT

Dempster-Shafer theory offers an alternative to traditional probabilistic theory for the mathematical representation of uncertainty. The significant innovation of this framework is that it allows for the allocation of a probability mass to sets or intervals. An important aspect of this theory is the combination of evidence obtained from multiple sources and the modeling of conflict between them. Plash Dutta and Tazid Ali [6,7] proposed methods to combine Fuzzy Focal elements and their corresponding Basic Probability Assignments of two variables. Using evidence theory here we make an investigation for interval focal elements by combining focal elements and their corresponding Basic Probability Assignments (BPA) under Modified Arithmetic operations rather than ordinary arithmetic operations with the help of a numerical example.

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#### Introduction

Probability theory is proposed only for randomness uncertainty and it is inappropriate to represent epistemic uncertainty. To overcome the constraint of probabilistic method, Dempster put forward a theory in 1976 and now it is known as evidence theory (or) Dempster-Shafer Theory. The Dempster-Shafer theory of evidence, one of the most popular uncertainty theories used in many areas, such as expert systems, pattern classification, information fusion [3], which was first developed by Dempster[1] and later extended and refined by shafer[8], does not need prior information and can quantify uncertainty induced by "unknown" (or) "incorrect" information. One of the advantages of evidence theory is that focal elements and their corresponding basic probability assignments of variables can be combined. In [7], authors have also considered all the arithmetic operations between fuzzy focal elements by taking the operation for the corresponding basic probability assignment (bpa) of the resulting focal elements based on the operation between the focal elements. In this paper, we extend the methods proposed by authors in [7] to combine interval focal elements and their basic probability assignments of two variables by using modified interval arithmetical operations and acquired better results.

The paper is organized as follows: The Dempster-Shafer Theory (DST) is given in Section 2 and Section 3 deals with basic definitions of triangular fuzzy numbers, the fuzzy arithmetic operations on interval numbers. The concept of Algebraic Combination of Focal Elements is provided in Section 4. Finally, in Section 5, the effectiveness of the proposed method is illustrated by means of an example and some concluding remarks are given in section 6.

### **Dempster-Shafer Theory (DST)**

Dempster-Shafer Theory (DST) is a mathematical theory of evidence. In a finite discrete space, Dempster-Shafer theory can be interpreted as a generalization of probability theory where probabilities assigned to sets as opposed to mutually exclusive singletons. In traditional probability theory, evidence is associated with only one possible event. In Dempster-Shafer Theory, evidence can be associated with multiple possible events Eg: sets of events.

A frame of discernment (or simply a frame) usually denoted as  $\Theta$  is a set of mutually exclusive and exhaustive propositional hypotheses one and only one of which is true [8]. Evidence theory is based on two dual non additive measures, namely Belief measure



and Plausibility measure. There is one important function is Dempster-Shafer theory to define Belief measure and plausible measure which is known as Basic Probability Assignments.

A function  $m: 2^{\Theta} \rightarrow [0,1]$  is called Basic Probability Assignments on the set  $\Theta$  if it is satisfies the following conditions (*i*)  $m(\phi) = 0$ (*ii*)  $\sum_{A \subseteq \Theta} m(A) = 1$ 

where  $\phi$  is an empty set and A is any subset of  $\Theta$ .

The Basic Probability Assignment function (or mass function) is a primitive function. Given a frame,  $\Theta$ , for each source of evidence, a mass function assigns a mass to every subset of  $\Theta$ , which represents the degree of belief that one of the hypotheses in the subset is true, given the source of evidence. A subset A of a frame  $\Theta$  is called the focal elements of m, if m(A) > 0. The lower bound, Belief for a set A is defined as the sum of all the basic probability assignments of the proper subsets  ${}^{(B)}$  of the set of interest  ${}^{(A)}(B \subseteq A)$ . The upper bound, Plausibility is the sum of all the basic probability assignments of set (B) that intersect the set of interest (A)  $(B \cap A \neq \phi)$ . Formally for all sets A that are elements of the power set  $(A \in P(X))$ , [Klir 1998],  $Bel(A) = \sum_{B/B \subseteq A} m(B)$  and  $Pl(A) = \sum_{B/B \cap A \neq \phi} m(B)$ 

The two measures, Belief and Plausibility are non additive. This can be interpreted as is not required for the sum of all the Belief measures to be one and similarly for the sum of all the Plausibility measures. Hence the, interval [Bel(A), Pl(A)] is the range of belief A.

#### The Dempster Rule of Combination: [4]

The Dempster rule of combination is critical to the original conception of the Dempster- Shafer theory. The measure of Belief and Plausibility are derived from the combined basic assignments. Dempster's rule combines multiple belief functions through their basic probability assignments (m). These belief functions are defined on the same frame of discernment, but are based on independent assignments or bodies of evidence. The Dempster rule of combination is purely a conjunctive operation (AND). The combination rule results in a belief function based on conjunctive pooled evidence [Shafer 1986, pg132].

The combination (is called the joint  $m_{12}$ ) calculated from the aggregation of two Basic Probability assignments  $m_1$  and  $m_2$  in the following manner:

$$m(C) = \frac{\sum_{A \cap B=C} m_1(A)m_2(B)}{1 - \sum_{A \cap B=\phi} m_1(A)m_2(B)} \quad \text{where } C \neq \phi \ , m_{12}(\phi) = 0$$

#### Preliminaries: [5]

In this section we discuss the basic concepts which are sequential.

#### **Triangular Fuzzy Number:**

A Triangular Fuzzy Number  $\tilde{A}$  is denoted as  $\tilde{A} = (a_1, a_2, a_3)$  and is defined by the membership function as

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{x - a_1}{a_2 - a_1} & \text{if } a_1 \le x \le a_2 \\ \frac{a_3 - x}{a_3 - a_2} & \text{if } a_2 \le x \le a_3 \\ 0 & \text{if otherwise} \end{cases}$$

## **Interval Numbers:**

If  $\tilde{A}$  is a triangular fuzzy number, we will let  $\tilde{A}_{\alpha} = [A_{\alpha}^{-}, A_{\alpha}^{+}]_{\text{be the closed interval which is a } \alpha_{-\text{cut for }} \tilde{A}_{\text{where }}$  $A_{\alpha}^{-}$  &  $A_{\alpha}^{+}$  are its left and right end points respectively.

Let A and B be two interval numbers defined by ordered pairs of real numbers with lower and upper bounds.

$$A = [a_1, a_2], \text{ where } a_1 \le a_2$$
$$B = [b_1, b_2], \text{ where } b_1 \le b_2$$

When  $a_1 = a_2$  and  $b_1 = b_2$ , these interval numbers degenerate to a scalar real number.

## Arithmetic Operations on Interval numbers: [6]

For the intervals  $A = [a_1, a_2]$  and  $B = [b_1, b_2]$  the following operations can be established. Addition:

$$A + B = [a_1, a_2] + [b_1, b_2] = [a_1 + b_1, a_2 + b_2]$$

Subtraction:

$$A - B = [a_1, a_2] - [b_1, b_2] = [a_1 - b_2, a_2 - b_1]$$

### **Multiplication:**

$$A \cdot B = [a_1, a_2][b_1, b_2] = \{\min(a_1b_1, a_1b_2, a_2b_1, a_2b_2), \max(a_1b_1, a_1b_2, a_2b_1, a_2b_2)\}$$

Where min(.) and max(.) produce the smallest and the largest number in the brackets correspondingly. **Inverse:** 

$$A^{-1} = [a_1, a_2]^{-1} = \left[\frac{1}{a_2}, \frac{1}{a_1}\right], \ 0 \notin [a_1, a_2]$$

half-width of interval number A. ie,

### **Division:**

$$A/B = [a_1, a_2]/[b_1, b_2] = \{\min(a_1/b_1, a_1/b_2, a_2/b_1, a_2/b_2), \max(a_1/b_1, a_1/b_2, a_2/b_1, a_2/b_2)\}$$

# Modified Arithmetic Operations on Interval numbers: [5]

Any interval number  $A = [a_1, a_2]_{is alternatively represented as} A = [m(A), w(A)]_{where} m(A)_{and} w(A)_{are the mid-point and}$ 

$$m(A) = \frac{a_1 + a_2}{2}$$
 & w(A) =  $\frac{a_2 - a_1}{2}$ 

Suppose  $A = [a_1, a_2]$  and  $B = [b_1, b_2]$  are two interval numbers .According to K.Ganesan and P.Veeramani [8], the modified arithmetic operations on interval numbers are given below.

## Addition:

$$A + B = [a_1, a_2] + [b_1, b_2]$$
  
= {(m(A) + m(B)) - k, (m(A) + m(B)) + k}  
where k = { $\frac{(b_2 + a_2) - (b_1 + a_1)}{2}$ } and a<sub>1</sub>, a<sub>2</sub>, b<sub>1</sub> & b<sub>2</sub> are any real numbers.

Subtraction:

$$A - B = [a_1, a_2] - [b_1, b_2]$$
  
= {(m(A) - m(B)) - k, (m(A) - m(B)) + k}  
where k = { $\frac{(b_2 + a_2) - (b_1 + a_1)}{2}$ } and a<sub>1</sub>, a<sub>2</sub>, b<sub>1</sub> & b<sub>2</sub> are any real numbers.

## **Scalar Multiplication:**

Let  $\lambda \in \mathbf{R}_{, \text{ then}} \quad \lambda A = \begin{cases} [\lambda a_1, \lambda a_2], \text{ for } \lambda \ge 0\\ [\lambda a_2, \lambda a_1], \text{ for } \lambda < 0 \end{cases}$ 

## **Multiplication:**

$$AB = [a_1, a_2][b_1, b_2]$$
  
= {(m(A)m(B))-k, (m(A)m(B))+k}  
where k = min {(m(A)m(B)) - \alpha, \beta - (m(A)m(B))}  
 $\alpha = min(a_1b_1, a_1b_2, a_2b_1, a_2b_2)$   
&  $\beta = max(a_1b_1, a_1b_2, a_2b_1, a_2b_2)$ 

Inverse:

$$A^{-1} = [a_1, a_2]^{-1} = \left[\frac{1}{a_2}, \frac{1}{a_1}\right]$$
$$= \left[\frac{1}{m(A)} - k, \frac{1}{m(A)} + k\right]$$
where  $k = \min\left\{\frac{1}{a_2}\left(\frac{a_2 - a_1}{a_1 + a_2}\right), \frac{1}{a_1}\left(\frac{a_2 - a_1}{a_1 + a_2}\right)\right\}$  and  $0 \notin [a_1, a_2]$ 

**Division:** 

$$A / B = [a_1, a_2] / [b_1, b_2] = [a_1, a_2] \left[ \frac{1}{b_2}, \frac{1}{b_1} \right]$$
  
= {(m(A)m(B)) - k, (m(A)m(B)) + k}   
where k = min {(m(A)m(B)) - \alpha, \beta - (m(A)m(B))}   
& & \beta = max(a\_1 / b\_1, a\_1 / b\_2, a\_2 / b\_1, a\_2 / b\_2)

Algebraic Combination of Focal Elements: [7]

Let  $X_1$  and  $X_2$  be two variables whose values are represented by Dempster- Shafer structure with focal elements  $A_1, A_2, A_3, \dots, A_n$  and  $B_1, B_2, B_3, \dots, B_m$  which are considered as intervals and their corresponding Basic Probability Assignments (BPA) are as follows:

$$m(A_i) = a_i$$
 and  $m(B_i) = b_j$ ,  $i = 1, 2, 3, ..., k$   $j = 1, 2, 3, ..., m$  respectively.  
where  $\sum_{i=1}^{n} a_i = 1$  and  $\sum_{j=1}^{m} b_j = 1$ 

Initially we combine all the fuzzy focal elements using fuzzy arithmetic which will produce 'nm' number of fuzzy focal elements and there after the corresponding basic probability assignments of resulting fuzzy focal elements will be calculated as follows **Addition of Fuzzy Focal Elements:** 

$$m(C_{ij}) = m(A_i + B_j) = \frac{m(A_i) + m(B_j)}{\sum_i \sum_j (m(A_i) + m(B_j))}.$$
(4.1)

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Subtraction of Fuzzy Focal Elements:

$$m(C_{ij}) = m(A_i - B_j) = \frac{m(A_i)(1 - m(B_j))}{\sum_i \sum_j (m(A_i)(1 - m(B_j)))}$$
(4.2)

**Multiplication of Fuzzy Focal Elements:** 

$$\mathbf{m}(\mathbf{C}_{ij}) = \mathbf{m}(\mathbf{A}_i \mathbf{B}_j) = \frac{\mathbf{m}(\mathbf{A}_i)\mathbf{m}(\mathbf{B}_j)}{\sum_i \sum_j (\mathbf{m}(\mathbf{A}_i)\mathbf{m}(\mathbf{B}_j))}$$
(4.3)

**Division of Fuzzy Focal Elements:** 

$$\mathbf{m}(\mathbf{C}_{ij}) = \mathbf{m}(\mathbf{A}_i / \mathbf{B}_j) = \frac{\mathbf{m}(\mathbf{A}_i) / \mathbf{m}(\mathbf{B}_j)}{\sum_i \sum_j (\mathbf{m}(\mathbf{A}_i) / \mathbf{m}(\mathbf{B}_j))}$$
(4.4)

Finally, we arrange all the focal elements in increasing order of the left end point.

### Numerical Example [6]:

Suppose Basic Probability Assignments (BPA) of two parameters is assigned by an expert and which are given in the following tables:

## Table 1 Bpa of the first parameter

Focal	BPA
Elements	
[5,7]	0.15
[7,15]	0.20
[16,20]	0.35
[18,22]	0.30

## **Addition of Focal Elements:**

Focal	Focal	BPA
Elements	Elements	
(ordinary)	(Modified)	
[30,37]	[30,37]	0.022
[32,43]	[32,43]	0.027
[34,41]	[34,41]	0.03
[35,44]	[35,44]	0.06
[36,47]	[36,47]	0.035
[37,50]	[37,50]	0.07
[39,51]	[39,51]	0.044
[41,48]	[41,48]	0.044
[41,57]	[41,57]	0.05
[43,50]	[43,50]	0.039

#### Table 2 Bpa of the second parameter

Focal	BPA
Elements	
[25,28]	005
[29,32]	0.12
[30,35]	0.43
[34,42]	0.25
[42,47]	0.15

Focal	Focal	BPA
Elements	Elements	
(ordinary)	(Modified)	
[45,51]	[45,51]	0.052
[46,54]	[46,54]	0.087
[47,54]	[47,54]	0.046
[47,56]	[47,56]	0.033
[48,57]	[48,57]	0.081
[49,62]	[49,62]	0.038
[50,62]	[50,62]	0.066
[52,64]	[52,64]	0.061
[58,67]	[58,67]	0.055
[60,69]	[60,69]	0.05

## Table 3. Basic Probability Assignments of resulting focal elements using algebraic addition

Number of focal elements of the first parameter is four and the second parameter is five respectively. After adding the focal elements using modified interval arithmetic we get twenty numbers of focal elements. Now the corresponding basic probability

assignments of resulting focal elements are calculated using (4.1) and arranging all the focal elements in increasing order of the left end point are given in the following table 3.

#### **Subtraction of Focal Elements:**

Subtracting all the focal elements using modified interval arithmetic we get twenty members of focal elements. Now, the corresponding Basic Probability Assignments of resulting focal elements are calculated (4.2) and arranging all the focal elements in increasing order of the left end point are given in the following table 4.

Focal	Focal	BPA
Elements	Elements	
(ordinary)	(Modified)	
[-42,-33]	[-42,-33]	0.032
[-40,-27]	[-40,-27]	0.042
[-37,-25]	[-37,-25]	0.028
[-35,-19]	[-35,-19]	0.038
[-31,-22]	[-31,-22]	0.074
[-30,-21]	[-30,-21]	0.021
[-29,-20]	[-29,-20]	0.064
[-28,-15]	[-28,-15]	0.028
[-27,-20]	[-27,-20]	0.033
[-26,-14]	[-26,-14]	0.066

Focal	Focal	BPA
Elements	Elements	
(Ordinary)	(Modified)	
[-25,-14]	[-25,-14]	0.044
[-24,-12]	[-24,-12]	0.056
[-23,-16]	[-23,-16]	0.0356
[-21,-10]	[-21,-10]	0.048
[-19,-10]	[-19,-10]	0.05
[-17,-8]	[-17,-8]	0.043
[-16,-9]	[-16,-9]	0.077
[-14,-7]	[-14,-7]	0.066
[-12,-5]	[-12,-5]	0.083
[-10,-3]	[-10,-3]	0.071

### Table 4. Basic Probability Assignments of resulting focal elements using algebraic subtraction

### **Multiplication of Focal Elements:**

Multiplying all the focal elements using modified interval arithmetic we get twenty members of focal elements. Now, the corresponding Basic Probability Assignments of resulting focal elements are calculated (4.3) and arranging all the focal elements in increasing order of the left end point are given in the following table 5.

Focal	Focal	BPA
Elements	Elements	
(ordinary)	(modified)	
[125,252]	[125,245]	0.0075
[145,288]	[145,282]	0.018
[150,315]	[150,305]	0.0645
[170,378]	[170,362]	0.0375
[175,420]	[175,408]	0.01
[203,480]	[203,468]	0.024
[210,423]	[210,413]	0.0225
[210,525]	[210,505]	0.086
[238,630]	[238,598]	0.05
[294,705]	[294,685]	0.03

Focal	Focal	BPA
Elements	Elements	
(ordinary)	(Modified)	
[400,560]	[400,527.5]	0.0175
[450,616]	[450,610]	0.015
[464,640]	[464,603.5]	0.042
[480,700]	[480,657.5]	0.1505
[522,704]	[522,698]	0.036
[540,770]	[540,760]	0.0129
[544,840]	[544,786]	0.0875
[612,924]	[612,908]	0.075
[672,940]	[672,885.5]	0.0525
[756,1034]	[756,1024]	0.045

Table 5. Basic Probability Assignments of resulting focal elements using algebraic multiplication

#### **Division of Focal Elements:**

Dividing all the focal elements using modified interval arithmetic we get twenty members of focal elements. Now, the corresponding Basic Probability Assignments of resulting focal elements are calculated (4.4) and arranging all the focal elements in increasing order of the left end point are given in the following table 6.

Focal	Elements	Focal	Elements	BPA
(ordinary)		(modified)		
[0.10638	3,0.21429]	[0.10638	8,0.20922]	0.02420
[0.11904	,0.26471]	[0.11904	4,0.25350]	0.0145
[0.14285	5,0.3]	[0.14285	5,0.29049]	0.0089
[0.14895	,0.35715]	[0.14895	5,0.34699]	0.0322
[0.1562,0	0.31035]	[0.1562,	0.3040]	0.0302
[0.16667	,0.44118]	[0.1666]	7,0.41877]	0.01936
[0.17857	',0.36]	[0.1785]	7,0.35143]	0.0726
[0.2,0.5]		[0.2,0.48	8096]	0.01125
[0.21875	,0.51725]	[0.21875	5,0.50431]	0.04033
[0.25,0.6	5]	[0.25,0.5	58286]	0.09679

Focal	Elements	Focal	Elements	BPA
(ordinary)		(modified)		
[0.34042,	0.4762]	[0.34042	2,0.44858]	0.0568
[0.38095,	0.58824]	[0.38095	,0.55043]	0.0339
[0.38297,	0.52381]	[0.38297	,0.51875]	0.0484
[0.42857,	0.64706]	[0.42857	,0.63585]	0.0290
[0.45714,	06667]	[0.45714	,062620]	0.0197
[0.5,0.689	966]	[0.5,0.65	032]	0.0706
[0.51429,	0.7334]	[0.51429	,0.72381]	0.0169
[0.56250,	0.75863]	[0.56250	,0.75216]	0.0605
[0.57142,	0.8]	[0.57142	2,0.75358]	0.01694
[0.64286,	0.88]	[0.64286	,0.87142]	0.01452

Table 6. Basic Probability Assignments of resulting focal elements using algebraic Division

#### **Conclusion:**

Evidence theory can handle both aleatory and epistemic uncertainty. Three important functions in evidence theory, the basic probability assignment function (bpa), Belief function (Bel) and Plausibility function (Pl) are used to quantify the given variable. As already stated in this paper interval focal elements and their Basic Probability Assignments of two variables are combined by Modified Interval Arithmetical operations. From the tables (3&4) in section 5, it is noticed that the results obtained when using ordinary arithmetic operations on intervals are the same as that of the Modified arithmetic operations. However, from tables (5&6), it is observed that Modified arithmetic operations have the advantage of simple calculations and high accuracy in multiplication and division. Those results are promising and interesting as it being addressed for the first time.

### **References:**

[1] A.P.Dempster, "Upper and Lower Probabilities induced by a Multi-Valued mapping", Annals of Mathematical Statistics 38(1967) 325-339.

[2] Ganesan.K and Veeramani.P, "On Arithmetic Operations of Interval Numbers", Uncertainty, Fuzziness and Knowledge based Systems, 13, 619-631, 2005.

[3] He you, wang Guohong, Guanxin, "Information Fusion Theory with Applications" Belling: Publishing house of Electronics Industry, Jan'2010[in Chinese].

[4] Kari Sentz & Scott Ferson, "Combination of Evidence in Dempster-Shafer Theory", Sand 2002-0835 Unlimited Release Printed April 2002. 13, 14 &16.

[5] A.Nagoorgani and R.Irene Hepzibah, "Modified Interval Arithmetic in Fuzzy Quadratic Programming Problem", Proceedings of International conference on Mathematics Methods and Computation, Jamal Mohammed College, Tiruchirappalli, Tamilnadu, India, Allied Publishers Pvt.Ltd. Chennai, pp 472- 477,ISBN 978-81-8424-466-3, 2009.

[6] Palash Dutta & Tazid Ali "Arithmetic Operations of Focal Elements and their corresponding Basic Probability Assignments in Evidence Theory", International Journal of Mathematical Archive 3(3) 1136-1140, 2012.

[7] Palash Dutta & Tazid Ali "Arithmetic Operations of Fuzzy Focal Elements in Evidence Theory", International Journal of Latest Trends in Computing, 2 (4), pp: 528-534, 2011.

[8] Shafer.G, "A Mathematical theory of Evidence", Princeton University: Princeton, 1976.