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Reliability based design optimization of hollow shaft using integrated probabilistic response surface methodology

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ABSTRACT

Classical reliability based design procedures require tedious calculations and time consuming. The goal of reliability of mechanical component adequately performs its intended function when operating under specified environmental conditions. Mechanical component design by safety factors using nominal values without considering uncertainties may lead to designs that are unsafe, or too conservative and thus not efficient. Design of a hollow shaft is one of complex and time consuming design procedure. This paper presents development of mathematical models to predict the outer diameter of a typical hollow shaft. This paper presents unique method to investigate engineering problem, its analysis, mathematical modeling and optimization with the help of RSM-response surface methodology and design of experiments (DOE). Response surface methodology, which is a statistical approach of design of experiments, is being applied with combined probabilistic design to optimize the design responses in the case of simultaneous variations of its design parameters. The technique is proved to be efficient and general purpose modeling a variety of components.

Introduction

A shaft is a rotating member, usually of circular cross section used to transmit power or motion. Shafts form the important elements of machines. Shafts support rotating parts like gears and pulleys and in turn, they are themselves supported by bearing resting in the rigid machine housings. Shafts are subjected to torque due to power transmission and bending moment due to reactions on the members that are supported by them. Shafts are made to have circular cross section and could be either solid or hollow. A hollow shaft has greater strength and stiffness than solid shaft of equal weight.

The design of a shaft may require the interrelated considerations of a number of factors, such as material and heat treatment, strength for power and loading requirements, stiffness, bearing performance, gear operation, critical speeds, weight and space limitations, and stress considerations.

Failure of a shaft usually necessitates a costly and time consuming major overhaul. A stress analysis at a specific point on a shaft can be made using only the shaft geometry in the vicinity of that point. Shafts are generally made of ductile materials and the maximum shear stress theory which gives results on the safe side is simple to apply and in consequence, is widely used to determine the shaft diameter.

In this section of the present work, a hollow circular shaft subjected to combined bending and torsion is designed using the probabilistic design procedure on the basis of strength.

Design procedures

Deterministic design

When a hollow shaft is subjected to combined bending (M) and torsion (T), according to maximum stress theory, maximum shear stress induced in the shaft is

$$\tau_{\max} = \frac{16}{\pi d_0^{\,3}(1 = k^4)}\sqrt{M^2 + T^2} \qquad \text{--- (1)}$$

and

$$d_0 = \left(\frac{16}{\pi d_0^{\,3}(1 = k^4)\tau_{\max}}\sqrt{M^2 + T^2}\right)^{1/3} \qquad \text{--- (2)}$$

Introducing factor of safety,

$$d_0 = \left(\frac{16n}{\pi d_0^{\,3}(1 = k^4)\tau_{\max} S}\sqrt{M^2 + T^2}\right)^{1/3} \qquad (3)$$

Equation (3) is used to determine the shaft diameter.

Probabilistic design

Probability of failure of a shaft is defined as the probability that induced stress in the shaft exceeds the strength of the shaft material. Hence reliability is a function of material strength and the external load acting on the shaft. The bending moment (M), torsion (T), induced stress and strength of the material are assumed as random variables. All the random variables are assumed to follow normal distribution. The design parameter is the outside diameter of the hollow shaft which is also considered to be probabilistic in nature. Ratio of inside to outside diameter (k) is assumed to be known.

Induced shear stress $\tau = K\frac{T_e}{d_0^{\,3}}$ ---- (4)

Where $K = \frac{16}{\pi(1-k^4)}$

Equivalent torque, $T_c = (M^2 + T^2)^{0.5}$

Approximate mean, standard deviation and coefficient of variation of induced stress can be obtained from the relationships

$$\bar{f} \approx f\left(\bar{x}_1, \bar{x}_2, ...\bar{x}_n\right)$$

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And

$$\sigma_f \approx \left[\sum_{x=1}^{n} (\partial f / \partial x_i)^2 (\sigma_{xi})^2 \right]^{1/2}$$

Which holds when the dispersion of each random variable, $C = \sigma_x / \bar{x}$ is less than 0.2

$$\bar{\tau} = K \frac{\bar{T}_e}{\bar{d}_0^{\,3}} \qquad \text{------ (5)}$$

$$\sigma_t = \frac{K}{\bar{d}_0^{\,3}} \left[\sigma_{te}^{\,2} + \left(\frac{3\bar{T}_e \sigma_{d0}}{d_0} \right)^2 \right]^{0.5} \qquad \text{---- (6)}$$

$$C_\tau^{\,2} = C_{te}^{\,2} + 9C_{d0}^{\,2} \qquad \text{----- (7)}$$

Mean and standard deviation of $\bar{T}_e$ are

$$\bar{T}_e = \left[\left(\bar{M}^2 + \bar{T}^2 \right)^2 + 2\left(\bar{M}^2 \sigma_T^{\,2} + \bar{T}^2 \sigma_M^{\,2} + \sigma_M^{\,2} \sigma_T^{\,2} \right) \right]^{0.25} \qquad \text{---- (8)}$$

$$\sigma_{Te} = \left[\left(\bar{M}^2 + \sigma_M^{\,2} + \bar{T}^2 + \sigma_T^{\,2} \right) - \bar{T}_e^{\,2} \right]^{0.5} \qquad \text{---- (9)}$$

The standard normal variate (Z) is term of the expected values and the standard deviations of the random variables ‘S’ and ‘τ’ is

$$Z = -\frac{\bar{S} - \bar{\tau}}{\left(\sigma_s^{\,2} + \sigma_\tau^{\,2} \right)^{0.5}} \qquad \text{----- (10)}$$

$$\bar{d}_0^{\,6} + \frac{2K\bar{T}_e}{\bar{S}(Z^2 C_s^{\,2} - 1)} \bar{d}_0^{\,3} + \frac{(K\bar{T}_e)^2 (Z^2 C_\tau^{\,2} - 1)}{\bar{S}^2 (Z^2 C_s^{\,2} - 1)} = 0 \qquad \text{----- (11)}$$

The quadratic equation in $\bar{d}_0^{\,3}$ is solved for the mean outside diameter using a computer program developed in ‘C’.

Hypothetical case:

The following numerical data is used for the computation. Mean values of bending moment, torsion and shear strength are taken as 800000 N-mm, 200000 N-mm and 170 N/mm^2 respectively. Coefficient of variation of M, T, S and d_0 is taken as 0.01 the value of $k = d_i/d_o = 0.75$.

Results

a. By changing reliability values from 0.9 to 0.999999 and keeping all the other variables constant, the outside diameter (d_o) is obtained in the table (1) and are compared with the results obtained from deterministic design. R Vs $\bar{d}_o$ plot is shown in figure 1.

Table 1: variation of $\bar{d}_o$ with R

R	0.9	0.99	0.999	0.9999	0.99999	0.999999
$\bar{d}_o$	33.5188	33.8886	34.1554	34.3729	34.5603	34.7262

$C_{d0} = C_M = C_T = C_S = 0.01$

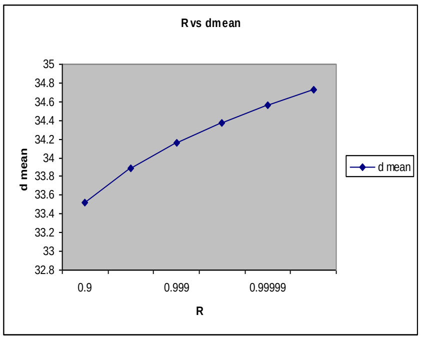

Figure 1: reliability Vs outer diameter

b. By changing the value of C_M from 0.01 to 0.1 and keeping all the other variables constant, the outside diameter (d_o) is obtained in the table (2) and are compared with the results obtained from deterministic design. C_M Vs $\bar{d}_o$ plot is shown in figure 2.

Table 2: variation of $\bar{d}_o$ with C_M

C_M	0.01	0.03	0.05	0.07	0.09	0.1
$\bar{d}_o$	34.3729	34.7261	35.25	35.8318	36.4279	36.7255

$C_{d0} = C_T = C_S = 0.01$, R = 0.9999.

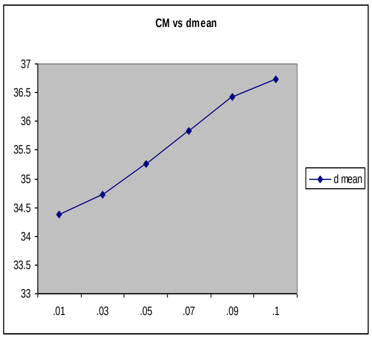

Figure 2: C_M Vs outer diameter

c. By changing the value of C_T from 0.01 to 0.1 and keeping all the other variables constant, the outside diameter (d_o) is obtained in the table (3) and are compared with the results obtained from deterministic design. C_T Vs $\bar{d}_o$ plot is shown in figure 3.

Table 3: variation of $\bar{d}_o$ with C_T

C_T	0.01	0.03	0.05	0.07	0.09	0.1
$\bar{d}_o$	34.3729	34.3748	34.3284	34.3839	34.3913	34.3956

$C_{d0} = C_M = C_S = 0.01$, R = 0.9999.

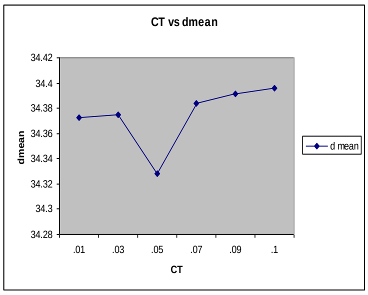

Figure 3: C_T Vs outer diameter

d. By changing the value of C_s from 0.01 to 0.1 and keeping all the other variables constant, the outside diameter (d_o) is obtained in the table (4) and are compared with the results obtained from deterministic design. C_S Vs $\bar{d}_o$ plot is shown in figure 4.

Table 4: variation of $\bar{d}_o$ with C_S

C_S	0.01	0.03	0.05	0.07	0.09	0.1
$\bar{d}_o$	34.3729	34.8904	35.7259	36.7782	38.0339	38.7457

$C_{d0} = C_T = C_M = 0.01$, R = 0.9999.

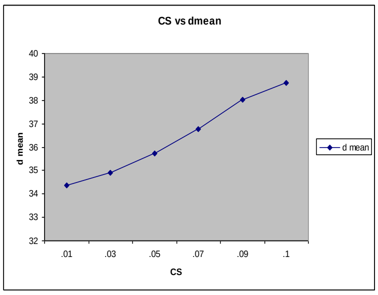

Figure 4: C_S Vs outer diameter

e. By changing the value of C_{do} from 0.01 to 0.1 and keeping all the other variables constant, the outside diameter (d_o) is obtained in the table (5) and are compared with the results obtained from deterministic design. C_{do} Vs $\bar{d}_o$ plot is shown in figure 5.

Table 5: variation of $\bar{d}_o$ with C_{do}

C_{do}	0.01	0.03	0.05	0.07	0.09	0.1
$\bar{d}_o$	34.3729	34.4469	38.3554	40.0974	41. 7020	42.4603

$C_M = C_T = C_S = 0.01$, $R = 0.9999$.

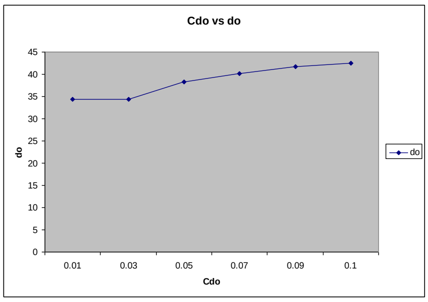

Figure 5: C_{do} Vs outer diameter

d_o obtained from deterministic design = 41.655mm.

Constant parameters:

$\bar{M}$ =800000, N-mm, $\bar{T}$ =200000 N-mm, $\bar{S}$ =170 N/mm^2, k=0.75.

Reliability Based Response Surface Design Optimization

Response surface methodology is proved to be an efficient tool for obtaining optimum conditions for designing of mechanical components. The design parameters are considered to be random and no parameter, in practice, will be constant, while the other parameter is subjected to change. Therefore, simultaneous random variation in the design parameters is considered in the present research, which appreciably changes the design response.

The experiments are conducted for the design matrix obtained depending upon the number of factors and their levels. The design responses are extracted from the C programs. The experimental values are used to predict a model, which optimizes the design parameters within the predefined range. The model converges to single or more optimized outputs, based upon complexity of the problem.

The output data from the probabilistic design is utilized for getting the optimum response values (minimized value of face width), with maximized range of reliability. The input design parameters taken for designing the hollow shafts are Reliability(R), coefficient of variation of bending moment (C_M), Coefficient of variation of torsion (C_T), coefficient of variation of bending strength (C_S) and coefficient of variation of outer diameter(C_{do}), for which the outside diameter (d_o) is the design response. Central composite design is being selected for producing the design matrix, and is given in the table (7). The ranges of the design parameters are tabulated as

Table 6: Maximum and minimum ranges of design parameters

Design parameter	**Low level**	**High level**
Reliability R	0.9	0.999999
Coefficient of variation of bending moment (C_M)	0.01	0.1
Coefficient of variation of torsion (C_T)	0.01	0.1
coefficient of variation of bending strength (C_S)	0.01	0.1
coefficient of variation of outer diameter(C_{do})	0.01	0.1

Table 7: Design matrix

Run order	**R**	**C_M**	**C_T**	**C_S**	**C_{do}**	**d_o**
1	0.9999	0.1	0.01	0.1	0.01	39.7571
2	0.9999	0.01	0.01	0.01	0.1	38.6969
3	0.94995	0.055	0.055	0.055	0.055	36.2441
4	0.9	0.1	0.1	0.1	0.1	37.6176
5	0.9	0.01	0.01	0.01	0.1	37.0609
6	0.9	0.01	0.1	0.01	0.01	33.5577
7	0.9999	0.01	0.1	0.1	0.1	44.425
8	0.9999	0.1	0.01	0.01	0.1	42.7666
9	0.9999	0.1	0.1	0.1	0.01	39.7633
10	0.9	0.01	0.1	0.1	0.1	37.4551
11	0.94995	0.055	0.1	0.055	0.055	36.255
12	0.9999	0.1	0.1	0.1	0.1	44.753
13	0.9999	0.055	0.055	0.055	0.055	39.692
14	0.9	0.01	0.1	0.01	0.1	37.0649
15	0.94995	0.055	0.055	0.1	0.055	37.901
16	0.9999	0.1	0.01	0.1	0.1	44.748
17	0.9999	0.1	0.1	0.01	0.1	42.771
18	0.94995	0.055	0.052	0.055	0.055	36.243
19	0.9999	0.01	0.1	0.1	0.01	38.705
20	0.94995	0.055	0.055	0.055	0.055	36.2441
21	0.831148209	0.055	0.055	0.055	0.055	35.116
22	0.94995	0.055	0.055	0.055	0.1	40.553
23	0.9999	0.1	0.1	0.01	0.01	36.711
24	0.9	0.01	0.01	0.1	0.01	34.7757
25	0.94995	0.052	0.055	0.055	0.055	36.2317
26	0.9	0.1	0.01	0.01	0.1	37.2336
27	0.9	0.1	0.01	0.01	0.01	34.489
28	0.9	0.1	0.01	0.1	0.1	37.6137
29	0.9999	0.01	0.1	0.01	0.01	34.3873
30	0.9	0.01	0.1	0.1	0.01	34.781
31	0.94995	0.055	0.055	0.052	0.055	36.2203
32	0.94995	0.1	0.055	0.055	0.055	37.0121
33	0.9999	0.01	0.01	0.1	0.01	39.6969
34	0.9999	0.01	0.1	0.01	0.1	42.418
35	0.94995	0.055	0.055	0.055	0.055	36.2441
36	0.94995	0.055	0.055	0.055	0.055	36.2441
37	0.9	0.01	0.01	0.1	0.1	37.4511
38	0.9999	0.1	0.01	0.01	0.01	36.7044
39	0.94995	0.055	0.055	0.055	0.055	36.2441
40	0.9	0.1	0.1	0.1	0.01	35.2623
41	0.9	0.01	0.01	0.01	0.01	33.5472
42	0.9999	0.01	0.01	0.1	0.1	44.4204
43	0.94995	0.055	0.055	0.055	0.052	36.1179
44	0.9	0.1	0.1	0.01	0.1	37.2375
45	0.94995	0.055	0.055	0.055	0.055	36.2441
46	0.9	0.1	0.01	0.1	0.01	35.2579
47	0.9	0.1	0.1	0.01	0.01	34.4941
48	0.9999	0.01	0.01	0.01	0.01	34.3648
49	0.94995	0.055	0.055	0.055	0.055	36.24411
50	0.94995	0.055	0.055	0.055	0.055	36.24411

Analysis of Variance (ANOVA)

ANOVA is a collection of statistical models, and their associated procedures, in which observed variance in a particular variable is partitioned into components due to different sources of variation. ANOVA provides a statistical test whether or not the means of several groups or all are equal. F-test and p-test are conducted to test the validity and significance of the model developed through regression. Table 8 shows the ANOVA conducted on model developed for hallow shaft.

Table 8: ANOVA table

Source	Sum of Squares	df	Mean Square	F Value	p-value Prob > F	
Model	388.01	20	19.4008	12.1327	< 0.0001	significant
A-r	151.7	1	151.749	94.899	< 0.0001	
B-cm	6.08	1	6.08025	3.80241	0.0609	
C-ct	0.187	1	0.18728	0.11712	0.7346	
D-cs	31.56	1	31.5649	19.7398	0.0001	
E-cdo	140.5	1	140.561	87.9029	< 0.0001	
AB	1.687	1	1.68728	1.0551	0.3128	
AC	0.234	1	0.23416	0.1464	0.7047	
AD	15.01	1	15.0141	9.38939	0.0047	
AE	15.59	1	15.5955	9.75301	0.0040	
BC	0.234	1	0.23471	0.14678	0.7044	
BD	2.125	1	2.12530	1.32910	0.2584	
BE	0.258	1	0.25826	0.16151	0.6907	
CD	0.701	1	0.70110	0.43845	0.5131	
CE	0.685	1	0.68509	0.42843	0.5179	
DE	1.324	1	1.32405	0.82802	0.3703	
A^2	6.18629	1	6.18629	3.86873	0.0588	
B^2	2.12166	1	2.12166	1.32683	0.2588	
C^2	0.93160	1	0.93160	0.58259	0.4515	
D^2	4.14028	1	4.14028	2.58921	0.1184	
E^2	13.7987	1	13.7987	8.62934	0.0064	
Residual	46.3724	29	1.59905			
Lack of Fit	46.3724	22	2.10783			
Pure Error	0	7				
Cor Total	434.389	49				

Observations from ANOVA (table-8):

- The Model F-value of 12.13 implies the model is significant. There is only a 0.01% chance that a "Model F-Value" this large could occur due to noise.
- Values of "Prob > F" less than 0.0500 indicate model terms are significant. In this case A, D, E, AD, AE, E^2 are significant model terms.
- Values greater than 0.1000 indicate the model terms are not significant. If there are many insignificant model terms (not counting those required to support hierarchy), model reduction may improve the model.

Table 9: R squared results

Std. Dev.	1.264535	R-Squared	0.893247
Mean	37.62566	Adj R-Squared	0.819624
C.V. %	3.360833	Pred R-Squared	0.523691
PRESS	206.9036	Adeq Precision	12.22854

R-squared results (table 9):

R^2 is a measure of amount of reduction in the variability of response obtained by regressor variables in the model. Always, there exists a condition that $0<R^2<1$, and also for significance of the model, R^2should tend to unity. The following are the observations from the R-square table (table 9).

- The "Pred R-Squared" of 0.5237 is not as close to the "Adj R-Squared" of 0.8196 as one might normally expect. This may indicate a large block effect or a possible problem with your model and/or data.
- "Adeq Precision" measures the signal to noise ratio. A ratio greater than 4 is desirable. Your ratio of 12.229 indicates an adequate signal.

Mathematical Model:

$d_0=-5.28+40.55*R+9.33*C_M+1.05*C_T\ +22.5*C_S+47.5*C_{do}$

Table (10) gives the predicted values for their corresponding experimental values:

Table 10: Predicted Values of Outer Diameter (d_o)

Run Order	R	c_m	c_t	c_s	c_{do}	d_o	d_{op}
1	0.9999	0.01	0.1	0.1	0.1	44.425	42.0796
2	0.9999	0.1	0.01	0.01	0.1	42.7666	41.1292
3	0.9999	0.1	0.1	0.1	0.01	39.7633	38.9818
4	0.9	0.01	0.1	0.1	0.1	37.4551	37.9338
5	0.95	0.06	0.16	0.06	0.06	36.255	37.9142
6	0.9999	0.1	0.1	0.1	0.1	44.753	42.8879
7	1.0688	0.06	0.06	0.06	0.06	39.692	42.8704
8	0.9	0.01	0.1	0.01	0.1	37.0649	36.1518
9	0.95	0.06	0.06	0.16	0.06	37.901	39.9202
10	0.9999	0.1	0.01	0.1	0.1	44.748	42.9112
11	0.9999	0.1	0.1	0.01	0.1	42.771	41.1059
12	0.95	0.06	0.05	0.06	0.06	36.243	37.9688
13	0.9999	0.01	0.1	0.1	0.01	38.705	38.1736
14	0.95	0.06	0.06	0.06	0.06	36.2441	37.9402
15	0.8311	0.06	0.06	0.06	0.06	35.116	33.0058
16	0.95	0.06	0.06	0.06	0.16	40.553	42.2802
17	0.9999	0.1	0.1	0.01	0.01	36.711	37.1998
18	0.9	0.01	0.01	0.1	0.01	34.7757	34.0512
19	0.95	0.05	0.06	0.06	0.06	36.2317	36.9524
20	0.9	0.1	0.01	0.01	0.1	37.2336	36.9834
21	0.9	0.1	0.01	0.01	0.01	34.489	33.0774
22	0.9	0.1	0.01	0.1	0.1	37.6137	38.7654
23	0.9999	0.01	0.1	0.01	0.01	34.3873	36.3916
24	0.9	0.01	0.1	0.1	0.01	34.781	34.0278
25	0.95	0.06	0.06	0.05	0.06	36.2203	35.7622
26	0.95	0.16	0.06	0.06	0.06	37.0121	38.8382
27	0.9999	0.01	0.01	0.1	0.01	39.6969	38.197
28	0.9999	0.01	0.1	0.01	0.1	42.418	40.2976
29	0.95	0.06	0.06	0.06	0.06	36.2441	37.9402
30	0.95	0.06	0.06	0.06	0.06	36.2441	37.9402
31	0.9	0.01	0.01	0.1	0.1	37.4511	37.9572
32	0.9999	0.1	0.01	0.01	0.01	36.7044	37.2233
33	0.95	0.06	0.06	0.06	0.06	36.2441	37.9402
34	0.9	0.1	0.1	0.1	0.01	35.2623	34.836
35	0.9	0.01	0.01	0.01	0.01	33.5472	32.2692
36	0.9999	0.01	0.01	0.1	0.1	44.4204	42.103
37	0.95	0.06	0.06	0.06	0.05	36.1179	33.1662
38	0.9	0.1	0.1	0.01	0.1	37.2375	36.96
39	0.95	0.06	0.06	0.06	0.06	36.2441	37.9402
40	0.9	0.1	0.01	0.1	0.01	35.2579	34.8594
41	0.9	0.1	0.1	0.01	0.01	34.4941	33.054
42	0.9999	0.01	0.01	0.01	0.01	34.3648	36.415
43	0.95	0.06	0.06	0.06	0.06	36.2441	37.9402
44	0.95	0.06	0.06	0.06	0.06	36.2441	37.9402
45	0.9999	0.01	0.1	0.1	0.1	44.425	42.0796
46	0.9999	0.1	0.01	0.01	0.1	42.7666	41.1292
47	0.9999	0.1	0.1	0.1	0.01	39.7633	38.9818
48	0.9	0.01	0.1	0.1	0.1	37.4551	37.9338
49	0.95	0.06	0.16	0.06	0.06	36.255	37.9142
50	0.9999	0.1	0.1	0.1	0.1	44.753	42.8879

Optimization of Hollow Shafts:

Hollow shaft is designed for optimum outer diameter (d_o) with the design parameters: R, C_M, C_T, C_S and C_{do}, and the optimum value of the response extracted by predicting the model. As the main criterion is to have minimum outer diameter combined with higher reliability, the model is evaluated for the optimum face width and is given in the table (11).

Table 11: Optimum values

R	C_M	C_T	C_S	C_{do}	d_o
0.9984	0.020	0.020	0.020	0.020	35.1156

Tables 12- 16 give the comparison of probabilistic output versus RSM values, which shows a proven reduction in outer diameter.

Table 12: Probabilistic Versus Response Values of Outer Diameter 'd_o' with Variation of 'R'

R	0.9	0.99	0.999	0.9999	0.99999	0.999999
$\bar{d}_o$ (probabilistic)	33.5188	33.8886	34.1554	34.3729	34.5603	34.7262
$\bar{d}_o$ (RSM)	32.0188	33.6683	34.0332	34.067	34.0734	34.075

Table 13: Probabilistic Versus Response Values of Outer Diameter 'd_o' with Variation of ' C_M '

C_M	0.01	0.03	0.05	0.07	0.09	0.1
$\bar{d}_o$ (probabilistic)	34.3729	34.7261	35.25	35.8318	36.4279	36.7255
$\bar{d}_o$ (RSM)	34.0697	34.2563	34.442	35.6295	35.8161	36.0994

Table 14: Probabilistic Versus Response Values of Outer Diameter 'd_o' with Variation of ' C_T '

C_T	0.01	0.03	0.05	0.07	0.09	0.1
$\bar{d}_o$ (probabilistic)	34.3729	34.3748	34.3284	34.3839	34.3913	34.3956
$\bar{d}_o$ (RSM)	34.0697	34.0907	34.1117	34.1327	34.1537	34.1642

Table 15: Probabilistic Versus Response Values of Outer Diameter 'd_o' with Variation of ' C_S '

C_S	0.01	0.03	0.05	0.07	0.09	0.1
$\bar{d}_o$ (probabilistic)	34.3729	34.8904	35.7259	36.7782	38.0339	38.7457
$\bar{d}_o$ (RSM)	34.0697	34.5197	34.9697	35.4197	36.8694	37.0947

Table 16: Probabilistic Versus Response Values of Outer Diameter 'd_o' with Variation of ' C_{do} '

C_{do}	0.01	0.03	0.05	0.07	0.09	0.1
$\bar{d}_o$ (probabilistic)	34.3729	34.4469	38.3554	40.0974	41.702	42.4603
$\bar{d}_o$ (RSM)	34.0697	34.0197	36.9697	38.9197	39.8697	40.3447

Graphical analysis:

Predicted versus experimental response:

Figure (6) shows the plot of scatter of predicted values of outside diameter d_o, from its experimental values.

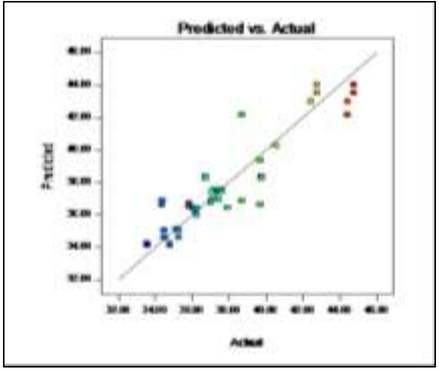

Figure 6: Predicted versus actual values of d_0

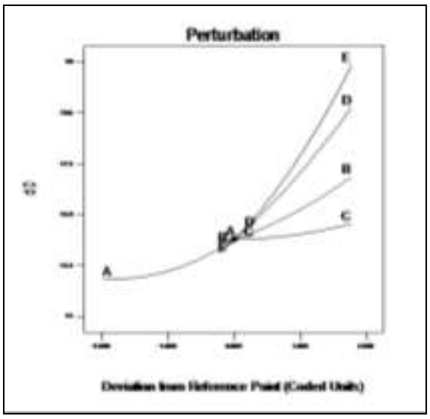

Figure 7: Perturbation curve

Perturbation curve:

Figure (7) shows the convergence of all the parameters at the prescribed optimum response (d_o).

Interaction plot:

Figure (8) indicates the relationship between the most influencing parameters: reliability and coefficient of variation of bending moment (C_M) at optimum value of face width d_o = 36.9904.

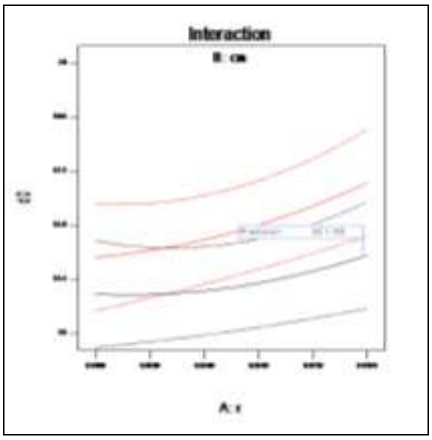

Figure 8: Interaction plot

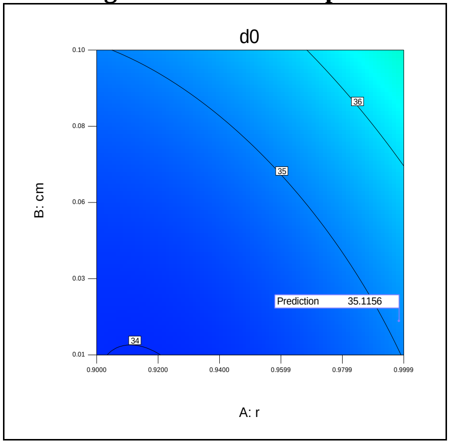

Figure 9: Contour plot for d_o

Contour plot:

Figure (9) is the contour graph to represent the optimum outer diameter (d_o) across R and C_M.

3D surface plot:

The three dimensional variation of the response surface with the most influencing input parameters (R and C_M) within the range of minimum and maximum values is given in the 3D surface plot in the figure (10).

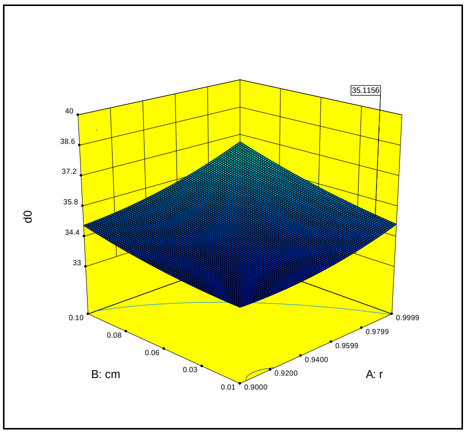

Figure 10: 3D surface plot for optimum d_o

Ramped plot:

A desirability of 0.963 indicates the satisfactory response within the safe range of its responses, as shown in the figure (11).

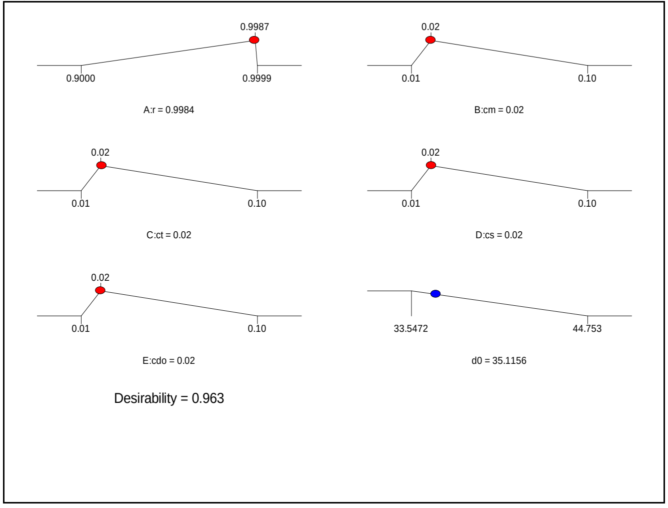

Figure 11: Ramped plot for the optimized outer diameter (d_o) of the hollow shaft with its corresponding parameters

Summary and Conclusions

Optimization is quite often associated almost exclusively with the use of mathematical techniques to model and analyze decision problems and these mathematical and stochastic models are usually tailored to fit to specific real life problems. A known fact is that it is difficult to conceive a model that reflects the reality as close as possible and simple for analysis. For this reason, different models each representing one or more problem situations are developed. During the past three decades a substantial body of literature has been developed on reliability models.

Design of hollow shaft with probabilistic nature of its elements, reliability, coefficient of variation of bending moment, coefficient of variation of torsion, coefficient of variation of shear stress and coefficient of variation of outer diameter to give out the outer diameter, which is also a probabilistic element of criterion.

Response surface method has been applied to model the elements with a relationship (linear/quadratic) for predicting the responses. This application has given the optimal values of the responses satisfying the prescribed constraints of the input parameters.

Ultimately, integration of probabilistic design with response surface design has proven to be an efficient and a simple method for reliability based design optimization of the mechanical components.

Scope for future work

- In the present work, the machine elements are idealized with only a few parameters. In future, effect of all the other parameters may be studied.
- In the present work, the random parameters are assumed to follow normal distribution. Other types of distributions may also be worked out.
- Apart from Response Surface Methodology, other non-traditional techniques (GA, Neural Networks, etc) may be studied for application.
- Reliability based design of a complete machine requires consideration of reliability design of individual elements. In future works, the same may be extended to the reliability based design of complete machine.

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