



# Denoising of Images Based on Different Wavelet Thresholding by Using Various Shrinkage Methods using Basic Noise Conditions

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## ARTICLE INFO

### Article history:

Received: 5 January 2013;

Received in revised form:

20 September 2014;

Accepted: 29 September 2014;

### Keywords

De-noising,  
Spatial domain methods,  
Wavelet shrinkage,  
Optimal threshold selection.

## ABSTRACT

Wavelet transforms enable us to represent signals with a high degree of scarcity. Wavelet thresholding is a signal estimation technique that exploits the capabilities of wavelet transform for signal denoising. The aim of this paper is to study various thresholding techniques such as Sure Shrink, Visu Shrink and Bayes Shrink and determine the best one for image denoising. This paper presents an overview of various threshold methods for image denoising. Wavelet transform based denoising techniques are of greater interest because of their performance over Fourier and other spatial domain techniques. Selection of optimal threshold is crucial since threshold value governs the performance of denoising algorithms. Hence it is required to tune the threshold parameter for better PSNR values. In this paper, we present various wavelet based shrinkage methods for optimal threshold selection for noise removal.

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## Introduction

In general, an image may be contaminated by noise during acquiring and transmission. The noise present in the images may appear as additive or multiplicative components which have been modelled in a number of ways in the literature [1],[17] such as Gaussian noise, Speckle noise, Salt & Pepper noise, Impulse noise etc... As the occurrence of noisy pixels in the image is random in nature, their distributions are modelled using probabilistic methods [20] [24]. In most of the real time applications such as medical imaging, satellite image data analysis, remote applications etc., the noisy components have to be removed to ensure faithful information retrieval from the images. A common defect in the imaging system is unwanted non linearity in the sensor and display system. Post processing correction of sensor signals and pre-processing correction of display signals can reduce degradations substantially [1]. Hence pre-processing is essential in any information analysis and retrieval system. Denoising is one of the pre-processing techniques which have drawn much attention of the researchers over a few decades. In this paper, present a detailed survey of various noise removal techniques, with a focus on threshold computing methods is presented since choosing the threshold is crucial in the process of denoising. This paper is organized as follows. Section 2 presents denoising procedure and classification of denoising methods. Section 3 discusses about the wavelet based denoising techniques. Various threshold methods and the tradeoffs involved in selecting an optimal threshold are presented in Section 4. Finally, discussions on observations and conclusion are presented in Section 5.

### Methods of Denoising

If  $f(x,y)$  be the uncorrupted image of size  $N \times N$  and  $n(x,y)$  be the noise function, then the noisy image observation  $g(x,y)$  with additive noise. The process of denoising is nothing but the estimation of the information from noisy observation. With this background, the state of art denoising methods can be categorized as follows.

### Spatial Filtering Techniques

Spatial filtering is the method of choice in situations when only additive noise is present. This category consists of mean filter and the order statistics filter such as Median filter, Maximum and Minimum filter, Midpoint and Alpha trimmed median filter. Arithmetic and Geometric mean filters are well suited for random noise like Gaussian or uniform noise. The Contra-harmonic filter is well suited for impulse noise, but it requires the prior knowledge about the noise (light or dark). As found in the literature [1],[17], median filter can perform well in removing impulse noise while the number of passes of the median filter has to be kept as low as possible, since larger number of passes may result in blurred images. The process of spatial filtering consists of moving the filter mask (Fig: 1) from point to point in an image. At each point  $(x,y)$  the response of filter at that point is calculated. The mask may be of any size of interest (3X3, 5X5, 7X7 etc...). Also, it has to be noted that size of the filter mask affects the performance of the filter [15].

Another class of filters which fall under spatial filters is adaptive filter, which method changes behavior based on the statistical characteristics of the image inside the filter region defined by  $m \times n$  rectangular window. These filters can offer superior denoising performance with the cost of increased complexity [17] [24].

**Adaptive median filter is the prime variant of adaptive filter. Filter mask size is altered according to the parameters calculated**

In the mask area considered originally. It performs well for the impulse noise with low spatial density and seeks to preserve details while smoothing non-impulse noise too. Researchers have shown interest to evolve adaptive iterative median filter which outperforms even for high density noises [26].

### Frequency domain filtering

Frequency domain filtering can be used for periodic noise reduction and removal. This category of filters include band pass filter, band stop filter, Notch (Reject/Pass) filters. The

appropriate filter can be chosen with the prior knowledge of noise distribution. The various Fourier domain filtering techniques such as Inverse filter, Wiener filter and least square filter are found in literature. A simple method of removing multiplicative noise like speckle noise too has been proposed namely homomorphic filtering [1] [17]. Fourier transform has been found to be an important image processing tool for image processing and analysis. The major advantage of Fourier domain analysis is that, it can explore the geometric characteristics of a spatial domain image [2]. It has been used for the removal of additive noises from the images. Unlike Fourier transform, Wavelet transform shows localization in both time and frequency and hence it has proved itself to be an efficient tool for a number of image processing applications including noise removal [19]. Fourier transform based methods are less useful because, they cannot work on non-stationary signals and they can capture only global features. But in the real scenario, as the images are only piecewise smooth and the noise distributions are random in nature, Fourier transform cannot perform well for the stochastic noise, but wavelets can do. Hence wavelet based noise removal has attracted much attention of the researchers for several years [4], [6]. A detailed study on wavelet based Denoising techniques is presented in the next section

**Wavelet denoising**

Wavelet transform is the mathematical tool used for various image processing applications such as noise removal, feature extraction, compression and image analysis. The general method of wavelet based denoising is that, the noisy image may first be transformed to wavelet domain [2] [6].

The transformed image appears as four sub-bands (A, V, H, and D) as shown in Fig 1 based on the level of decomposition 'j'. 2D discrete wavelet transform leads to decomposition of approximate coefficients at level 'j' into four components i.e. the approximation at level 'j+1' and details in three orientations (Horizontally, Vertically and diagonally) [25]. Since the noisy components are of high frequency, the three higher bands may contain the noisy components [25], and proper threshold may be applied to smooth the noisy wavelet coefficients followed by the inverse 2D-DWT may be applied to reconstruct the denoised image. Selection of optimal threshold is crucial for the performance of denoising algorithm. Threshold is selected based on the image and noise priors such as mean and variance [10] [23]. Selection of optimal threshold along with various types of wavelet threshold methods is presented in the next section.

**Fig.1 One DWT decomposition step**



**Wavelet based threshold methods**

**Sure Shrink**

Sure Shrink is more explicitly adaptive to unknown smoothness and has better large-sample MSE properties. This method is a subband adaptive threshold scheme, based on Stein's unbiased estimator for risk (SURE) (quadratic loss function) [6-8]. One gets an estimate of the risk for a particular threshold value t. minimizing the risks in 't' gives a selection of the threshold value. Sure Shrink is a thresholding by applying subband adaptive threshold, a separate threshold is computed for each detail subband based upon SURE (Stein's unbiased estimator for risk), a method for estimating the loss  $2 \mu^{\hat{}} - \mu$  in an unbiased fashion. In our case let wavelet coefficients in the jth subband be  $\{ X_i : i = 1, \dots, d \}$ ,  $\mu^{\hat{}}$  is the soft threshold. applied

to the image data, resulting in an estimate of the mean vector. This estimate is sparse and much less noisy than the raw image data [14]. The SURE principle just described has a serious drawback in situations of extreme sparsity of the wavelet coefficients. In such cases the noise contributed o the SURE profile by the many coordinates at which the signal is zero, swamps the information contributed to the SURE profile by the few coordinates where the signal is nonzero. Consequently, Sure Shrink uses a Hybrid scheme [16].

**Bayes Shrink**

Bayes Shrink has attracted much attention since it sets different thresholds for every subband. Here sub-bands are frequency bands that differ from each other in level and direction. The relationship between the wavelet transforms of the degraded image, uncorrupted image and generalized Gaussian noise with distribution:

$N(0, \sigma^2)$  (Y, X and V respectively), can be modeled as:

$$Y = X + V \tag{1}$$

Since huge information about the noise is available at the diagonal coefficients of first level wavelet decomposition (HH1) the noise variance 'σ' is calculated using the robust estimator.  $W_m$  are the wavelet coefficients in each scale and M is the total number of wavelet coefficients. With this background, the threshold using Bayes shrink is calculated.

The Bayes shrink method is effective for images corrupted by Gaussian noise. Bayes shrink is less sensitive to the presence of noise in the areas around the edges [9] [11]. However, the presence of noise in flat regions of the image is perceptually more noticeable by the human visual system. Bayes shrink performs little Denoising in high activity sub-regions to preserve the sharpness of edges but completely denoised the flat sub-parts of the image.

The risk function values are equal to the risk in coefficient values. A mere least square estimate does not denoise the original image [21]. Hence to estimate wavelet coefficients.

**Bivariate Shrink**

New shrinkage function which depends on both coefficient and its parent yield improved results for wavelet based image denoising. Here, we modify the Bayesian estimation problem as to take into account the statistical dependency between a coefficient and its parent. Then,

$$y_1 = w_1 + n_1 \tag{2}$$

$$y_2 = w_2 + n_2 \tag{3}$$

Where  $y_1$  and  $y_2$  are noisy observations of  $w_1$  and  $w_2$  and  $n_1$  and  $n_2$  are noise samples.

Then, mathematically it can be written as-

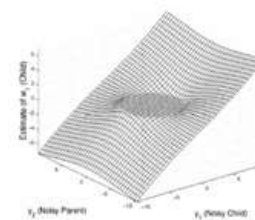
$$y = w + n \tag{4}$$

$$y = (y_1, y_2)$$

$$w = (w_1, w_2) \tag{5}$$

$$n = (n_1, n_2) \tag{6}$$

**Fig.2 New Bivariate shrinkage function**



**Neigh Shrink**

In the spatial domain, it is well known that an adaptive Wiener method based on estimation from local information is

very efficient for digital image enhancement. In the wavelet domain, despite the de-correlating properties of the wavelet transform, as pointed out in the introduction, there still exist significant residual statistical dependencies between neighbor wavelet coefficients. Our goal is to exploit this dependency to improve the estimation of a coefficient given its noisy observation and a context (spatial and scale neighbors).

One of the simplest wavelet shrinkage rules for an N x N image is the universal threshold-

$$\lambda = \sqrt{2\sigma^2 \log N^2} \tag{7}$$

The universal threshold grows asymptotically and removes more noise coefficients as N tends to infinity. The universal threshold is designed for smoothness rather than for minimizing the errors. So  $\lambda$  is more meaningful when the signal is sufficiently smooth or the length of the signal is close to infinity. Natural image, however, is usually neither sufficiently smooth nor composed of infinite number of pixels. In fact, if we suppose that an optimal threshold which minimize MSE (or maximized PSNR),  $\lambda$  is  $\alpha \lambda$ ,  $\alpha$  is always much less than 1.0 for natural image. Especially we got very similar  $\alpha$  value for different kinds and size of images when we applied soft thresholding rule.

**Trade off between Threshold, PSNR and Complexity**

Selection of optimal threshold determines the efficiency of the Denoising algorithm [10]. The common measure of quality in images in peak signal to noise ratio are defined as

$$PSNR = 10 \log_{10} (255)^2 / MSE \text{ (db)} \tag{8}$$

Here MSE is the mean square error whose magnitude quantifies the presence of noise and the performance of Denoising algorithm. As discussed in section - IV wavelet based shrinkage algorithms give better estimate of the noise priors and hence the threshold with the expense of high computational complexity. It is very crucial to select the threshold value with less computational complexity and with significant improvements in PSNR.

**Evaluation Criteria**

The above said methods are evaluated using the quality measure Peak Signal to Noise ratio which is calculated using the formula:

$$PSNR = 10 \log_{10} (255)^2 / MSE \text{ (db)} \tag{9}$$

Where, MSE is the mean squared error between the original image and the reconstructed de-noised image. It is used to evaluate the different de-noising scheme like Neigh shrink and Modified Neigh shrink.

**Experiments**

Quantitatively assessing the performance in practical application is complicated issue because the ideal image is normally unknown at the receiver end. So this paper uses the following method for experiments. One original image is applied with Gaussian noise with different variance. The methods proposed for implementing image de-noising using wavelet transform take the following form in general. The image is transformed into the orthogonal domain by taking the wavelet transform. The detail wavelet coefficients are modified according to the shrinkage algorithm. Finally, inverse wavelet is taken to reconstruct the de-noised image. In this paper, different wavelet bases are used in all methods. For taking the wavelet transform of the image, readily available MATLAB routines are taken. In each sub-band, individual pixels of the image are shrunk based on the threshold selection. A de-noised wavelet transform is created by shrinking pixels. The inverse wavelet transform is the de-noised image.

**Results and Discussions**

For the above mentioned three methods, image de-noising is performed using wavelets from the second level to fourth level decomposition and the results are shown in figure (3) and table if formulated for second level decomposition for different noise variance as follows. It was found that three level decomposition and fourth level decomposition gave optimum results. However, third and fourth level decomposition resulted in more blurring. The experiments were done using a window size of 3X3, 5X5 and 7X7. The neighborhood window of 3X3 and 5X5 are good choices.

Fig. 1 Gaussian Noise with Sure Shrink Method

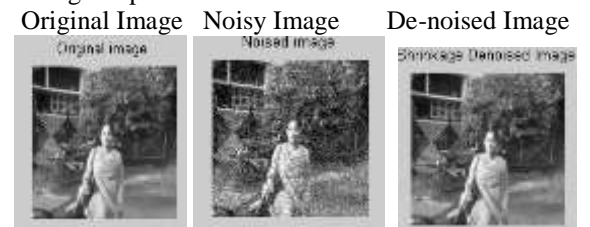


Original Image Noisy Image De-noised Image

Fig. 2 Salt & Pepper Noise with Sure Shrink Method



Original Image Noisy Image De-noised Image



Original Image Noisy Image De-noised Image

Results based on PSNR values obtained by applying on different methods.

**Table 1. Type of noise: Gaussian Noise**

	Lena	Barbara	Mandrill	My_Image
Sure Shrink	24.9186	24.8543	26.1505	26.444
Bayes Shrink	23.9419	23.4202	24.369	24.7382
Neigh Shrink	26.5673	27.8746	24.8382	31.0067
Bivariate Shrink	71.9643	71.4069	96.2018	77.7025

**Table 2. Type of noise: Salt and Pepper Noise**

	Lena	Barbara	Mandrill	My_Image
Sure Shrink	18.5611	19.2412	18.7047	20.9524
Bayes Shrink	18.567	19.1534	18.5042	20.8096
Neigh Shrink	22.6607	22.6526	23.4103	22.625
Bivariate Shrink	75.7063	71.4103	69.1896	72.0309

**Table 3. Type of noise: Speckle Noise**

	Lena	Barbara	Mandrill	My_Image
Sure Shrink	19.4768	21.2934	20.7625	22.9878
Bayes Shrink	16.9754	18.5228	17.3399	20.754
Neigh Shrink	24.5477	23.3882	23.3804	23.7186
Bivariate Shrink	75.6929	71.4178	69.1993	71.9989

**Conclusion**

In this paper, the image de-noising using discrete wavelet transform is analyzed. The experiments were conducted to study the suitability of different wavelet bases and also different window sizes. Among all discrete wavelet bases, coiflet performs well in image de-noising. Experimental results show that Bivariate Shrink Method gives better result than Sure

Shrink, Bayes Shrink and Neigh Shrink methods when applied on series of images.

#### Future Scope

In this paper three types of noise are involved and applied to number of images after introducing a particular type of shrink method. This work can be further elaborated in other types of noise like Brownian noise, Poisson noise etc. to produce a wide range of denoising methods.

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