



Reliability investigation of utensils processing unit

Shikha Bansal^{1,*} and S.C.Agarwal²

¹Department of Mathematics, SRM University, NCR Campus, Modinagar, India.

²Department of Mathematics, M.M (P.G) College, Modinagar, India.

ARTICLE INFO

Article history:

Received: 12 July 2014;

Received in revised form:

21 August 2014;

Accepted: 3 September 2014;

Keywords

Exponential time distribution,

Weibull time distribution,

Redundancy,

Boolean Function Technique.

ABSTRACT

This investigation deals with the reliability of utensils processing unit. Utensils processing unit consists of six subsystems, viz., hot rolling machine, cold rolling machine, cutting machine, pressing machine, polishing machine and packaging machine, connected in 1-out-of-6: F arrangement system. Mathematical formulation of the model is carried out by employing Boolean Function Technique. Various measures of system effectiveness such as reliability and mean time to failure are obtained. Numerical results are also presented to demonstrate the validity of analyzed results. The failure rate is exponentially distributed.

© 2014 Elixir All rights reserved

Introduction

Reliability is an important factor in the planning, design and operation of engineering systems, but the increase of demands of modern society and automation of their products, the industrial systems are becoming complex day by day. As long as man has built things, he has wanted to make them as reliable as possible. In order to increase the effectiveness of the system, redundancy plays an important role

Many researchers [5, 3, 2] used various techniques such as Supplementary Variable Technique, Regenerative Point Technique, Fuzzy logics, etc. But these methods involve complex computations to calculate the reliability of the systems. Keeping these facts in view, in this paper we have discussed the logical probability method of reliability calculation which describe the structure of complex system/equipments and the features of its. In this paper we proposed Boolean function technique to find the reliability of Utensils Manufacturing Plant. This plant consists of six subsystems, viz., hot rolling machine, cold rolling machine, cutting machine, pressing machine, polishing machine and packaging machine, working in series. The system configuration has been shown in Fig1

Initially, the raw material is melted in hot rolling machine and then cooled to relieve internal stresses and soften the metal with the help of cold rolling machine. After this process cutting machine is used to obtain the desired shape or size of the product. Further the product is pressed according to shape and size of utensils of different dies by pressing machine. After pressing, polishing machine is used for the purpose of shining the products and finally the products are placed in the packaging machine in order to package the utensils which yield the final product. Utensils Manufacturing Plant consists of the following six main subsystems:

[1] Hot rolling machine (A) consists of one unit. The system fails when this subsystem fails.

[2] Cold rolling machine (B) consists of one unit. It is subjected to major failure only.

[3] Cutting machine (C) consists of two units working in parallel. Complete failure occurs when both units fail.

[4] Pressing machine (D) consists of two units working in parallel. Complete failure occurs when both units fail.

[5] Polishing machine (E) consists of two units; one working and other is in standby with perfect switching.

[6] Packaging machine (F) consists of one unit. It is subjected to major failure only.

Notations

x_1, x_2	:	States of hot and cold rolling machine
x_3, x_4	:	States of cutting machine
x_5, x_6	:	States of pressing machine
x_7, x_9	:	States of polishing machine
x_8	:	States of perfect Switching devices
x_i	:	$\begin{cases} 0, & \text{in bad state} \\ 1, & \text{in good state } (i=1,2,\dots,10) \end{cases}$
x'_i	:	Negation of $x'_i \forall i=1,2,\dots,10$
\wedge / \vee	:	Conjunction / Disjunction.
R_i	:	Reliability of i^{th} unit of the system $\forall i=1,2,\dots,10$
Q_i	:	$1-R_i$
R_S	:	Reliability of the whole system.
$R_{SW}(t) / R_{SE}(t)$:	Reliability of the whole system when failures follow Weibull / Exponential time distribution.

Formulation Of Mathematical Model

By employing Boolean function technique, the conditions of capability for successful operation of the system in terms of logical matrix are expressed as shown below:

$$F(x_1, x_2, \dots, x_9) = \begin{vmatrix} x_1 & x_2 & x_3 & x_5 & x_7 & x_{10} \\ x_1 & x_2 & x_3 & x_5 & x_8 & x_9 & x_{10} \\ x_1 & x_2 & x_3 & x_6 & x_7 & x_{10} \\ x_1 & x_2 & x_3 & x_6 & x_8 & x_9 & x_{10} \\ x_1 & x_2 & x_4 & x_5 & x_7 & x_{10} \\ x_1 & x_2 & x_4 & x_5 & x_8 & x_9 & x_{10} \\ x_1 & x_2 & x_4 & x_6 & x_7 & x_{10} \\ x_1 & x_2 & x_4 & x_6 & x_8 & x_9 & x_{10} \end{vmatrix} \dots(1)$$

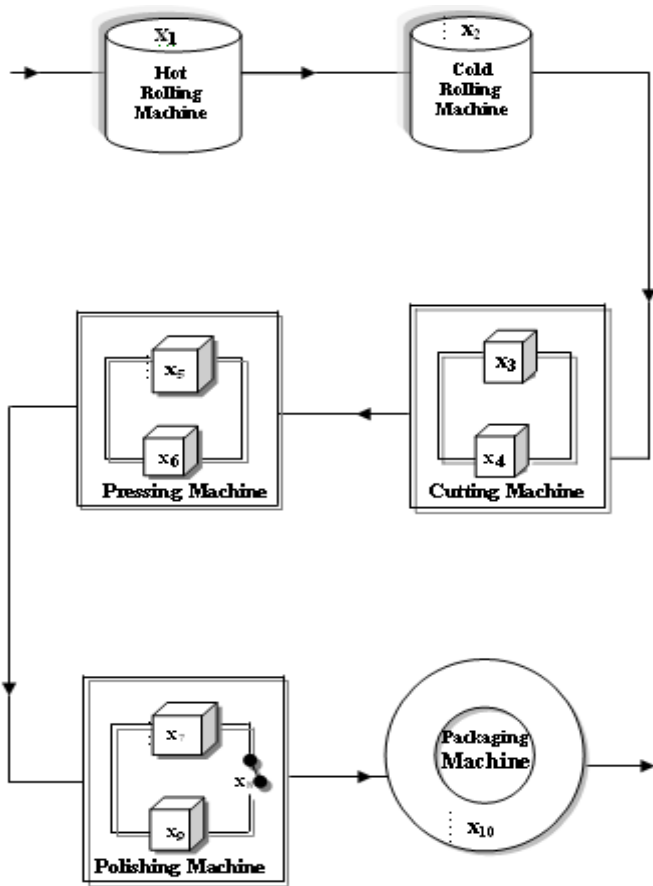


Fig 1: System Configuration of Utensils Processing Unit

Solution Of The Model

By using algebra of logics, equation (1) may be written as :

$$F(x_1, x_2, \dots, x_{10}) = |x_1 x_2 x_{10} \wedge f(x_3, x_4, \dots, x_9)| \quad ..(2)$$

Where

$$f(x_3, x_4, \dots, x_9) = \begin{vmatrix} x_3 & x_5 & x_7 \\ x_3 & x_5 & x_8 & x_9 \\ x_3 & x_6 & x_7 \\ x_3 & x_6 & x_8 & x_9 \\ x_4 & x_5 & x_7 \\ x_4 & x_5 & x_8 & x_9 \\ x_4 & x_6 & x_7 \\ x_4 & x_6 & x_8 & x_9 \end{vmatrix} = \begin{vmatrix} M_1 \\ M_2 \\ M_3 \\ M_4 \\ M_5 \\ M_6 \\ M_7 \\ M_8 \end{vmatrix} \quad ..(3)$$

where,

$$M_1 = x_3 \quad x_5 \quad x_7 \quad ..(4)$$

$$M_2 = x_3 \quad x_5 \quad x_8 \quad x_9 \quad ..(5)$$

$$M_3 = x_3 \quad x_6 \quad x_7 \quad ..(6)$$

$$M_4 = x_3 \quad x_6 \quad x_8 \quad x_9 \quad ..(7)$$

$$M_5 = x_4 \quad x_5 \quad x_7 \quad ..(8)$$

$$M_6 = x_4 \quad x_5 \quad x_8 \quad x_9 \quad ..(9)$$

$$M_7 = x_4 \quad x_6 \quad x_7 \quad ..(10)$$

$$M_8 = x_4 \quad x_6 \quad x_8 \quad x_9 \quad ..(11)$$

By orthogonalization algorithm, equation (3) may be written as

$$f(x_1, x_2, \dots, x_9) = \begin{vmatrix} M_1 \\ M'_1 & M_2 \\ M'_1 & M'_2 & M_3 \\ M'_1 & M'_2 & M'_3 & M_4 \\ M'_1 & M'_2 & M'_3 & M'_4 & M_5 \\ M'_1 & M'_2 & M'_3 & M'_4 & M'_5 & M_6 \\ M'_1 & M'_2 & M'_3 & M'_4 & M'_5 & M'_6 & M_7 \\ M'_1 & M'_2 & M'_3 & M'_4 & M'_5 & M'_6 & M'_7 & M_8 \end{vmatrix} \quad ..(12)$$

Using algebra of logic, one can determine the following:

$$M'_1 M'_2 = \begin{vmatrix} x'_3 \\ x_3 & x'_5 \\ x_3 & x_5 & x'_7 \end{vmatrix} \wedge |x_3 \quad x_5 \quad x_8 \quad x_9|$$

$$= |x_3 \quad x_5 \quad x'_7 \quad x_8 \quad x_9| \quad ..(13)$$

Similarly,

$$M'_1 M'_2 M'_3 = |x_3 \quad x'_5 \quad x_6 \quad x_7| \quad ..(14)$$

$$M'_1 M'_2 M'_3 M'_4 = |x_3 \quad x'_5 \quad x_6 \quad x'_7 \quad x_8 \quad x_9| \quad ..(15)$$

$$M'_1 M'_2 M'_3 M'_4 M'_5 = |x'_3 \quad x_4 \quad x_5 \quad x_7| \quad ..(16)$$

$$M'_1 M'_2 M'_3 M'_4 M'_5 M'_6 = |x'_3 \quad x_4 \quad x_5 \quad x'_7 \quad x_8 \quad x_9| \quad ..(17)$$

$$M'_1 M'_2 M'_3 M'_4 M'_5 M'_6 M'_7 = |x'_3 \quad x_4 \quad x'_5 \quad x_6 \quad x_7| \quad ..(18)$$

$$M'_1 M'_2 M'_3 M'_4 M'_5 M'_6 M'_7 M'_8 = |x'_3 \quad x_4 \quad x'_5 \quad x_6 \quad x'_7 \quad x_8 \quad x_9| \quad ..(19)$$

Inserting equations (13) through (19), into (12), one may obtain:

$$f(x_3, x_4, \dots, x_9) = \begin{vmatrix} x_3 & x_5 & x_7 \\ x_3 & x_5 & x'_7 & x_8 & x_9 \\ x_3 & x_5 & x_6 & x_7 \\ x_3 & x'_5 & x_6 & x'_7 & x_8 & x_9 \\ x_3 & x_4 & x_5 & x_7 \\ x'_3 & x_4 & x_5 & x'_7 & x_8 & x_9 \\ x'_3 & x_4 & x'_5 & x_6 & x_7 \\ x_3 & x_4 & x_5 & x_6 & x'_7 & x_8 & x_9 \end{vmatrix} \quad (20)$$

Using (20), equation (2) becomes

$$F(x_1, x_2, \dots, x_{10}) = \begin{vmatrix} x_1 & x_2 & x_3 & x_5 & x_7 & x_{10} \\ x_1 & x_2 & x_3 & x_5 & x'_7 & x_8 & x_9 & x_{10} \\ x_1 & x_2 & x_3 & x'_5 & x_6 & x_7 & x_{10} \\ x_1 & x_2 & x_3 & x'_5 & x_6 & x'_7 & x_8 & x_9 & x_{10} \\ x_1 & x_2 & x_3 & x_4 & x_5 & x_7 & x_{10} \\ x_1 & x_2 & x'_3 & x_4 & x_5 & x'_7 & x_8 & x_9 & x_{10} \\ x_1 & x_2 & x'_3 & x_4 & x'_5 & x_6 & x_7 & x_{10} \\ x_1 & x_2 & x'_3 & x_4 & x_5 & x_6 & x'_7 & x_8 & x_9 & x_{10} \end{vmatrix} \quad ..(21)$$

Finally, the probability of successful operation, i.e. Reliability of the whole system is given by

$$\begin{aligned}
 R_S &= P_r \{F(x_1, x_2, \dots, x_{10}) = 1\} \\
 &= R_1 R_2 R_{10} [R_3 R_5 R_7 + Q_3 R_4 R_5 R_7 + R_3 R_5 Q_7 R_8 R_9 + Q_3 R_4 Q_5 R_6 Q_7 R_8 R_9 \\
 &\quad R_3 Q_5 R_6 R_7 + R_3 Q_5 R_6 Q_7 R_8 R_9 + Q_3 R_4 R_5 Q_7 R_8 R_9 + Q_3 R_4 Q_5 R_6 R_7] \\
 &= R_1 R_2 R_{10} [R_3 R_5 R_7 + R_3 R_6 R_7 + R_4 R_5 R_7 + R_4 R_6 R_7 - R_3 R_5 R_8 R_9 - R_3 R_5 R_6 R_7 \\
 &\quad R_3 R_6 R_8 R_9 - R_3 R_4 R_5 R_7 + R_4 R_5 R_8 R_9 - R_3 R_4 R_6 R_7 - R_4 R_5 R_6 R_7 \\
 &\quad R_4 R_6 R_8 R_9 - R_3 R_5 R_7 R_8 R_9 - R_3 R_5 R_6 R_8 R_9 - R_3 R_6 R_7 R_8 R_9 \\
 &\quad - R_3 R_4 R_5 R_8 R_9 - R_4 R_5 R_7 R_8 R_9 + R_3 R_4 R_5 R_6 R_7 - R_3 R_4 R_6 R_8 R_9 \\
 &\quad - R_4 R_5 R_6 R_8 R_9 - R_4 R_6 R_7 R_8 R_9 + R_3 R_5 R_6 R_7 R_8 R_9 + R_3 R_4 R_5 R_7 R_8 R_9 \\
 &\quad R_3 R_4 R_5 R_6 R_8 R_9 + R_3 R_4 R_6 R_7 R_8 R_9 + R_4 R_5 R_6 R_7 R_8 R_9 \\
 &\quad - R_3 R_4 R_5 R_6 R_7 R_8 R_9] \dots(22)
 \end{aligned}$$

Some Particular Cases

Case I: When reliability of each component is R

Equation (22) yields:

$$R_S = 4R^6 - 7R^8 + 5R^9 - R^{10} \dots(23)$$

Case II: When failure rates follow Weibull time distribution:

Let λ_i be the failure rate corresponding to system state $x_i, \forall i=1, 2, \dots, 10$, then reliability of considered system at an instant 't', is given by

$$R_{SW}(t) = \sum_{i=1}^{14} \exp\{-\alpha_i t^p\} - \sum_{j=1}^{13} \exp\{-\beta_j t^p\} \dots(24)$$

where, p is a positive parameter, and

- $\alpha_1 = c + \lambda_3 + \lambda_5 + \lambda_7$
- $\alpha_2 = c + \lambda_3 + \lambda_6 + \lambda_7$
- $\alpha_3 = c + \lambda_4 + \lambda_5 + \lambda_7$
- $\alpha_4 = c + \lambda_4 + \lambda_6 + \lambda_7$
- $\alpha_5 = c + \lambda_3 + \lambda_5 + \lambda_8 + \lambda_9$
- $\alpha_6 = c + \lambda_3 + \lambda_6 + \lambda_8 + \lambda_9$
- $\alpha_7 = c + \lambda_4 + \lambda_5 + \lambda_8 + \lambda_9$
- $\alpha_8 = c + \lambda_4 + \lambda_6 + \lambda_8 + \lambda_9$
- $\alpha_9 = c + \lambda_3 + \lambda_4 + \lambda_5 + \lambda_6 + \lambda_7$
- $\alpha_{10} = c + \lambda_3 + \lambda_5 + \lambda_6 + \lambda_7 + \lambda_8 + \lambda_9$
- $\alpha_{11} = c + \lambda_3 + \lambda_4 + \lambda_5 + \lambda_7 + \lambda_8 + \lambda_9$
- $\alpha_{12} = c + \lambda_3 + \lambda_4 + \lambda_5 + \lambda_6 + \lambda_8 + \lambda_9$
- $\alpha_{13} = c + \lambda_3 + \lambda_4 + \lambda_6 + \lambda_7 + \lambda_8 + \lambda_9$
- $\alpha_{14} = c + \lambda_4 + \lambda_5 + \lambda_6 + \lambda_7 + \lambda_8 + \lambda_9$
- $\beta_1 = c + \lambda_3 + \lambda_5 + \lambda_6 + \lambda_7$
- $\beta_2 = c + \lambda_3 + \lambda_4 + \lambda_5 + \lambda_7$
- $\beta_3 = c + \lambda_3 + \lambda_4 + \lambda_6 + \lambda_7$
- $\beta_4 = c + \lambda_4 + \lambda_5 + \lambda_6 + \lambda_7$
- $\beta_5 = c + \lambda_3 + \lambda_5 + \lambda_7 + \lambda_8 + \lambda_9$
- $\beta_6 = c + \lambda_3 + \lambda_5 + \lambda_6 + \lambda_8 + \lambda_9$

- $\beta_7 = c + \lambda_3 + \lambda_6 + \lambda_7 + \lambda_8 + \lambda_9$
- $\beta_8 = c + \lambda_3 + \lambda_4 + \lambda_5 + \lambda_8 + \lambda_9$
- $\beta_9 = c + \lambda_4 + \lambda_5 + \lambda_7 + \lambda_8 + \lambda_9$
- $\beta_{10} = c + \lambda_3 + \lambda_4 + \lambda_6 + \lambda_8 + \lambda_9$
- $\beta_{11} = c + \lambda_4 + \lambda_5 + \lambda_6 + \lambda_8 + \lambda_9$
- $\beta_{12} = c + \lambda_4 + \lambda_6 + \lambda_7 + \lambda_8 + \lambda_9$
- $\beta_{13} = c + \lambda_3 + \lambda_4 + \lambda_5 + \lambda_6 + \lambda_7 + \lambda_8 + \lambda_9$

Where, $c = \lambda_1 + \lambda_2 + \lambda_{10}$

Case III: When failure rates follow exponential time distribution

Exponential distribution is nothing but a particular case of Weibull distribution for p=1 and is very useful in various practical problems. Therefore, the reliability of considered system as a whole at an instant 't' is expressed as:

$$R_{SE}(t) = \sum_{i=1}^{14} \exp\{-\alpha_i t\} - \sum_{j=1}^{13} \exp\{-\beta_j t\} \dots(25)$$

Also, an important reliability parameter, viz., M.T.T.F., in this case is given by

$$M.T.T.F = \int_0^{\infty} R_{SE}(t) dt = \sum_{i=1}^{14} \left(\frac{1}{\alpha_i} \right) - \sum_{j=1}^{13} \left(\frac{1}{\beta_j} \right) \dots(26)$$

where, α_i and β_j 's have been mentioned earlier
NUMERICAL EXAMPLE

- (A) Setting $\lambda_i (i=1, 2, \dots, 10) = 0.001, p = 2$ and $t = 0, 1, 2, \dots$ in equation (24) and (25), one may obtain Table 1 and fig 2
- (B) Setting $\lambda_i (i=1, 2, \dots, 10) = \lambda = 0, 0.1, 0.2, \dots, 1.0$ in equation (26), one may get Table 2 and fig 3

Table 1

Time (t)	R _{SE} (t)	R _{SW} (t)
0	1	1
1	0.997001	0.997001
2	0.994002	0.988009
3	0.991005	0.973049
4	0.988009	0.952177
5	0.985014	0.925497
6	0.982021	0.893185
7	0.979029	0.855514
8	0.976038	0.812875
9	0.973049	0.765788
10	0.970062	0.714913

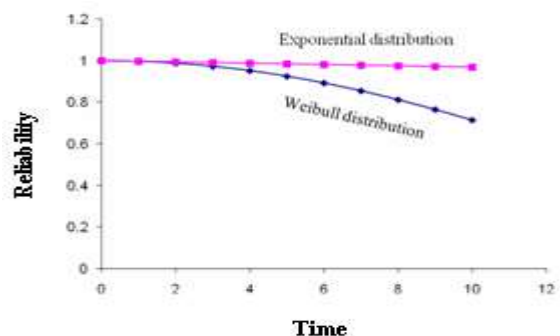


Fig 2: Reliability V/S Time

Table 2

λ	MTTF
0	∞
0.1	2.472222
0.2	1.236111
0.3	0.824074
0.4	0.618056
0.5	0.494444
0.6	0.412037
0.7	0.353175
0.8	0.309028
0.9	0.274691
1	0.247222

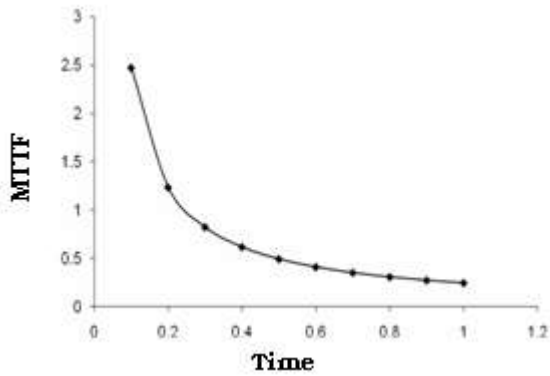


Fig 3: MTTF V/S Time

Interpretation of the results

[1] Table 1 and graph “Reliability V/S Time” (Fig 2) reveals that the reliability of the complex system decreases approximately at a uniformly rate in case of exponential time distribution, but it decreases very rapidly when failure rates follow weibull distribution.

[2] Table 2 and graph “MTTF V/S Failure Rate” (Fig 3) yield that MTTF of the system decreases catastrophically in the beginning but later it decreases approximately at a uniform rate.

References

[1] Ekata, S.B.Singh, *Operational readiness of global mobile satellite communication system under partial and complete failure*, Journal of Engineering Science and Technology, Vol. 6, No. 1, (2011), 82-93.

[2] K.Kumar, J.Singh, P.Kumar, *Fuzzy reliability and fuzzy availability of the serial process in butter-oil processing plant*, journal of mathematics and statistics, vol 15 (1), (2009), 65-71.

[3] S.Chander, R.K.Bhardwaj, *Reliability and economic analysis of 2-out-of-3 redundant system with priority to repair*, African Journal of Mathematics and Computer Science Research Vol. 2(11), (2009), 230-236.

[4] S.G.Choudhary, K.B.Mishra, *Evaluation of fuzzy reliability of a non-series parallel network* Microelectron. Reliabil. 32. (1992). 1-4.

[5] S. Shakuntla , A.K. Lal , S.S. Bhatia , Jai Singh, *Reliability analysis of Polytube industry using supplementary variable technique*, Applied Mathematics and Computation, 218 (2011) ,3981–3992