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Magnetohydrodynamic oscillatory Stokes flow past a porous sphere

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ABSTRACT

In this paper, the magnetohydrodynamic oscillatory Stokes flow past a porous sphere has been discussed. A uniform magnetic field is applied transversely to the flow field. Flow outside the porous region is governed by unsteady Stokes equation and Darcy's law is used in the porous region. Drag and torque are calculated using Faxen's law. It is observed that the increase or decrease of the drag and torque depends on the effect of magnetic field with variable permeability. The effect of magnetic field on various flow quantities like drag and torque has been observed for uniform oscillatory flow, doublet in uniform flow and oscillating Stokeslet. Graphs are plotted for various measures and results are obtained using these graphs.

1. Introduction

Oscillatory flow in porous media has gained importance lately because of its wide range of applications in the fields of industry, geophysical applications such as enhanced oil recovery, reservoir, land erosion, chemical engineering etc. With such a vast array of potential applications, it is fairly evident that a lot of people have put in their minds and thoughts, trying to invent more innovations in this prevalent field. Initially the work was done by Hasimoto [8] on a sphere theorem on the Stokes equation for axisymmetric viscous flow. Similar work was followed by Collins [3]. Beavers and Joseph [1] derived the boundary conditions at the permeable wall. The boundary condition for the porous medium was developed by Saffman [25]. Many researchers [13, 24, 26] modified the boundary conditions of the porous material. Higdon et al. [9] and Qin et al. [20] discussed the Stokes flow past a porous material. A singularity method for unsteady lineralized flow was derived by Pozrikidis [17,18]. Flow through porous particles and porous sphere was examined by many authors [5, 15, 16, 21]. They used the Stokes equation, Brinkman equation and Darcy's law for the free flow region and the porous region. Raja Sekhar and Amaranath [22] discussed the Stokes flow past a porous sphere with an impermeable core. Again Raja Sekhar, Padmavathi and Amaranath [23] gave a theoretical explanation to complete general solution of Brinkman equation.

Dragon et al. [6] studied the mass transport in a flexible tube with oscillatory flow in the year 1991. Oscillatory Stokes flow in porous media was discussed by Chapman et al. [2]. Many studies were gone through the field of oscillatory flow through porous media in the year 2000. Later, Graham and Higdon [7] worked on the oscillatory forcing of flow through porous media. Looker and Carnie [12] discussed the hydrodynamics of an oscillating porous sphere. Enhancements in the chemical and process industry using oscillation was derived by Nr et al. [14]. Axial dispersion in packed beds of spheres was done by Crittenden et al. [4].The forces on a porous particle for an oscillating flow was studied by Vainshtein and Shapire [28]. Faxen derived the law for drag and torque in the year 1924. After that, Howells [10] had done the research on drag due to the motion of Newtonian fluid through the small fixed rigid bodies. Similarity between Faxen's relation and singularity solution like fluid-fluid, fluid-solid and solid-solid dispersions were derived by Kim and Lu [11]. Cell model calculation of drag parameters in spheres was discussed by Umnova et al. [27]. Recently Prakash, Raja Sekhar and Kohr [19] derived the Faxen's law for an arbitrary oscillatory Stokes flow past a porous sphere. They derived the drag and torque for the oscillatory Stokes flow and discussed the results with some examples like uniform oscillatory flow, oscillating Stokeslet, linear oscillatory shear flow. Hydrodynamics of a porous sphere with Newtonian fluid was used to derive the Faxen law.

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So far no research has been done on the effect of magnetic field for the oscillatory flow using Padmavathi's solution in the porous region. In this paper, Magnetohydrodynamic oscillatory viscous incompressible Stokes flow past a porous sphere is considered. The flow inside the porous sphere is governed by Darcy's law and the flow outside the porous sphere is governed by unsteady Stokes equation. Continuity of the pressure field and continuity of the normal velocity components are used as the boundary condition. Saffman's boundary condition is used for the tangential components of the velocity field. Faxen's law for the magnetohydrodynamic oscillatory unsteady Stokes flow is derived. Drag and torque is obtained for the oscillatory uniform flow, doublet and the oscillatory Stokeslet. Results are discussed using the graphs.

2. Mathematical Formulation:

Consider an arbitrary oscillatory magnetohydrodynamic flow of a viscous incompressible fluid past a porous sphere of radius 'a'. A uniform magnetic field is applied transversely to the flow field with magnetic induction B_0. Let us assume that the flow inside the porous sphere $(r < a)$ is governed by the Darcy's law

$$\nabla p_1 = -\left(\frac{\mu}{k}\boldsymbol{q}_1 + \sigma B_0^2 \boldsymbol{q}_1\right) \tag{1}$$

$$\nabla . \boldsymbol{q}_1 = 0 \tag{2}$$

Where p_1 is the pressure inside the porous sphere, $\boldsymbol{q}_1$ is the velocity, μ is the coefficient of viscosity , k is the permeability of the porous medium, σ is the electrical conductivity and B_0 is the magnetic induction.

The flow outside the porous sphere $(r > a)$ is governed by the unsteady Stokes flow

$$\rho \frac{\partial q}{\partial t} = -\nabla p + \mu \nabla^2 \boldsymbol{q} - \sigma B_0^2 \boldsymbol{q} \tag{3}$$

$$\nabla . \boldsymbol{q} = 0 \tag{4}$$

where ρ is the density of the fluid, p and $\boldsymbol{q}$ represent the pressure and velocity of the fluid outside the sphere. Let us introduce the oscillatory flow with frequency ω then the velocity and pressure fields as $\boldsymbol{q} = \boldsymbol{q}_2 e^{-i\omega t}$ and $p = p_2 e^{-i\omega t}$

Thus the equation of the flow outside the porous sphere is transformed to

$$\nabla p_2 = \left(i\omega\rho \boldsymbol{q}_2 + \mu\nabla^2 \boldsymbol{q}_2 - \sigma B_0^2 \boldsymbol{q}_2\right) \tag{5}$$

$$\nabla . \boldsymbol{q}_2 = 0 \tag{6}$$

The physical quantities are non-dimensionalized by using the transformation

$$\tilde{r} = \frac{r}{a}, \quad p^x = \frac{p_{1,2}}{\mu \boldsymbol{q}_0 a / k}, \quad \boldsymbol{q}^y = \frac{\boldsymbol{q}_{1,2}}{\boldsymbol{q}_0}, \quad M^2 = \frac{\sigma B_0^2}{\mu} \tag{7}$$

where $x = i, e$ and $y = i, E$ · $\boldsymbol{q}_0$ is the velocity of the basic flow and M is the magnetic parameter. Hence the nondimensional form for the flow inside the porous region is

$$\nabla p^i = -\lambda_i \boldsymbol{q}^i \tag{8}$$

$$\nabla . \boldsymbol{q}^i = 0 \tag{9}$$

where $\lambda_i = \left(1 + kM^2\right)$

And the equation for the flow outside the porous region is

$$\nabla p^e = \left(\nabla^2 - \lambda_e^2\right)\boldsymbol{q}^e \tag{10}$$

$$\nabla.\boldsymbol{q}^e = 0 \tag{11}$$

where $\boldsymbol{q}^e = \frac{k}{a^2}\boldsymbol{q}^E$, $\lambda_e^2 = \left(M_1^2 - i\omega a^2/\gamma\right)$, $M_1^2 = \frac{\sigma B_0^2 a^2}{\mu}$, and $Da = \frac{k}{a^2}$ is the Darcy number.

3. Boundary conditions:

Let $\left(\boldsymbol{q}_r, \boldsymbol{q}_\theta, \boldsymbol{q}_\phi\right)$ be the velocity components in spherical coordinate system and the corresponding stress components be

$$T_{rr} = -p + 2\mu\frac{\partial \boldsymbol{q}_r}{\partial r} \tag{12}$$

$$T_{r\theta} = \mu\left[\frac{1}{r}\frac{\partial \boldsymbol{q}_r}{\partial \theta} - \frac{\boldsymbol{q}_\theta}{r} + \frac{\partial \boldsymbol{q}_\theta}{\partial r}\right] \tag{13}$$

$$T_{r\phi} = \mu\left[\frac{1}{r\sin\theta}\frac{\partial \boldsymbol{q}_r}{\partial \phi} - \frac{\boldsymbol{q}_\phi}{r} + \frac{\partial \boldsymbol{q}_\phi}{\partial r}\right] \tag{14}$$

Beavers and Joseph proposed a condition for porous liquid interface. Saffman suggested the boundary condition at the surface of a porous medium. Looker and Carnie applied Saffman’s condition to oscillatory flow, under low frequency. Prakash and Raja sekhar proposed the following condition for oscillatory Stokes flow past a porous sphere.

i) Continuity of the pressure field on $r = 1$

$$p^e = p^i \tag{15}$$

ii) Continuity of the normal velocity component

$$q_r^e = q_r^i Da \tag{16}$$

iii) Saffman’s boundary condition for the tangential components of the velocity field

$$q_\theta^e = \frac{\sqrt{Da}}{\alpha}\frac{\partial q_\theta^e}{\partial r} \tag{17}$$

$$q_\phi^e = \frac{\sqrt{Da}}{\alpha}\frac{\partial q_\phi^e}{\partial r} \tag{18}$$

α is the dimensionless slip coefficient.

4. Method of solution:

Padmavathi et al. proposed the solution for the porous region involving Brinkman’s equation. The general form for the velocity components are

$$q_r = -\frac{1}{r}LA \tag{19}$$

$$q_\theta = \frac{1}{r}\frac{\partial}{\partial \theta}\frac{\partial}{\partial r}(rA) + \csc\theta\frac{\partial B}{\partial \phi} \tag{20}$$

$$q_\phi = \frac{1}{r\sin\theta}\frac{\partial}{\partial \phi}\frac{\partial}{\partial r}(rA) - \frac{\partial B}{\partial \theta} \tag{21}$$

where $L = \frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right) + \csc^2\theta\frac{\partial^2}{\partial\phi^2}$ is the transverse part of the Laplacian.

Let us assume the velocity corresponding to the basic flow is

$$\boldsymbol{q}^0 = CurlCurl\left(rA^0\right) + Curl\left(rB^0\right) \tag{22}$$

And
$$A^0(r,\theta,\phi)=\sum_{n=1}^{\infty}\left(\alpha_n r^n+\beta_n f_n(\lambda_e r)\right)S_n(\theta,\phi) \tag{23}$$

$$B^0(r,\theta,\phi)=\sum_{n=1}^{\infty}\left(\gamma_n f_n(\lambda_e r)\right)T_n(\theta,\phi) \tag{24}$$

With
$$S_n(\theta,\phi)=\sum_{m=0}^{n}P_n^m(\xi)\left(A_{mn}\cos m\phi+B_{mn}\sin m\phi\right) \tag{25}$$

$$T_n(\theta,\phi)=\sum_{m=0}^{n}P_n^m(\xi)\left(C_{mn}\cos m\phi+D_{mn}\sin m\phi\right) \tag{26}$$

where $f_n(\lambda_e r)$ is the modified spherical Bessel function of the first kind and $P_n^m(\xi)$ is the associated Legendre polynomial.

Velocity components and the pressure for the magnetohydrodynamic oscillatory flow in the free flow region is

$$\boldsymbol{q}^e=CurlCurl\left(rA^e\right)+Curl\left(rB^e\right) \tag{27}$$

$$p^e=P_0+\frac{\partial}{\partial r}\left(r(\nabla^2-\lambda_e^2)A^e\right) \tag{28}$$

where
$$A^e=\sum_{n=1}^{\infty}\left(\alpha_n r^n+\beta_n f_n(\lambda_e r)+\alpha_n' r^{-(n+1)}+\beta_n' g_n(\lambda_e r)\right)S_n(\theta,\phi). \tag{29}$$

$$B^e=\sum_{n=1}^{\infty}\left(\gamma_n f_n(\lambda_e r)+\gamma_n' g_n(\lambda_e r)\right)T_n(\theta,\phi) \tag{30}$$

where $g_n(\lambda_e r)$ is the modified spherical Bessel function of the second kind. α_n', β_n' and γ_n' are unknown constants to be determined using the boundary conditions.

The velocity component for the modified flow in the region $r>1$ becomes

$$q_r^e=\sum_{n=1}^{\infty}n(n+1)\left(\alpha_n r^{n-1}+\frac{\beta_n}{r}f_n(\lambda_e r)+\alpha_n' r^{-(n+2)}+\frac{\beta_n'}{r}g_n(\lambda_e r)\right)S_n(\theta,\phi) \tag{31}$$

$$\begin{aligned}q_\theta^e=\sum_{n=1}^{\infty}\Bigg\{\Bigg[&(n+1)\alpha_n r^{n-1}+\beta_n\left(\lambda_e f_{n+1}(\lambda_e r)+\frac{n+1}{r}f_n(\lambda_e r)\right)-n\alpha_n' r^{-(n+2)}\\ &-\beta_n'\left(\lambda_e g_{n+1}(\lambda_e r)-\frac{n+1}{r}g_n(\lambda_e r)\right)\Bigg]\frac{\partial S_n}{\partial\theta}+\csc\theta\frac{\partial T_n}{\partial\phi}\left[\gamma_n f_n(\lambda_e r)+\gamma_n' g_n(\lambda_e r)\right]\Bigg\}\end{aligned} \tag{32}$$

$$\begin{aligned}q_\phi^e=\sum_{n=1}^{\infty}\Bigg\{\Bigg[&(n+1)\alpha_n r^{n-1}+\beta_n\left(\lambda_e f_{n+1}(\lambda_e r)+\frac{n+1}{r}f_n(\lambda_e r)\right)-n\alpha_n' r^{-(n+2)}\\ &-\beta_n'\left(\lambda_e g_{n+1}(\lambda_e r)-\frac{n+1}{r}g_n(\lambda_e r)\right)\Bigg]\csc\theta\frac{\partial S_n}{\partial\phi}-\left[\gamma_n f_n(\lambda_e r)+\gamma_n' g_n(\lambda_e r)\right]\frac{\partial T_n}{\partial\theta}\Bigg\}\end{aligned} \tag{33}$$

In the porous region the pressure field is harmonic and finite at the origin. The velocity and pressure components become

$$\boldsymbol{q}^i=-\delta_n r^{n-1}\left(n+\frac{\partial}{\partial\theta}+\csc\theta\frac{\partial}{\partial\phi}\right)S_n(r,\theta) \tag{34}$$

$$p^i=P_0+\sum_{n=1}^{\infty}\delta_n r^n S_n(r,\theta) \tag{35}$$

The unknown constants can be determined in terms of the known constants by using the boundary conditions given in equations (15)-(18).

The velocity components for the porous region $r<1$ is

$$\boldsymbol{q}_r^i = -\sum_{n=1}^{\infty} n\delta_n r^{n-1} S_n \tag{36}$$

Now using the above boundary conditions (15)-(18) on the permeable boundary $r=1$ can be written as

$$\lambda_e^2 (n+1)\alpha_n r^n - \lambda_e^2 n\alpha_n^{'} r^{-(n+1)} + \delta_n r^n = 0 \tag{37}$$

$$n(n+1)\left(\alpha_n r^{n-1} + \frac{\beta_n}{r} f_n(\lambda_e r) + \alpha_n^{'} r^{-(n+2)} + \frac{\beta_n^{'}}{r} g_n(\lambda_e r)\right)\lambda_i + l^2 nr^n \delta_n = 0 \tag{38}$$

$$\begin{aligned}&\left(\alpha(n+1)r^{n-1} - l(n^2-1)r^{n-2}\right)\alpha_n - \left(\alpha nr^{-(n+2)} + l(n+2)nr^{-(n+3)}\right)\alpha_n^{'} + \left((\alpha r^{-1}(n+1)\right.\\ &\quad \left. - l(\lambda_e^2 + (n^2-1)r^{-2}))f_n + (\alpha\lambda_e + l\lambda_e r^{-1}) f_{n+1}\right)\beta_n + \left((\alpha(n+1)r^{-1}\right.\\ &\left. - l(\lambda_e^2 + (n^2-1)r^{-2}))g_n - (\alpha\lambda_e + l\lambda_e r^{-1})g_{n+1}\right)\beta_n^{'} = 0\end{aligned} \tag{39}$$

$$\left((\alpha - nl) f_n - l\lambda_e f_{n+1}\right)\gamma_n + \left((\alpha - nl) g_n + l\lambda_e g_{n+1}\right)\gamma_n^{'} = 0 \tag{40}$$

The unknown coefficients $\alpha_n^{'}, \beta_n^{'}, \gamma_n^{'}$ and δ_n in the above equations be determined in terms of the known coefficients α_n, β_n and γ_n by using Mathematica 8.0

5. Faxen's law for the porous sphere:

Faxen derived the laws for drag and torque in the year 1927. That is for an unbounded arbitrary stokes flow, the forces drag D and torque T acting on the rigid sphere of radius $'a'$ are

$$D = 6\pi\mu a[\ ]_0 \tag{41}$$

$$T = 4\pi\mu a^3 [\nabla\times q_0]_0 \tag{42}$$

where $[\ ]_0$ is the evaluation at the centre of the sphere. The force D exerted on the porous sphere in the region $r>1$ and the torque T are given by

$$D = \int_0^{2\pi}\int_0^{\pi} \left[T_{rr}^e \hat{e}_r + T_{r\theta}^e \hat{e}_\theta + T_{r\phi}^e \hat{e}_\phi\right]_{r=1} a^2 \sin\theta d\theta d\phi \tag{43}$$

$$T = \int_0^{2\pi}\int_0^{\pi} \left[rT_{r\theta}^e \hat{e}_\phi - rT_{r\phi}^e \hat{e}_\theta\right]_{r=1} a^2 \sin\theta d\theta d\phi \tag{44}$$

where $T_{rr}^e, T_{r\theta}^e$ and $T_{r\phi}^e$ are the normal, tangential and azimuthal stress components acting on the surface of the sphere $r=1$ respectively. $\hat{e}_r, \hat{e}_\theta$ and $\hat{e}_\phi$ are the unit vectors corresponding to the spherical coordinates $(r,\theta,\phi)\cdot$ $T_{rr}^e, T_{r\theta}^e$ and $T_{r\phi}^e$ are computed using the expressions given in equations (12)-(14) and are used in equations (43) and (44).

We now derive the Faxen's law for the drag and torque acting on a porous sphere in an unbounded magnetohydrodynamics arbitrary oscillatory Stokes flow.

$$D = \frac{8}{3}\pi\left[A_{11}\hat{i} + B_{11}\hat{j} + A_{10}\hat{k}\right]E \tag{45}$$

$$T = \frac{8}{3}\pi\left[C_{11}\hat{i} + D_{11}\hat{j} + C_{10}\hat{k}\right]F \tag{46}$$

where
$$E = \lambda_e^2 \left[\frac{\alpha_1^{'}}{2} - \alpha_1 - \beta_1 f_1 - \beta_1^{'} g_1 \right].$$

$$F = \gamma_1 f_1 + \gamma_1^{'} g_2$$

where $A_{11}, B_{11}, A_{10}, C_{11}, D_{11}$ and C_{10} are known constants.

6. Examples

i) **Uniform oscillatory flow**

An uniform flow along Z direction past a porous sphere, then the corresponding

$$A_0 = \frac{1}{2} r \cos\theta \tag{47}$$

$$B_0 = 0 \tag{48}$$

Comparing this A_0 and B_0 with the basic flow equations

$$\alpha_1 = \frac{1}{2} \tag{49}$$

$$\beta_1 = 0 \tag{50}$$

Hence the drag and torque can be written as

$$D = \frac{4\pi}{3} \lambda_e^2 \left(\alpha_1^{'} - 2\beta_1^{'} g_1 - 1 \right) \hat{k} \tag{51}$$

$$T = 0 \tag{52}$$

The variation of drag with various measures has been plotted. The drag is found to decrease with increase in l values as seen in fig.1. It is also affected by the oscillating frequency, due to which the higher the frequency, the lower is the drag. Similarly, the drag decreases with increasing l value but drag increases for increasing slip coefficient, as observed in fig.2. Contrary to this behaviour, the drag rises with increment in magnetic parameter and this increment is very high with respect to l, as found in fig.3. With the introduction of magnetic field, the drag is again found to decrease with increase in l value. As found in fig.4, higher the permeability higher is the drag.

When drag is plotted against permeability, it increases for increasing permeability as shown in the figures. For increasing values of l, the drag is found to decrease in fig.5.The magnetic parameter seems to increase with drag and higher the magnetic parameter, higher is the drag, rightly shown in fig.6. It is also observed from fig.7 that for increasing frequency the drag decreases but for lower values of frequency, the drag initially increases before facing a dip in the value due to increase in permeability.

ii) **Doublet in a uniform flow**

A doublet of strength m is in a uniform flow, at $(0,0,c)$. The basic flow is given by (A_0, B_0) where

$$A_0 = \frac{m}{c^2} + \left(\frac{m}{c^3} - \frac{1}{2}\right) r\cos\theta + \sum_{n=2}^{\infty} \frac{r^n}{c^{n+1}} P_n(\xi) \tag{53}$$

$$B_0 = 0 \tag{54}$$

In this case, the corresponding coefficients are given by

$$\alpha_1 = \frac{m}{c^3} - \frac{1}{2} \text{ and } \alpha_n = \frac{1}{c^{n+1}} \text{ for } n \geq 2 \tag{55}$$

$$\beta_1 = 0 \text{ for } \quad n \geq 1 \tag{56}$$

The drag and torque can be written as

$$D = \frac{4\pi}{3}\lambda_e^2\left(\alpha_1^{'} - 2\beta_1^{'}g_1 + (2m - c^3)/c^3\right)\hat{i} \tag{57}$$

$$T = 0 \tag{58}$$

For doublet, the behaviour of drag with the variation of magnetic parameter, slip coefficient and permeability is almost similar to that for the uniform flow.The drag is found to increase with respect to the increase in slip coefficient, as seen in fig.8. It is also affected by the oscillating frequency, due to which the higher the frequency, the lower is the drag, as found in fig.9. Also, the higher the permeability the higher is the drag, besides which the increase in magnetic parameter results in increase in drag, as seen in fig.10.

iii) **Oscillating Stokeslet**

Let us consider the velocity q_0

$$q^0 = CurlCurl\left(rA^0\right) + Curl\left(rB^0\right)$$

$$q^0 = 2gradA^0 + r\frac{\partial}{\partial r}gradA^0 - r\nabla^2 A^0 + (r\times\nabla)B^0$$

$$\left[q^0\right]_0 = \left[2gradA^0\right]_0 = 2(\alpha_1 + \frac{\lambda_e}{3}\beta_1)(A_{11}\hat{i} + B_{11}\hat{j} + A_{10}\hat{k}) \tag{59}$$

$$\left[\nabla^2 q^0\right]_0 = 2\frac{\lambda_e^3}{3}\beta_1(A_{11}\hat{i} + B_{11}\hat{j} + A_{10}\hat{k}) \tag{60}$$

$$\left[\nabla\times q^0\right]_0 = 2\frac{\lambda_e}{3}\gamma_1(C_{11}\hat{i} + D_{11}\hat{j} + C_{10}\hat{k}) \tag{61}$$

$$D = \frac{4\pi}{3\alpha_1}\left[\lambda_e^2\left[q^0\right]_0 - \left[\nabla^2 q^0\right]_0\right]\left[-\beta_1 f_1 - \beta_1^{'}g_1 - \alpha_1 + \alpha_1^{'}/2\right] \tag{62}$$

$$T = \frac{4\pi}{\lambda_e}\left[\nabla\times q^0\right]_0\left[f_1 + \gamma_1^{'}g_2/\gamma_1\right] \tag{63}$$

Consider an oscillatory Stokeslet of strength m at $(0,0,c)$ whose axis is along the positive axis.The velocity of such an oscillatory Stokeslet in Cartesian form is given by

$$u = \frac{b_1}{8\pi\mu}\left[\frac{A_1(\lambda_e r)}{r} + \frac{x^2}{r^3}A_2(\lambda_e r)\right] \tag{64}$$

$$v = \frac{b_1}{8\pi\mu}\left[\frac{xy}{r^3}A_2(\lambda_e r)\right] \tag{65}$$

$$w = \frac{b_1}{8\pi\mu}\left[\frac{x(z-c)}{r^3}A_2(\lambda_e r)\right] \tag{66}$$

where

$$A_1(\lambda_e r) = 2e^{-\lambda_e r}\left[1 + \frac{1}{(\lambda_e r)} + \frac{1}{(\lambda_e r)^2}\right] - \frac{2}{(\lambda_e r)^2}$$

$$A_2(\lambda_e r) = -2e^{-\lambda_e r}\left[1 + \frac{3}{(\lambda_e r)} + \frac{3}{(\lambda_e r)^2}\right] + \frac{6}{(\lambda_e r)^2}$$

$$r=\sqrt{x^2+y^2+(z-c)^2}$$

$$\left[q^0\right]_0=\frac{b_1}{4\pi\mu}\left[\left(\frac{1}{c}+\frac{1}{\lambda_e c^2}+\frac{1}{\lambda_e^2 c^3}\right)e^{-\lambda_e c}-\frac{1}{\lambda_e^2 c^3}\right]\hat{i} \tag{67}$$

$$\left[\nabla^2 q^0\right]_0=\frac{b_1}{4\pi\mu}\left[\frac{\lambda_e^2}{c}+\frac{\lambda_e}{c^2}+\frac{1}{c^3}\right]e^{-\lambda_e c}\hat{i} \tag{68}$$

$$\left[\nabla\times q^0\right]_0=\frac{b_1}{4\pi\mu}\left[\frac{\lambda_e}{c}+\frac{1}{c^2}\right]e^{-\lambda_e c}\hat{j} \tag{69}$$

When the basic flow is an oscillatory Stokeslet the drag and torque becomes

$$D=\frac{4\pi}{3\alpha_1}\left\{\frac{\lambda_e^2 b_1}{4\pi\mu}\left[\left(\frac{1}{c}+\frac{1}{\lambda_e c^2}+\frac{1}{\lambda_e^2 c^3}\right)e^{-\lambda_e c}-\frac{1}{\lambda_e^2 c^3}\right]\hat{i}-\frac{b_1}{4\pi\mu}\right.$$
$$\left.\left[\frac{\lambda_e^2}{c}+\frac{\lambda_e}{c^2}+\frac{1}{c^3}\right]e^{-\lambda_e c}\hat{i}\right\}\left[-\beta_1 f_1-\beta_1' g_1-\alpha_1+\alpha_1'/2\right]$$

$$D=\frac{b_1}{3\alpha_1}\left[1-e^{-\lambda_e c}\left(1+\lambda_e c+\lambda_e^2 c^2\right)\left(1+\frac{1}{c^3}\right)\right]\left[-\beta_1 f_1-\beta_1' g_1-\alpha_1+\frac{\alpha_1'}{2}\right] \tag{70}$$

$$T=\frac{4\pi}{\lambda_e}\left[\frac{b_1}{4\pi}\left(\frac{\lambda_e}{c}+\frac{1}{c^2}\right)e^{-\lambda_e c}\hat{j}\right]\left[f_1+\gamma_1'\frac{g_2}{\gamma_1}\right]$$

$$T=\frac{b_1}{\lambda_e}\left[\left(\frac{1+\lambda_e c}{c^2}\right)e^{-\lambda_e c}\hat{j}\right]\left[f_1+\gamma_1'\frac{g_2}{\gamma_1}\right]. \tag{71}$$

In the case of oscillatory Stokeslet, the drag decreases with increasing frequency and increases with increasing l as shown in fig.11. Furthermore, the drag decreases gradually with increasing slip coefficient as seen in fig.12. It is also observed from fig.13 that for increasing permeability the drag decreases due to the effect of magnetic parameter. This magnetic effect makes the drag decrease gradually as found in fig.14 though there is a definite rise in drag observed for the lower values of magnetic parameter. For various parameters, drag decreases in the case of an oscillatory Stokeslet.

While observing torque in oscillatory Stokeslet, it is found that torque increases for increasing l. But against increasing frequency, the torque is found to decrease in fig.15. Also for increasing c value, the torque decreases but for various magnetic parameters, though the torque decreases steadily, it is constant for $c>3$ as seen in fig.16. This is better observed in fig.17 where it is clear that the torque decreases for increasing c and for various slip coefficient, due to the effect of magnetic parameters, but for greater values of c, the torque becomes constant. With the effect of oscillating frequency in fig.18, the torque decreases and the effect is the same as observed in the previous figure. Also, torque increases gradually for increasing b values but decreases for various c values.

7. Results and Conclusion:

The effect of a uniform magnetic field on an oscillatory Stokes flow past a porous sphere of radius 'a' is considered. Unsteady Stokes equation is used outside the porous sphere and Darcy's law is used for inside the porous sphere. At the porous liquid interface appropriate boundary conditions are applied. Padmavathi's solution is used to solve the governing equations. Drag and torque are derived by using Faxen's law. Examples like uniform oscillatory flow, doublet in a uniform flow and oscillating Stokeslet are discussed. The graphs are plotted for various parameters and the results are discussed by using the figures.

Graphs are plotted for uniform flow using various parameters. Generally drag decreases when its variation with l values. More over drag decreases for varying oscillating frequency. For varying slip coefficient permeability and magnetic parameter, drag increases. That is drag increases if the parameters are high except oscillating frequency. Variation of drag with permeability is also considered. It is a known thing that if the permeability increases then drag increases. Here also drag increases for all the parameter except the oscillating frequency.

In the case of doublet drag behaves like a uniform flow. All the parameter effects are same like uniform flow. Another example is oscillatory stokeslet, it plays an important role in porous flows. Variation of drag with several parameters are discussed. Figures 11 to 14 shows the effect of drag for various parameters. Mostly drag decreases for frequency, slip coefficient, magnetic parameter. Figures 15 to 19 represents the effect of torque for various parameters. Torque decreases in all the cases except frequency against l values. Even though torque decreases steadily, it is constant for the high values.

Figures:

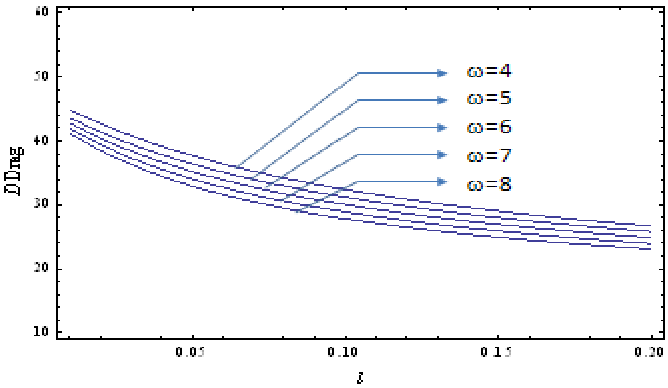

Fig.1. Variation of drag with l for different frequency in the case of uniform flow

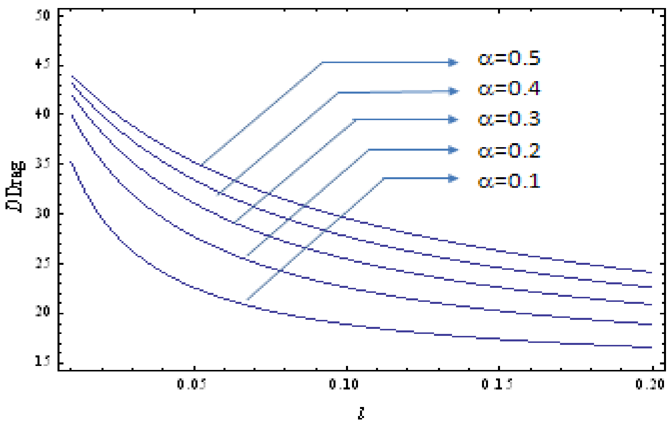

Fig.2. Variation of drag with l for different slip coefficient in the case of uniform flow

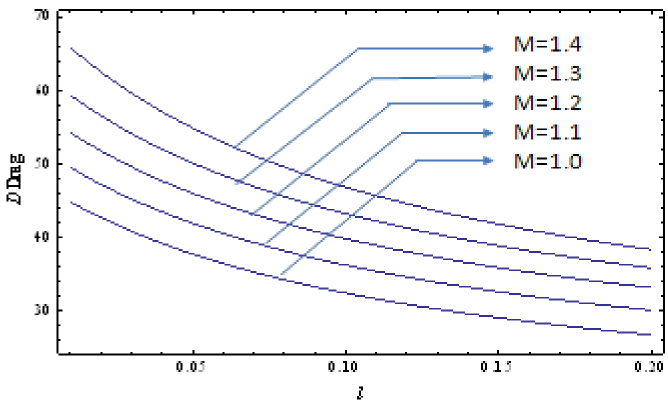

Fig.3. Variation of drag with l for different magnetic parameter in the case of uniform flow

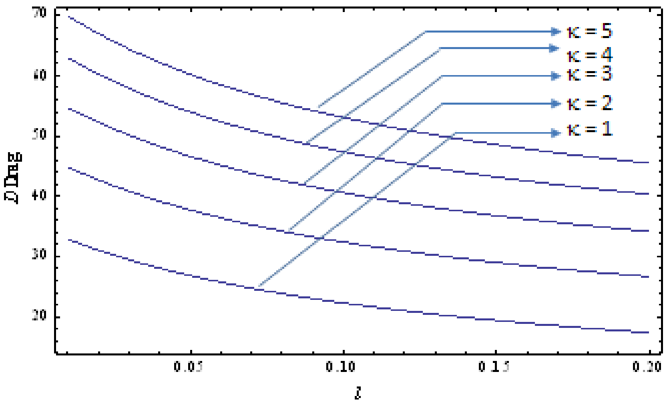

Fig.4. Variation of drag with l for different permeability in the case of uniform flow

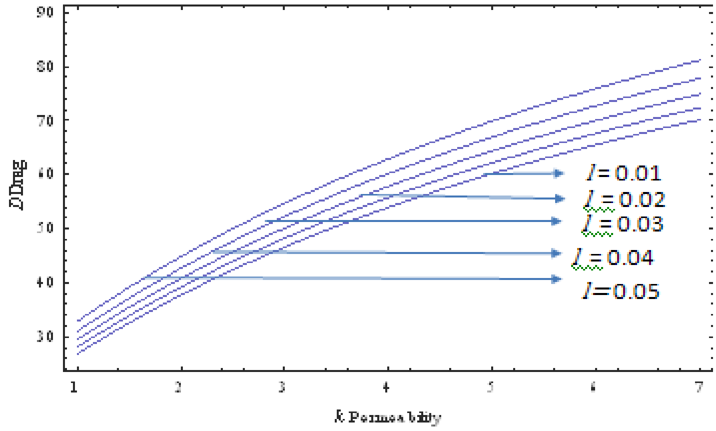

Fig.5. Variation of drag with permeability for different l in the case of uniform flow

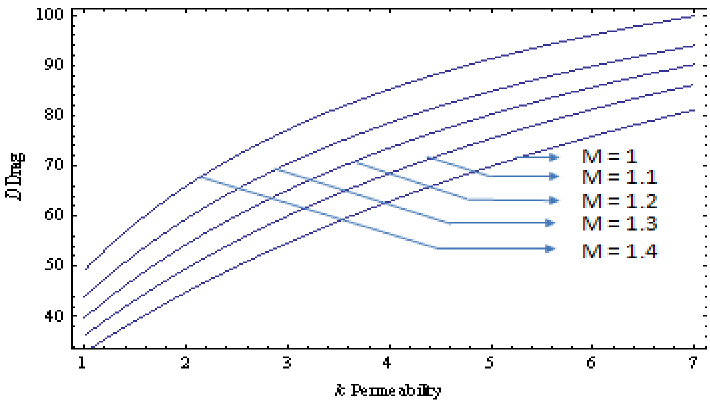

Fig.6. Variation of drag with permeability for different magnetic parameter in the case of uniformflow

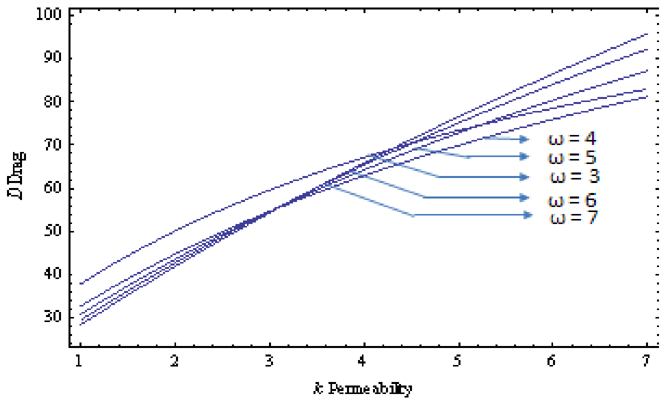

Fig.7. Variation of drag with permeability for different frequency in the case of uniform flow

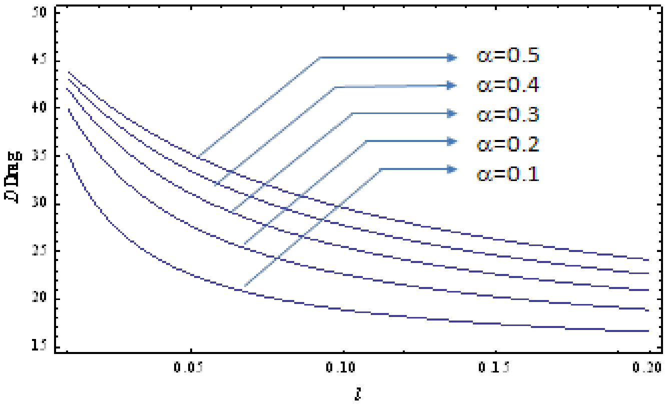

Fig.8. Variation of drag with l for different slip coefficient in the case of doublet in uniform flow

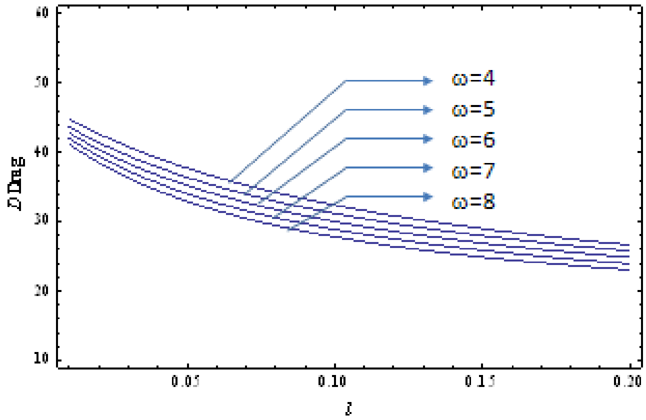

Fig.9. Variation of drag with l for different frequency in the case of doublet in uniform flow

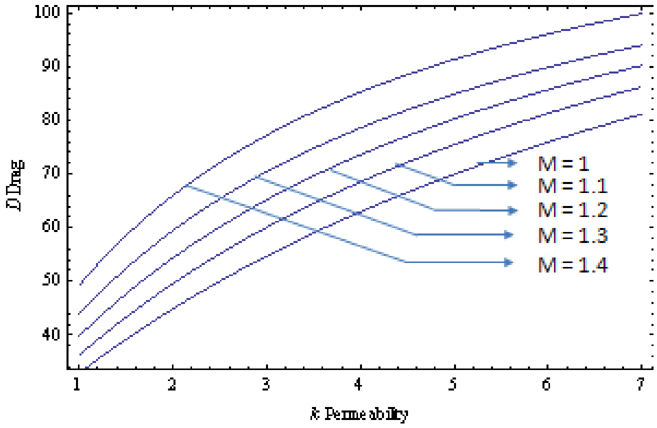

Fig.10. Variation of drag with permeability for different magnetic parameterin the case of doublet in uniform flow

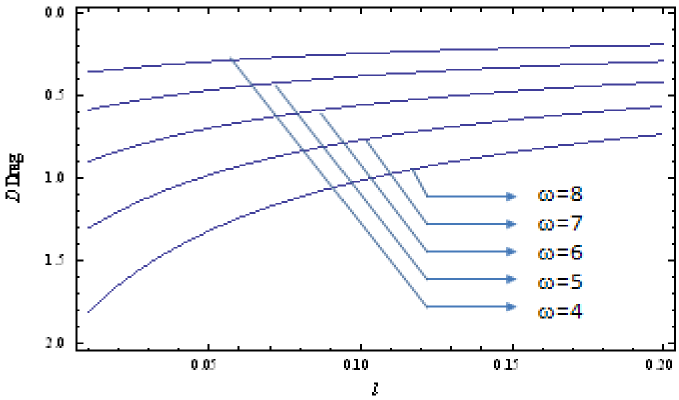

Fig.11. Variation of drag with l for different frequency in the case of oscillating stokeslet

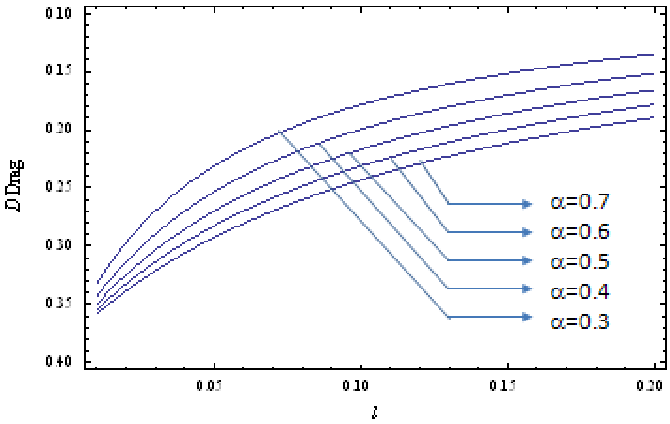

Fig.12. Variation of drag with l for different slip coefficient in the case of oscillating stokeslet

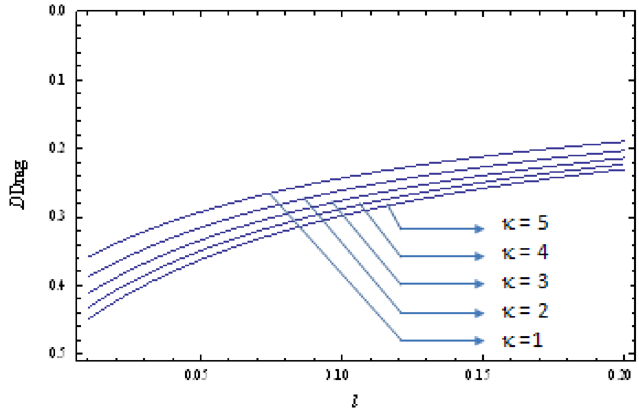

Fig.13. Variation of drag with l for different permeability in the case of oscillating stokeslet

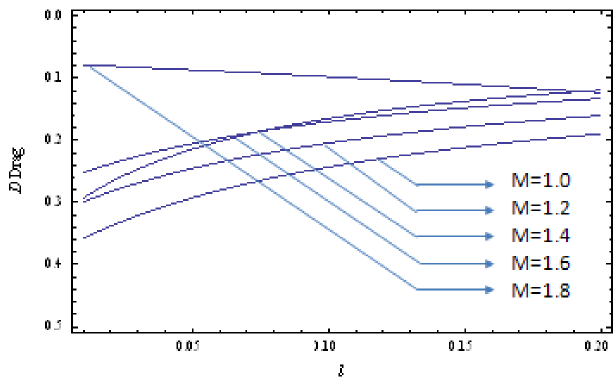

Fig.14. Variation of drag with l for different magnetic parameter in the case of oscillating stokeslet

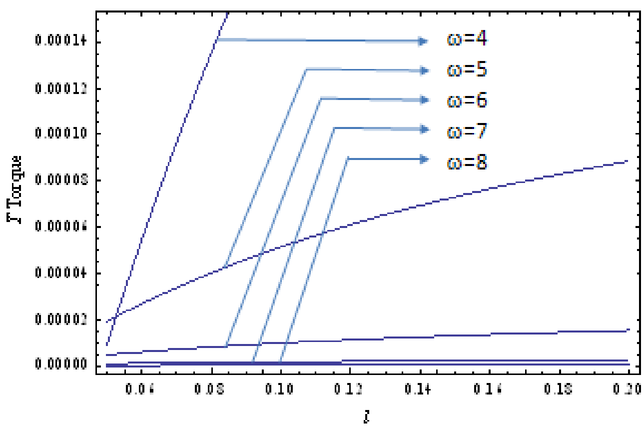

Fig.15. Variation of torque with l l for different frequency in the case of oscillating stokeslet

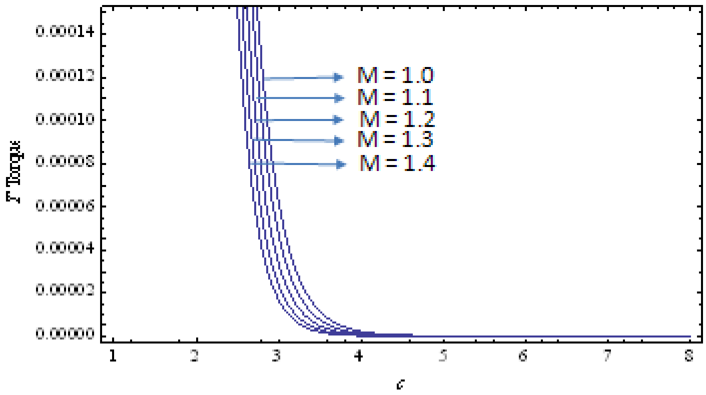

Fig.16. Variation of torque with c for different magnetic parameter in the case of oscillating stokeslet

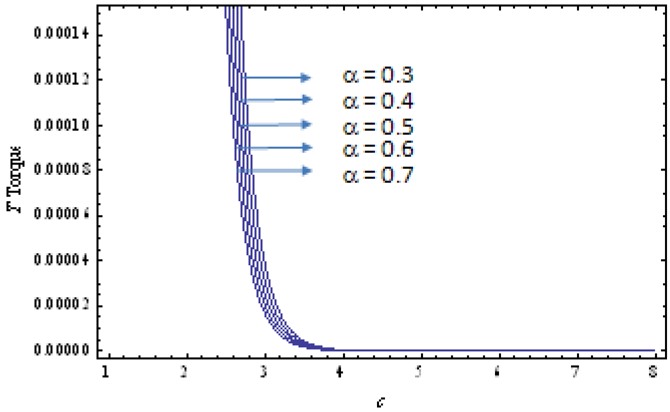

Fig.17. Variation of torque with c for different slip coefficient in the case of oscillating stokeslet

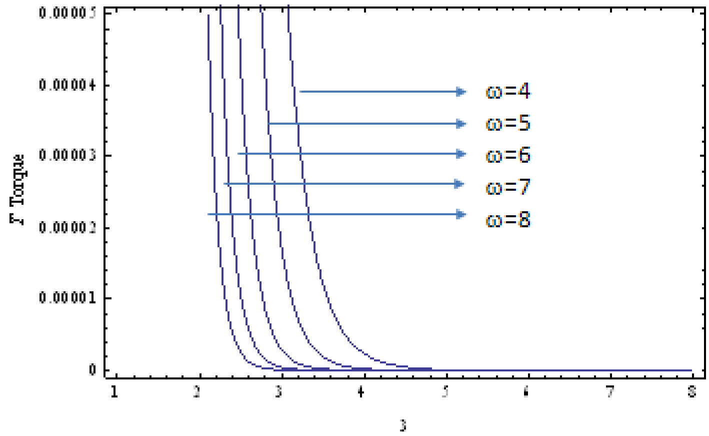

Fig.18. Variation of torque with c for different frequency in the case of oscillating stokeslet

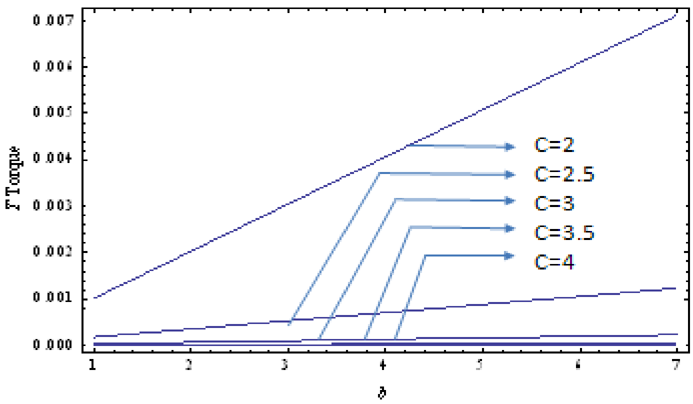

Fig.19. Variation of torque with b for different c values in the case of oscillating stokeslet

Appendix

$$\alpha_n' = \left[(1+n)\left(-g_{1+n}(\mathrm{l}\lambda_e+\alpha\lambda_e)(\alpha_n\lambda_e^2-(\alpha_n+f_n\beta_n)\lambda_i)+g_n\big(f_{1+n}\beta_n(\mathrm{l}\lambda_e+\alpha\lambda_e)\lambda_i+\alpha_n\lambda_e^2(l-ln^2+\alpha+n\alpha-l\lambda_e^2+l\lambda_i)\big)\right)\right]/[n(1+n)(l(2+n)+\alpha)g_n\lambda_i+\left(-g_{1+n}(\mathrm{l}\lambda_e+\alpha\lambda_e)+g_n\big((1+n)\alpha-l(-1+n^2+\lambda_e^2)\big)\right)(n\lambda_e^2+(1+n)\lambda_i)]$$

$$\beta_n' = 1/D_1\Big[-(1+n)(1+2n)\alpha_n(ln\lambda_e^2-(l+\alpha)\lambda_i) + \beta_n\Big(f_{1+n}(\mathrm{l}\lambda_e+\alpha\lambda_e)(n\lambda_e^2+(1+n)\lambda_i) + f_n\big(n\lambda_e^2(l-ln^2+\alpha+n\alpha-l\lambda_e^2)+(1+n)\big((1+2n)(l+\alpha)-l\lambda_e^2\big)\lambda_i\big)\Big)\Big]$$

$$\delta_n = \Big[(1+n)\lambda_e^2\Big(-g_{1+n}\big((1+2n)\alpha_n+nf_n\beta_n\big)(\mathrm{l}\lambda_e+\alpha\lambda_e) + g_n\Big(-nf_{1+n}\beta_n(\mathrm{l}\lambda_e+\alpha\lambda_e)+(1+2n)\alpha_n\big((1+n)(l+\alpha)-l\lambda_e^2\big)\Big)\Big)\lambda_i\Big]/D_1$$

$$\gamma_n' = -\frac{((-n\mathrm{l}+\alpha)f_n-f_{1+n}\mathrm{l}\lambda_e)\gamma_n}{(-n\mathrm{l}+\alpha)g_n+g_{1+n}\mathrm{l}\lambda_e}$$

$$D_1 = g_{1+n}(\mathrm{l}\lambda_e+\alpha\lambda_e)(n\lambda_e^2+(1+n)\lambda_i)+g_n(ln\lambda_e^4-(1+n)(1+2n)(l+\alpha)\lambda_i+(1+n)\lambda_e^2(n(l(-1+n)-\alpha) + l\lambda_i))$$

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