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The Jordan Canonical Forms of Complex s-orthogonal and s-skew symmetric Matrices

S.Krishnamoorthy[1,*] and K.Jaikumar[2]

[1]Department of Mathematics, Government Arts College (Autonomous), Kumbakonam, Tamil nadu, India.

[2]Department of Mathematics, Dharmapuram Adhinam Arts College Dharmapuram, Mayiladuthurai, Tamil nadu, India.

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ABSTRACT

We study the Jordan Canonical Forms of complex s-orthogonal and s-symmetric matrices, and consider some related results.

Introduction

The study of secondary symmetric and secondary orthogonal matrices was initiated by Anna Lee [1] and [2]. In this paper we present some extended results of [3] in the context of s-orthogonal and s-skew symmetric matrices. We denote the space of $n \times n$ matrices and complex matrices by M_n and £ respectively. The secondary transpose of A is defined by $A^s = VA^TV$ and $A^\Theta = VA^*V$, where 'V' is the fixed disjoint permutation matrix with units in its secondary diagonal.

Definition 1.1 [4]. Let $A \in$ £

a) The matrix A is called *s-symmetric*, if $A^s = A$. That is $A^TV = VA$.

b) The matrix A is called *s-skew symmetric*, if $A^s = -A$. That is $A^TV = -VA$.

c) The matrix A is called *s-orthogonal*, if $AA^s = A^sA = I$. That is $A^TVA = V$.

Basic Results

Our main objective is to present a new approach to the following classical characterization of the Jordan Canonical Forms of Complex s-orthogonal and s-skew symmetric matrices.

Theorem 2.1. A $n \times n$ complex matrix is similar to a complex s-orthogonal matrix if and only if its Jordan Canonical Form can be expressed as a direct sum of matrices of only the following five types

(a). $J_k(\lambda) \oplus J_k(\lambda^{-1})$ for $\lambda \in$ £ $\setminus \{-1, 0, 1\}$ and any k,

(b). $J_k(1) \oplus J_k(1)$ for any even k,

(c). $J_k(-1) \oplus J_k(-1)$ for any even k,

(d). $J_k(1)$ for any odd k, and

(e). $J_k(-1)$ for any odd k.

Theorem 2.2. A $n \times n$ complex matrix is similar to a complex s-skew symmetric matrix if and only if its Jordan Canonical Form can be expressed as a direct sum of matrices of only the following five types

(a). $J_k(\lambda) \oplus J_k(-\lambda)$ for $\lambda \in$ £ $\setminus \{0\}$ and any k,

(b). $J_k(0) \oplus J_k(0)$ for any even k, and

(c). $J_k(0)$ for any odd k.

The Complex s-orthogonal

Lemma 3.1. Let $A \in M_n$ be nonsingular. The following are equivalent

(a). A is similar to a complex s-orthogonal matrix

(b). A is similar to a complex s-orthogonal matrix via a complex s-symmetric similarity

Tele:
E-mail addresses: kjkdaac@gmail.com

(c). there exists a nonsingular complex s-symmetric S such that $A^s = SA^{-1}S^{-1}$ and

(d). there exists a nonsingular complex s-symmetric S such that $A^s SA = S$.

Proof: Assuming (a), suppose that X is nonsingular and $XAX^{-1} = L$ is complex s-orthogonal. The algebraic polar decomposition ensures that there is a nonsingular complex s-symmetric G and a complex s-orthogonal Q such that $X = QG$. Then $L = XAX^{-1} = QGAG^{-1}Q^s$, so $GAG^{-1} = Q^s LQ$ is a product of complex s-orthogonal matrices and hence is complex s-orthogonal.

Assuming (b), suppose that $A = GQG^{-1}$ for some complex s-symmetric G and complex s-orthogonal Q. Then $A^{-1} = GQ^sG^{-1}$ and $A^s = G^{-1}Q^sG = G^{-2}A^{-1}G^2$, which is (c) with $S = G^{-2}$.

Now assume (c) and write $S = Y^sY$ for some $Y \in M_n$ so $A^s = SA^{-1}S^{-1} = Y^sYA^{-1}Y^{-1}Y^{-s}$, or $(YAY^{-1})^s = Y^{-s}A^sY^s = YA^{-1}Y^{-1} = (YAY^{-1})^{-1}$; YAY^{-1} is therefore complex s-orthogonal and so (a) follows. The equivalence of (c) and (d) is clear.

Lemma 3.2. For any positive integer k and any $\lambda \neq 0$, $J_k(\lambda) \oplus J_k(\lambda^{-1})$ is similar to a complex s-orthogonal matrix.

Lemma 3.3. For any odd positive integer k, each of $J_k(1)$ and $J_k(-1)$ is similar to a complex s-orthogonal matrix.

Lemma 3.4. Let $r, k_1, ..., k_r$ be positive integers with k_1 even, and suppose that $k_1 > k_2 \geq ... \geq k_r$ if $r > 1$. Then neither $J_{k_1}(1) \oplus ... \oplus J_{k_r}(1)$ nor $J_{k_1}(-1) \oplus ... \oplus J_{k_r}(-1)$ is similar to a complex s-orthogonal matrix.

Theorem 3.5. Let $r, k_1, ..., k_r$ and $p, l_1, ..., l_p$ be positive integers with k_1 and l_1 even, suppose that $k_1 > k_2 \geq ... \geq k_r$ if $r > 1$ and that $l_1 > l_2 \geq ... \geq l_p$ if $p > 1$. Then $J_{k_1}(1) \oplus ... \oplus J_{k_r}(1) \oplus J_{l_1}(-1) \oplus ... \oplus J_{l_p}(-1)$ is not similar to a complex s-orthogonal matrix.

Lemma 3.6. Let $C \in M_k$ be similar to a complex s-orthogonal matrix. If $B \oplus C$ is similar to a complex s-orthogonal matrix for some $B \in M_n$, then B is similar to a complex s-orthogonal matrix.

Theorem 3.7. Let A be a complex s-orthogonal matrix. Then the even sized Jordan blocks of A corresponding to each of the Eigen values +1 and -1 are paired.

The s-skew symmetric

Lemma 4.1. A given $A \in M_n$ is similar to a complex s-skew symmetric matrix if and only if there is a non singular s-symmetric S such that $A^s = -SAS^{-1}$.

Lemma 4.2. For any positive integer k and any $\lambda \in £$, $J_k(\lambda) \oplus J_k(-\lambda)$ is similar to a s-skew symmetric matrix.

Lemma 4.3. For any odd positive integer k, $J_k(0)$ is similar to a s-skew symmetric matrix.

Lemma 4.4. Let $r, k_1, ..., k_r$ be positive integers with k_1 even, and suppose that $k_1 > k_2 \geq ... \geq k_r$ if $r > 1$. Then neither $J_{k_1}(0) \oplus ... \oplus J_{k_r}(0)$ is not similar to a s-skew symmetric matrix.

Lemma 4.5. Let C be similar to a complex s-skew symmetric matrix. If $B \oplus C$ is similar to a s-skew symmetric matrix, then B is also similar to a s-skew symmetric matrix.

Theorem 4.6. Let $A \in M_n$ be s-skew symmetric. Then the even sized singular Jordan blocks of A are paired.

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