Available online at www.elixirpublishers.com (Elixir International Journal)

Applied Mathematics

Elixir Appl. Math. 74 (2014) 26901-26905

ISSN: 2229-712X

Periodic solid burst error correcting codes

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ARTICLE INFO

Article history:

Received: 12 July 2014;

Received in revised form: 21 August 2014;

Accepted: 3 September 2014;

Keywords

Periodic errors,
Blockwise error correcting codes,
Burst error, Solid burst error,
Parity check matrix, Syndromes.

ABSTRACT

In this paper, we study a lower bound on the blockwise periodic error correcting perfect codes along with lower and upper bounds on periodic solid burst error correcting codes.

Introduction

In most memory and storage systems, the information is stored in various sub-blocks of the code length. So, when error occurs in such a system, it does in a few places of the same sub-block and the pattern of errors is known. Thus, when we consider error correction, we correct errors which occur in the same sub-block.One of the interesting problems in coding theory is the study of perfect code with respect to the Hamming metric. It is well known that perfect codes emerge from the well known Rao-Hamming Bound

$$q^{n-k} = \sum_{i=0}^{t} \binom{n}{i} (q-1)^i$$

where the parameters have the usual meanings e.g. the code is an (n,k) linear code over GF(q) and t denotes the number of errors which the code is capable to correct. Hamming introduced an infinite class of single error correcting perfect codes over GF(2). In addition to these codes, the only known perfect codes are Golay's (23,12) and (11,6) linear codes over binary and ternary fields respectively. Exploring more perfect codes has been a topic of vigorous research activity. However, such a study has led to a number of results about non-existence of different classes of perfect codes.

In coding theory, the errors are generally classified mainly in two categories - random and burst errors and a lot of work has been done on codes correcting / detecting such error. But,in some channels,it has also been observed that the occurrence of errors may follow a pattern, different from these two. The pattern is such that error repeats after certain fixed interval.

In communication channel like Astrophotography [1], Gyrosope [2], and Computed Tomography [3] where small mechanical error occurs periodically in the accuracy of the tracking in a motorized mount that results small movements of the target that can spoil long-exposure images, even if the mount is perfectly polar-aligned and appears to be tracking perfectly in short tests. It repeats at a regular interval - the interval being the amount of time it takes the mount's drive gear to complete one revolution. This type of error is termed as *periodic* error. Codes detecting and correcting such errors are studied by Tyagi and Das ([4], [5]). Periodic error may be defined as follows:

Definition. A periodic error of order s is an n-tuple whose non zero components are located at a gap of s positions and the number of its starting positions is among the first $s+1$ components, where $s = 1,2,3,...,(n-1)$.

For s =1, the *periodic* error of order 1 are the vectors where error may occur in 1st, 3rd, 5th,... positions or 2nd, 4th, 6th,… positions. For example, in a vector of length 8, *periodic* error vectors of order 1 are of the type 10101000, 00101000, 0010101, 10101010, 10001010, 01010101, 01000101, 00000101, 00000001 etc.

For s =2, the *periodic* error vectors of order 2 are those where error may occur in 1st, 4th, 7th,... positions or 2nd, 5th, 8th,... positions or 3rd, 6th, 9th,... positions. The *periodic* error vectors of order 2 may look like 10010010, 10000010, 00010010, 01001001, 01000001, 01000000, 00001001, etc in a vector of length 8.

For s = 3, in a code length 8, the *periodic* error vectors of order 3 are 10001000, 01000100, 00100010, 00010001, 10000000, 01000000 etc.

The nature of errors differs from channel to channel depending upon the behavior of channels or the kind of errors which occur during the process of transmission. With the possibility of occurrence of solid burst error in channels like semiconductor memory data [6], supercomputer storage system [7], one of the important areas in coding theory is to study solid burst error. A solid burst may be defined as follows:

Definition. A solid burst of length s is a vector with non zero entries in some s consecutive positions and zero elsewhere.

Schillinger [8] developed codes that correct solid burst errors. Shiva and Cheng [9] produced a paper for multiple solid burst error correcting codes in binary case with a very simple decoding scheme. Sharma and Dass [10] also studied solid burst error correcting perfect codes. P.K. Das [11] obtained blockwise solid burst error correcting codes.

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As usual, we consider a linear code to be a subspace of n-tuples over GF(q). The block of n digits, consisting of r check digits and $k = n – r$ information digits, is considered to be divided into b mutually exclusive sub-blocks. Each sub-block contains $t=n/b$ digits.

Note that an (n, k) linear code capable of correcting an error requires the syndromes of any two vectors to be different irrespective of whether they belong to the same sub-block or different sub-blocks. So, in order to correct s-periodic errors lying within a sub-block the following conditions need to be satisfied:

(a) The syndrome resulting from the occurrence of an s-periodic error must be distinct from the syndrome resulting from any other s-periodic errors within the *same* sub-block.

(b) The syndrome resulting from the occurrence of s-periodic errors within a single sub block must be distinct from the syndrome resulting from any s-periodic error in *any other* sub block.

This paper is organized into two sections. In section 1, blockwise periodic error correcting perfect codes are studied whereas in section 2, periodic solid burst error correcting codes are obtained.

Section 1

Das and Tyagi [12] have studied codes that are capable of correcting periodic errors in different sub-blocks of a code length and obtained lower and upper bounds on the number of parity-check digits. The lower bound given by Das and Tyagi is as follows:

Lower Bound (Das and Tyagi)

Theorem 1. *The number of parity check digits r in an* (n, k) *linear code subdivided into b sub-blocks of length t each, that corrects s-periodic error lying within a single sub-block is at least*

$$\log_q\left\{1+b\sum_{i=0}^{s}\left(q^{k_i}-1\right)\right\} \text{ where } k_i=\left\lceil\frac{t-i}{s+1}\right\rceil \tag{1}$$

Proof. Let there be an (n, k) linear code vector over GF(q) that corrects all s-periodic errors within a single corrupted sub-block. The maximum number of distinct syndromes available using r check bits is q^r. The proof proceeds by first counting the number of syndromes that are required to be distinct by condition (a) and (b) and then setting this number less than or equal to q^r.

First we consider a sub-block, say i-th sub-block of length t. Since the code is capable of correcting all errors which are s-periodic errors within a single sub-block, any syndrome produced by an s-periodic error in a given sub-block must be different from any such syndrome likewise resulting from s-periodic errors in the same sub-block by condition (a).

Also by condition (b), syndromes produced by s-periodic errors in different sub-blocks must be distinct. Thus the syndromes produced by s-periodic errors, whether in the same sub-block or in different sub-blocks should be distinct.

Since there are b sub-blocks and number of s-periodic errors in a vector of length t (Tyagi and Das [5]) is

$$\sum_{i=0}^{s}\left(q^{k_i}-1\right) \text{ where } k_i=\left\lceil\frac{t-i}{s+1}\right\rceil. \tag{2}$$

So we must have at least $1+b\sum_{i=0}^{s}\left(q^{k_i}-1\right)$ distinct syndromes, including the all zero syndrome.

Therefore we must have

$$q^r \geq 1+b\sum_{i=0}^{s}\left(q^{k_i}-1\right) \tag{3}$$

or

$$r \geq \log\left\{1+b\sum_{i=0}^{s}\left(q^{k_i}-1\right)\right\}$$

where $k_i=\left\lceil\frac{t-i}{s+1}\right\rceil$.

Remark. For $b = 1$, the bound reduces to

$$r \geq \log\left\{1+b\sum_{i=0}^{s}\left(q^{k_i}-1\right)\right\} \text{ where } k_i=\left\lceil\frac{t-i}{s+1}\right\rceil$$

which coincides with the necessary condition for the existence of a code correcting all s-periodic errors (refer Theorem 1, Tyagi and Das [5]).

Discussion. For the possibility of the existence of perfect codes, we must consider inequality in (1) as equality, viz.

$$q^r = 1+b\sum_{i=0}^{s}\left(q^{k_i}-1\right) \text{ where } k_i=\left\lceil\frac{t-i}{s+1}\right\rceil \tag{4}$$

For $q = 2$,

$$2^{n-k} = 1+b\sum_{i=0}^{s}\left(2^{k_i}-1\right) \text{ where } k_i=\left\lceil\frac{t-i}{s+1}\right\rceil \tag{5}$$

The two set of values of the parameters s,t,b,n,k_0,k_1,k_2 that satisfies equation (5)

	s	t	b	n	k_0	k_1	k_2
1.	2	4	3	12	2	1	1
2.	2	8	15	120	3	3	2

Case 1. For first set $s = 2$, $t = 4$, $b = 3$, we have from equation (5)

$$2^{n-k} = 1+3\left[\left(2^{k_0}-1\right)+\left(2^{k_1}-1\right)+\left(2^{k_2}-1\right)\right] \tag{6}$$

Taking as $n=12, k_0=2, k_1=1, k_2=1$, the equality (6) gives $k=8$. This may give rise to (12, 8) blockwise 2-periodic error correcting perfect code.

Consider the matrix

$$H=\begin{bmatrix}000001111110\\011010000111\\001101100011\\111000101100\end{bmatrix}_{4\times 12}$$

If we consider the matrix H as the parity check matrix for (12,8) blockwise periodic error correcting perfect code, then it can be verified from the error pattern – syndrome table that the code is a blockwise periodic error correcting perfect code.

ERROR PATTERN	SYNDROMES
1000 0000 0000	0001
0001 0000 0000	0010
1001 0000 0000	0011
0000 1000 0000	0100
0000 0001 0000	1000
0000 1001 0000	1100
0000 0000 1000	1001
0000 0000 0001	0110
0000 0000 1001	1111
0100 0000 0000	0101
0010 0000 0000	0111

0000 0100 0000	1010
0000 0010 0000	1011
0000 0000 0100	1101
0000 0000 0010	1110

Case 2. For second set s = 2, t = 8, b = 15, we have from equation (5)

$$2^{n-k}=1+15\left[\left(2^{k_0}-1\right)+\left(2^{k_1}-1\right)+\left(2^{k_2}-1\right)\right] \qquad (7)$$

Taking as $n=120, k_0=3, k_1=3, k_2=2$, the equality (6) gives $k=112$. This may also give rise to (120, 112) blockwise 2-periodic error correcting perfect code.

Section 2

The following theorem gives a lower bound on the necessary number of parity check digits required for a code that corrects solid burst periodic error of length b lying within a single block of length n. The proof is based on the technique used in Theorem 4.13, Peterson and Weldon [13].

Lower Bound

Theorem 1. *An* (n, k) *linear code over* GF(q) *is lying within a single block of length n. The number of parity check digits required for the code that corrects solid burst periodic error of length b lying within a single block of length n is at least*

$$\log_q\left[1+\sum_{i=0}^{b+s-1}\left(q^{k_i}-1\right)\right], \qquad (8)$$

where, $k_i=\left\lceil\dfrac{n-b+1-i}{b+s}\right\rceil, i=0,1,2\ldots,(b+s-1)$

Proof. let there be an (n,k) linear code vector over $\mathrm{GF}(q)$ that corrects all solid burst of length b lying within a single block of length n. The maximum number of distinct syndromes obtained by $(n-k)$ check bits is $q^{(n-k)}$.

Firstly, counting the number of syndromes that are required to be distinct by condition (a) and (b) and then inequality of this number less than or equal to $q^{(n-k)}$. For correction, all these solid burst periodic error vectors must belong to different cosets. The numbers of positions in which solid burst periodic error may occur are $k_0,k_1,k_2,\ldots,k_{b+s-1}$ corresponding to $b+s$ sequences. Therefore the total number of error patterns, including the pattern of all zero's, is

$$=1+\left[\binom{k_0}{1}(q-1)+\binom{k_0}{2}(q-1)^2+\ldots+\binom{k_0}{k_0}(q-1)^{k_0}\right]$$
$$+\left[\binom{k_1}{1}(q-1)+\binom{k_1}{2}(q-1)^2+\ldots+\binom{k_1}{k_1}(q-1)^{k_1}\right]$$
$$+\ldots\left[\binom{k_{b+s-1}}{1}(q-1)+\binom{k_{b+s-1}}{2}(q-1)^2+\ldots+\binom{k_{b+s-1}}{k_{b+s-1}}(q-1)^{k_{b+s-1}}\right]$$
$$=1+\left(q^{k_0}-1\right)+\left(q^{k_1}-1\right)+\ldots+\left(q^{k_{b+s-1}}-1\right)$$
$$=1+\sum_{i=0}^{b+s-1}\left(q^{k_i}-1\right)$$

To correct all such errors, we must have

$$q^{(n-k)}\geq 1+\sum_{i=0}^{b+s-1}\left(q^{k_i}-1\right)$$

Corollary 1. For q = 2, the number of parity check digits in a linear code that corrects all solid burst periodic error is at least

$$2^{(n-k)}\geq 1+\sum_{i=0}^{b+s-1}\left(2^{k_i}-1\right) \qquad (9)$$

where $k_i=\left\lceil\dfrac{n-b+1-i}{b+s}\right\rceil, i=0,1,2,\ldots,(b+s-1).$

Upper bound

Now, we derive an upper bound on the number of check digits required for the existence of such a code. The proof is based on the technique used to establish Varshamov-Gilbert-Sacks bound by constructing a parity check matrix for such a code (refer Sacks [14], also Theorem 4.7, Peterson and Weldon [13]). This technique not only ensures the existence of such a code but also gives a method for construction of the code.

Theorem 2. *An* (n, k) *linear code over* GF(q) *capable of correcting solid burst periodic error , occurring within a single block of length n can always be constructed using* $(n-k)$ *check digits where* $q^{(n-k)}$ *is the smallest integer satisfying the inequality*

$$q^{n-k}\geq q^{p+1}\sum_{i=1}^{b+s-1}q^{r_i}-(b+s-1)q^p \qquad (10)$$

where $p=\left\lceil\dfrac{n}{b+s}-1\right\rceil, r_i=\left\lceil\dfrac{n-b+1-i}{b+s}\right\rceil-1$, $i=1,2,3,4\ldots(b+s-1)$

Proof. The existence of such a code will be proved by constructing an $(n-k)\times n$ parity check matrix H. Select any non zero $(n-k)$ tuple as the first column h_1 of the matrix H. After having selected the first $(n-1)$ columns $h_1,h_2,h_3,\ldots,h_{n-1}$. The condition to add the n th column h_n as follows:

$$h_n\neq\sum_{i=1}^{p}a_{i1}h_{n-i(b+s)}+\sum_{i=0}^{r_1}b_{i1}h_{n-1-i(b+s)}$$
$$\neq\sum_{i=1}^{p}a_{i2}h_{n-i(b+s)}+\sum_{i=0}^{r_2}b_{i2}h_{n-2-i(b+s)}$$
$$\vdots$$
$$\neq\sum_{i=1}^{p}a_{i(b+s-1)}h_{n-i(b+s)}+\sum_{i=0}^{(b+s-1)}b_{i(b+s-1)}h_{n-(b+s-1)-i(b+s)}$$

where

$p=\left\lceil\dfrac{n}{b+s}-1\right\rceil, r_i=\left\lceil\dfrac{n-b+1-i}{b+s}\right\rceil-1$, $i=1,2,3,4\ldots(b+s-1)$

; $a_{ij}b_{ij}\in\mathrm{GF}(q)\,\forall\, i, j$.

This condition ensures that there shall not be a code vector which can be expressed as the linear combinations of two solid burst periodic errors.

Now we will evaluate total number of linear combinations of terms in, which is

$$=\left(q^{p+r_1+1}-1\right)+\left(q^{p+r_2+1}-1\right)+$$
$$\left(q^{p+r_3+1}-1\right)+\ldots+\left(q^{p+r_{b+s-1}+1}-1\right)+1-(b+s-1-1)\left(q^p-1\right)$$
$$=q^{p+r_1+1}+q^{p+r_2+1}+q^{p+r_3+1}+\ldots+$$
$$q^{p+r_{b+s-11}+1}-(b+s-1)+1-(b+s-2)\left(q^p-1\right)$$
$$=q^{p+1}\left(q^{r_1}+q^{r_2}+q^{r_3}+\ldots q^{r_{b+s-1}}\right)-(b+s-1)q^p$$

$$= q^{p+1}\sum_{i=1}^{b+s-1} q^{r_i} - (b+s-1)q^p$$

We know that total number of $(n-k)$ tuples available is $q^{(n-k)}$.
Thus the column h_n can added to H provided

$$q^{n-k} \geq q^{p+1}\sum_{i=1}^{b+s-1} q^{r_i} - (b+s-1)q^p .$$

Corollary 2. For q = 2, there shall always exist an (n,k) linear code that corrects all solid burst periodic error, provided that

$$2^{n-k} \geq 2^{p+1}\sum_{i=1}^{b+s-1} 2^{r_i} - (b+s-1)2^p \qquad (11)$$

where

$$p = \left\lceil \frac{n}{b+s} - 1 \right\rceil,\ r_i = \left\lceil \frac{n-b+1-i}{b+s} \right\rceil - 1 ,\ i = 1,2,3,4\ldots(b+s-1).$$

Discussion

To determine the values of n and k to explore the possibilities of the existence of solid burst periodic error correcting code over GF(2). These have been put in the form of Table.

Values of *n* and *k* for *q* = 2

s **(period)**	***b*** **(burst length)**	***n*** **(code length)**	***k***	**Codes**
2	2	10	5	(10,5)
	3	13	8	(13,8)
	4	16	11	(16,11)
3	2	12	7	(12,7)
	3	15	10	(15,10)
	4	18	13	(18,13)
4	2	14	9	(14,9)
	3	15	10	(15,10)
	4	17	12	(17,12)

Illustrations

1. Consider a (10,5) binary code with the 5×10 matrix H which has been constructed by the procedure given in the theorem 2 by taking s = 2, b = 2, n = 10, k = 5, q = 2

$$\mathbf{H} = \begin{bmatrix} 1000000111 \\ 0100011001 \\ 0010001011 \\ 0001011110 \\ 0000101000 \end{bmatrix}_{5\times10}$$

ERROR PATTERN	**SYNDROMES**
1100000000	11000
0000110000	01011
0000000011	01010
1100110000	10011
0000110011	00001
1100000011	10010
1100110011	11001
0110000000	01100
0000011000	00101
0110011000	01001
0011000000	00110
0000001100	11100
0011001100	11010
0001100000	00011
0000000110	00100
0001100110	00111

2. Consider a (12,7) binary code with the 5×12 matrix H which has been constructed by the procedure given in the theorem 2 by taking s = 3, b = 2, n = 12, k = 7, q = 2

$$\mathrm{H} = \begin{bmatrix} 100001001111 \\ 010001110111 \\ 001000011010 \\ 000100110011 \\ 000010000101 \end{bmatrix}_{5\times12}$$

ERROR PATTERN	**SYNDROMES**
110000000000	11000
000001100000	10010
000000000011	00101
110001100000	01010
000001100011	10111
110000000011	11101
110001100011	01111
011000000000	01100
000000110000	00100
011000110000	01000
001100000000	00110
000000011000	11010
001100011000	11100
000110000000	00011
000000001100	01101
000110001100	01110
000011000000	11001
000000000110	00111
000011000110	11110

3. Consider a (15,10) binary code with the 5×15 matrix H which has been constructed by the procedure given in the Theorem 2 by taking s = 3, b = 3, n = 15, k = 10, q = 2

$$\mathbf{H} = \begin{bmatrix} 100000001011001 \\ 010001110110000 \\ 001000011011010 \\ 000100110111110 \\ 000011000100101 \end{bmatrix}_{5\times15}$$

ERROR PATTERN	**SYNDROMES**
111000000000000	11100
000000111000000	10000
000000000000111	10100
111000111000000	01100
000000111000111	00100
111000000000111	01000
111000111000111	11000
011100000000000	01110
000000011100000	10001
011100011100000	11111
001110000000000	00111
000000001110000	00001
001110001110000	00110
000111000000000	01010
000000000111000	11101
000111000111000	10111
000011100000000	00010
000000000011100	01011
000011100011100	01001
000001110000000	01101
000000000001110	10011
000001110001110	11110

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