Available online at www.elixirpublishers.com (Elixir International Journal)

Materials Science

Elixir Materials Science 75 (2014) 27598-27602

ISSN: 2229-712X

Effects of variable viscosity and thermal conductivity on stagnation flow of a micropolar fluid towards a vertical permeable surface

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ARTICLE INFO

Article history:

Received: 29 August 2014;

Received in revised form: 20 September 2014;

Accepted: 7 October 2014;

Keywords

Viscosity, Thermal conductivity, Micro polar fluid, Stagnation point flow, Suction/injection, Boundary layer.

ABSTRACT

A computer oriented numerical approach to study the effects of variable viscosity and thermal conductivity of stagnation flow of a micropolar fluid towards a vertical permeable surface is investigated in this study. The external flow impinges normal to the heated plate and we have assumed the viscosity and thermal conductivity as the inverse linear function of temperature. The partial differential equations governing the problem under consideration have been transformed into a system of non-linear ordinary differential equations by the similarity transformation and solved them numerically by shooting method. Numerical results are carried out for various dimensionless parameters of the problem especially variable viscosity parameter, thermal conductivity parameter, micro-rotation parameter along with the Prandtl number. The results are presented graphically for velocity distribution, temperature distribution and micropolar distributions for various values of non-dimensional parameters. It is found that the effects of the parameters representing variable property of viscosity and thermal conductivity are significant.

Introduction

The theory of micropolar fluids was originally formulated by Eringen [1]. In essence, the theory introduces new material parameters, an additional independent vector field, the microrotation and new constitutive equations which must be solved simultaneously with the usual equations for Newtonian flow. The desire to model the non-Newtonian flow of fluid containing rotating micro-constituents prporous media, turbulent shear flows, and flowing capillaries and microchannels by Lukaszewiez [2].

We analyze the effect of the variable viscosity and the variable thermal conductivity on self-similar boundary layer flow of a micropolar fluid in a porous channel, where the flow is driven by uniform mass transfer through the channel walls. The corresponding Newtonian fluid model was first studied by Berman [3], who described an exact solution of the Navier-Stokes equations by assuming a self-similar solution and reducing the governing partial differential equations to a nonlinear ordinary differential equation of fourth order. The solution is of potential value in understanding more realistic flow in channels and pipes, and study of Berman's exact solution and generalizations of it have attracted numerous studies subsequently, for example Yuan [4], Robinson [5], Zaturska et. al. [6], Desseaux [7].Through the viscosity and thermal conductivity are assumed as constant properties but in actual these are temperature dependent (Schlichiting [8], Eckert[9]). Therefore, in this paper we consider the effect of variable viscosity and variable thermal conductivity on stagnation flow of a micropolar fluid towards a vertical permeable surface.

Formalation of the problem:

Consider a laminar two-dimensional stagnation flow of an incompressible micropolar fluid impinges normal to a vertical plate. It is assumed that the free stream velocity $U(x)$ and the temperature of the plate $T_w(x)$ vary linearly with the distance x from the stagnation point, i.e. $U(x)=ax$ and $T_w(x)=T_\infty+bx$, where a, b are positive constants. The steady laminar boundary layer equations governing the flow are

Equation of continuity

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial x}=0 \tag{1}$$

Equation of momentum

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$$u\frac{\partial u}{\partial x}+v\frac{\partial v}{\partial y}=U\frac{dU}{dx}+\left(\frac{\mu+k}{\rho}\right)\frac{\partial^2 u}{\partial y^2}+\frac{k}{\rho}\frac{\partial N}{\partial y}\pm g\beta(T-T_\infty) \qquad (2)$$

The angular momentum equation

$$\rho j\left(u\frac{\partial N}{\partial x}+v\frac{\partial N}{\partial y}\right)=\gamma\frac{\partial^2 N}{\partial y^2}-k\left(2N+\frac{\partial u}{\partial y}\right) \qquad (3)$$

The energy equation

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2} \qquad (4)$$

subject to the boundary conditions

$$u=0,\ v=V_w,\ N=-\frac{1}{2}\frac{\partial u}{\partial y},\ T=T_w(x) \text{ at } y=0 \qquad (5)$$

$$u\to U(x),\ N\to 0,\ T\to T_\infty \text{ as } y\to\infty$$

Where u and v are the velocity components along the x-axis respectively, T is the fluid temperature, N is the component of the microrotation vector normal to the x-y plane, ρ is the density, j is the microinertia density, μ is the dynamic viscosity, k is the gyro-viscosity, γ is the spin gradient viscosity and V_w is the uniform surface mass flux. The last term on the right hand side of equation (2) represents the influence of the thermal buoyancy force on the flow field with "+" and "-" signs pertaining respectively to the buoyancy assisting and the buoyancy opposing flow regions. We assume that

$$\gamma=\left(\mu+\frac{k}{2}\right)j=\mu\left(1+\frac{K}{2}\right)j$$, where $K=\frac{k}{\mu}$ is the material parameter.

To seek similarity solutions for equations (1)-(4) subject to the boundary conditions (5), introduce the following dimensionless similarity variables:

$$\eta=y\left(\frac{U}{vx}\right)^2,\ f(\eta)=\frac{\psi}{(vxU)^{\frac{1}{2}}},\ g(\eta)=\left(\frac{vx}{U^3}\right)^{\frac{1}{2}}N,\ \theta(\eta)=\frac{T-T_\infty}{T_w-T_\infty} \qquad (6)$$

where η is the independent similarity variable, $f(\eta)$ is dimensionless stream function, $g(\eta)$ is dimensionless microrotation, $\theta(\eta)$ is dimensionless temperature and ν is the kinemetic viscosity of the fluid. Further, ψ is the stream function which is defined in the usual way as $u=\frac{\partial\psi}{\partial y}$ and $v=-\frac{\partial\psi}{\partial x}$ so as to identically satisfy equation (1). Using (6), we get

$$u=Uf'(\eta),\ v=-(\nu a)^{\frac{1}{2}}f(\eta) \qquad (7)$$

The fluid viscosity is assumed to be inverse linear function of temperature as

$$\frac{1}{\mu}=\frac{1}{\mu_\infty}\left[1+\alpha\left(T-T_\infty\right)\right],\frac{1}{\mu}=a\left(T-T_r\right),\ a=\frac{\alpha}{\mu_\infty} \text{ and } T_r=T_\infty-\frac{1}{\alpha} \qquad (8)$$

where *a* and T_r are constants and their values depends on the reference state and the thermal property of the fluid. In general *a*>0 for liquids and *a*<0 for gases. T_r is transformed reference temperature related to viscosity parameter. α is constant based on thermal property and μ_∞ is the viscosity at $T=T_\infty$ similarly, consider the variation of thermal conductivity as,

$$\frac{1}{\lambda}=\frac{1}{\lambda_\infty}\left[1+\xi\left(T-T_\infty\right)\right],\frac{1}{\lambda}=b\left(T-T_k\right), b=\frac{\xi}{\lambda_\infty} \text{ and } T_k=T_\infty-\frac{1}{\xi} \tag{9}$$

where b and T_k are constants and their values depends on the reference state and thermal property of the fluid ξ is constant based on thermal property and λ_∞ is the viscosity at $T=T_\infty$.

Using equation (6), it can be easily verified that the continuity equation is satisfied automatically and using equation (6) - (9) in the equations (2),(3) and (4) become,

$$f'''=\frac{\theta_v-\theta}{K(\theta_v-\theta)+\theta_v}f'^2-\frac{\theta_v\theta'}{(\theta_v-\theta)\{K(\theta_v-\theta)+\theta_v\}}f''-\frac{\theta_v-\theta}{K(\theta_v-\theta)+\theta_v}\{ff''+Kg'+B\theta+1\} \tag{10}$$

$$g''=\frac{2}{2+K}\left\{\frac{1}{G}(2g+f'')+(f'g-fg')\right\} \tag{11}$$

$$\theta''=-\frac{\theta'^2}{\theta_c-\theta}-\frac{\theta_c-\theta}{\theta_c}P_r(f\theta'-f'\theta) \tag{12}$$

The corresponding boundary conditions are

$$f(0)=f_0,\ f'(0)=0,\ g(0)=-\frac{1}{2}f'',\theta(0)=1 \tag{13}$$

$$f'(\eta)\to 1,\ g(\eta)\to 0 \text{ as } \eta\to\infty$$

Results and discussion:

The equations (10)-(12) together with the boundary conditions (13) are solved for various values of the parameters involved in the equations using algorithms based on the shooting method. Results are presented for velocity distribution, microrotation distribution and temperature distribution with the variation of different parameters.

Initially solution was taken for constant values of taking Pr=0.70, G=0.51,K=2.00, ϴv =-10.00, ϴc =-10.00 with the viscosity parameter Vr ranging from -10 to -1 at the certain values of ϴc=-10.00 Similarly the solutions have been found with varying the thermal conductivity parameter ϴc ranging from -10 to -1 at the certain values of ϴv=-10 keeping other values remaining same. We have considered in some detail the influence of physical parameters on velocity distribution, micrototation distribution and temperature distribution which is shown in figures 1-5. The figures 1 and 2 show the variations in velocity and microrotation distribution with the variation of viscosity parameter ϴv. From the figures it is clear that the velocity decreases as ϴv increases. Figures 3 and 4 show the variations in temperature and velocity distribution with the variation of thermal conductivity parameter ϴc. From the figures it is seen that both temperature and velocity decreases as ϴc increases. From figure 5, it is clear that the velocity increases with the increasing value of B.

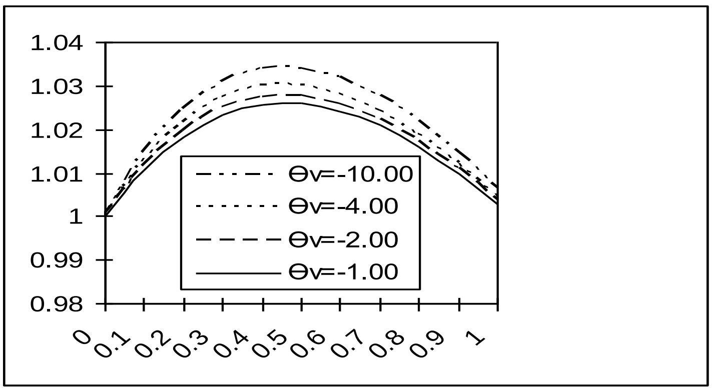

Fig.1. Velocity distribution profile (f′) with the variation of ϴv

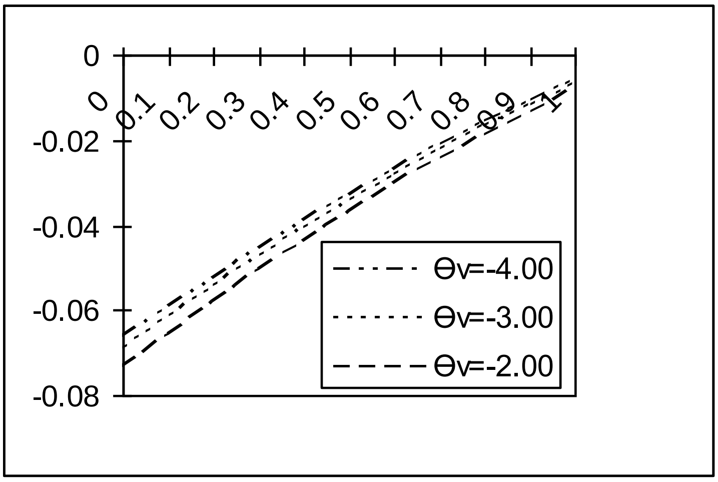

Fig.2. Microrotation distribution profile (g) with the variation of Θv

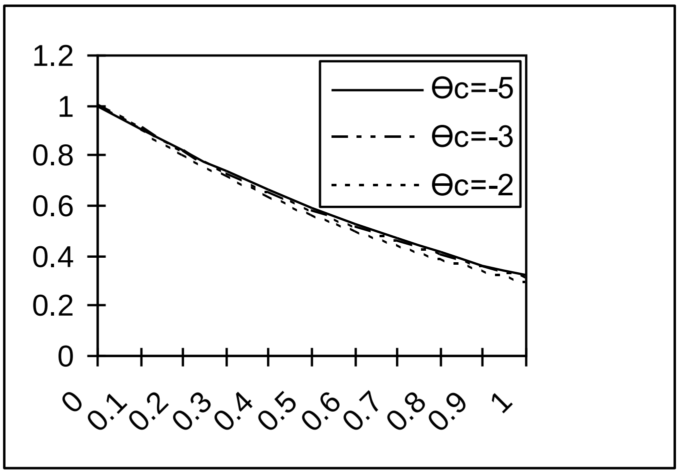

Fig 3. Temperature distribution profile (Θ) with the variation of Θc

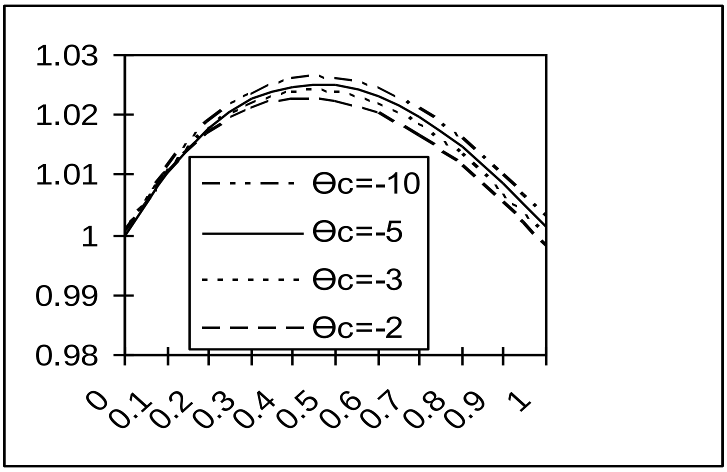

Fig.4. Velocity distribution profile (f′) with the variation of Θc

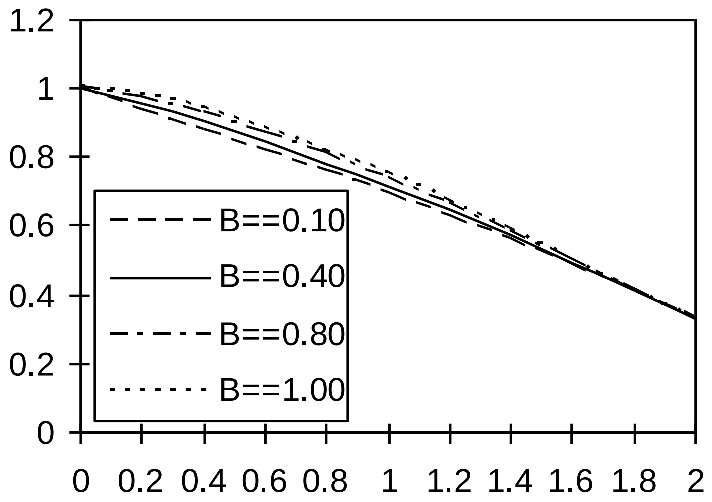

Fig 5. Velocity distribution profile (f′) with the variation of buoyancy parameter B

Conclusion

In this study, the stagnation flow of a micropolar fluid towards a vertical permeable surface is investigated when the viscosity and thermal conductivity are assumed to vary with temperature. The results presented demonstrate clearly that the viscosity parameter has a substantial effect on velocity and microrotation distribution, while the effect on temperature distribution is very pronounced due to the variation of thermal conductivity parameter. In addition, buoyancy parameter B takes a major role in the variation of velocity distribution. Thus the assumption of constant properties may lead severe errors in the design of fluid machinery and in various flow problems.

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