# Algorithm for graph with different vertices 

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#### Abstract

In this paper we will discuss the algorithm for new type of graph which vertices are different. The algorithm for each type of this graph also will be illustrated. Then we will illustrate different examples for each type.


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## Introduction

Definition of Graph: An (undirected) graph $G$ is defined by two finite sets. a non-void set $X$ of elements called vertices, a set $E$ (which can be empty) of elements called edges, with for each edge e two associated vertices, $x$ and $y$, distinct or not, called the end vertices of e [4].
Definition of Algorithm: In mathematics and computer science, an algorithm is an effective method expressed as a finite list of well-defined instructions for calculating a function .In simple words an algorithm is a step-by-step procedure for calculations.
Definition of Spanning tree :Spanning tree for a graph G is a subgraph of $G$ that contains every vertex of $G$ and is a tree [5].
Definition of link-graph : knot-graph (link-graph) is a graph in which each vertex is a link[1].
Definition of simple graph: A "simple" graph is a graph with no loops or multiple edges [6].
Definition of graph with graph vertices: Consider the geometric graph $\mathrm{G}(\mathrm{V}, \mathrm{E})$ where
$\mathrm{V}(\mathrm{G})=\left\{\left\{\mathrm{V}^{0}{ }_{1}, \mathrm{e}^{0}{ }_{1}\right\},\left\{\mathrm{v}^{1}{ }_{1}, \mathrm{e}^{1}{ }_{1}\right\} \ldots \ldots .\left\{\mathrm{v}^{\mathrm{n}}{ }_{1}, \mathrm{e}^{\mathrm{n}}{ }_{1}\right\},\left\{\mathrm{V}^{0}{ }_{2}, \mathrm{e}^{0}{ }_{2}\right\},\left\{\mathrm{V}^{1}{ }_{2}\right.\right.$, $\left.\mathrm{e}^{1}{ }_{2}\right\} \ldots . .\left\{\mathrm{V}^{\mathrm{n}}{ }_{2}, \mathrm{e}^{\mathrm{n}}{ }_{2}\right\},\left\{\mathrm{V}^{0}{ }_{\mathrm{n}}, \mathrm{e}^{0}{ }_{\mathrm{n}}\right\}$,
$\left\{\mathrm{V}_{\mathrm{n}}^{1}, \mathrm{e}_{\mathrm{n}}^{1}\right\} \ldots \ldots\left\{\mathrm{V}_{\mathrm{m}}^{\mathrm{n}}, \mathrm{e}^{\mathrm{n}}{ }_{\mathrm{m}}\right\}$ and $\mathrm{E}(\mathrm{G})=\left\{\mathrm{e}^{1}, \mathrm{e}^{2}, \mathrm{e}^{3}, \ldots \ldots \mathrm{e}^{\mathrm{n}}\right\}$. We are called this graph ( graph with complex vertices)[2].
Definition of graph with different vertices : The graph with different vertices is a graph which each vertex has different chap such as: link vertex, graph vertex, or simple vertex , whether this graph is simple or have multiedges (multigraph) [3].

1. Algorithm for Graph which its vertices has all three chaps (simple, knot or graph) :

## Input:

Connected graph with different vertices $\mathrm{V}_{\mathrm{n}}, \mathrm{V}_{\mathrm{nm}}$ such that $0 \leq \mathrm{n}$ $\leq \mathrm{k}, 0 \leq \mathrm{m} \leq \mathrm{k}, \mathrm{k}$ integer $\geq 1$.

## Algorithm body:

Crete a subgraph that visit $\mathrm{V}_{\mathrm{n}}, \mathrm{V}_{\mathrm{nm}}$ proceeding from vertex to vertex along internal spanning tree $\mathrm{T}\left(\mathrm{V}_{\mathrm{nm}}\right.$ not simple vertices) then along outer spanning tree $\mathrm{T}^{\top}$.

1. Initialized $\mathrm{T}, \mathrm{T}$ to have all vertices and no edge.
2. Visit $\mathrm{V}_{\mathrm{n}}, 0 \leq \mathrm{n} \leq \mathrm{k}, \mathrm{k}$ integer $\geq 1$.

If $V_{n}$ simple then,
2a. Attach the edges $\left\{\mathrm{e}_{\mathrm{n}}\right\}$
Return to step 2.
Output T.

## Else

3. Visit $\mathrm{V}_{\mathrm{n}}, \mathrm{V}_{\mathrm{nm}}, 0 \leq \mathrm{n}, \mathrm{m} \leq \mathrm{k}$.

If $V_{n}$ knot vertices then,
3a. Visit outer vertices $\mathrm{V}_{\mathrm{n}}$ then internal vertices $\mathrm{V}_{\mathrm{nm}}$.
3b. Attach the internal edges $\left\{\mathrm{e}_{\mathrm{nm}}\right\}$.
3c. Attach the outer edges $\left\{\mathrm{e}_{\mathrm{nm}}{ }^{\prime}\right\}$.
Return to step 3.
Output $\mathrm{T}^{\prime}, \mathrm{T}^{\prime \prime}$.
Else
4. Visit $\mathrm{V}^{\mathrm{n}}, \mathrm{V}^{\mathrm{nm}}$.

If $\mathrm{V}^{\mathrm{n}}$ is graph then,
4a. Visit outer vertices $\mathrm{V}_{\mathrm{n}}$ then internal vertices $\mathrm{V}^{\mathrm{n}}, \mathrm{V}^{\mathrm{nm}}$.
4 b . Attach the internal edges $\left\{\mathrm{V}^{\mathrm{nlm} 1}, \mathrm{~V}^{\text {nimi }}\right\}$ to $\mathrm{T}^{\text {III }}$ and visit $\mathrm{V}^{\mathrm{nlmk}}$.
4c. Assign $V^{n 1 m 1}$ to $V^{\text {nimk }}$.
Return to step 4.
Output $\mathrm{T}^{\text {III }}$.
End
Output T, $\mathrm{T}^{\prime}, \mathrm{T}^{\prime \prime}, \mathrm{T}^{\text {III. }}$.
End algorithm.
Example 1: For a graph shown in Fig.(1), which has different vertices ( simple, knot, graph) we can compute the algorithm to find the spanning tree as follows:


Fig 1.

## Input:

Connected graph with different vertices $\mathrm{V}_{\mathrm{n}}, \mathrm{V}_{\mathrm{nm}}$ such that $0 \leq \mathrm{n}$ $\leq \mathrm{k}, 0 \leq \mathrm{m} \leq \mathrm{k}, \mathrm{k}$ integer $\geq 1$.

## Algorithm body:

Crete a subgraph that visit $\mathrm{V}_{\mathrm{n}}, \mathrm{V}_{\mathrm{nm}}$ proceeding from vertex to vertex along internal spanning tree $\mathrm{T}\left(\mathrm{V}_{\mathrm{nm}}\right.$ not simple vertices) then along outer spanning tree $\mathrm{T}^{\dagger}$.

1. Initialized $\mathrm{T}, \mathrm{T}$ to have all vertices and no edge.
2. Visit $\mathrm{V}_{4}$ then

2a. Attach $\mathrm{e}_{1}$ to T .
Return to step 2.
Output T.
Else
3. Visit $V_{3}$ then internal vertices $V_{31}, v_{33}$.

3a. Attach the internal edges $\left\{\mathrm{e}_{31}, \mathrm{e}_{32} \mathrm{e}_{33}\right\}$.
3b. Attach the outer edges $\left\{\mathrm{e}_{31}, \mathrm{e}_{32}, \mathrm{e}_{33}\right\}$.
Return to step 3.
Output $\mathrm{T}^{\prime}, \mathrm{T}^{\prime \prime}$.
Else
4. Visit $V_{1}$ then $V^{0}$, then

4 a . Visit internal vertices $\mathrm{V}^{01}$ to $\mathrm{V}^{03}$.
4b. Attach the internal edges $\left\{\mathrm{V}^{\mathrm{o1}}, \mathrm{~V}^{04}\right\},\left\{\mathrm{V}^{\mathrm{o0}}, \mathrm{~V}^{04}\right\},\left\{\mathrm{V}^{02}\right.$, $\left.\mathrm{V}^{04}\right\},\left\{\mathrm{V}^{03}, \mathrm{~V}^{04}\right\}$ to $\mathrm{T}^{\text {IIII }}$.
4c. Attach the outer edge $\mathrm{e}^{1}$ to $\mathrm{T}^{\text {|lII }}$ and visit $\mathrm{V}^{1}$.
Return to step 4.
Output $\mathrm{T}^{\text {III }}, \mathrm{T}^{\text {IIII }}$.
End
Output $\mathrm{T}, \mathrm{T}^{\prime}, \mathrm{T}^{\prime \prime}, \mathrm{T}^{\prime \prime \prime}, \mathrm{T}^{\prime \prime \prime \prime}$.
End algorithm.
Then the tree will be:


Fig 2
Algorithm for Graph which its vertices has only two chaps: The algorithm when its vertices are simple and knot:
For a graph shown in Fig.(3) which its vertices are simple and knot the algorithm will be:

## $V_{2}$



Fig 3

## Input:

Connected graph with different vertices $\mathrm{V}_{\mathrm{n}}, \mathrm{V}_{\mathrm{nm}}$ such that $0 \leq \mathrm{n}$ $\leq \mathrm{k}, 0 \leq \mathrm{m} \leq \mathrm{k}, \mathrm{k}$ integer $\geq 1$.

## Algorithm body:

Crete a subgraph that visit $\mathrm{V}_{\mathrm{n}}, \mathrm{V}_{\mathrm{nm}}$ proceeding from vertex to vertex along internal spanning tree T ( $\mathrm{V}_{\mathrm{nm}}$ not simple vertices) then along outer spanning tree $T$.

1. Initialized $\mathrm{T}, \mathrm{T}^{\prime}$ to have all vertices and no edge.
2. Visit $V_{1}$ then

2a. Attach $\mathrm{e}_{1}$ to T .
Return to step 2.
Output T.
Else
3. Visit $V_{3}$ then

3a. Attach the internal edges $\left\{\mathrm{e}_{31}, \mathrm{e}_{32} \mathrm{e}_{33}\right\}$.
3b. Attach the outer edges $\left\{\mathrm{e}_{31}^{\}, \mathrm{e}_{32}^{〕}, \mathrm{e}_{33}^{\prime}\right\}$.
Return to step 3.
Output $\mathrm{T}^{\prime}$.
End
Output T, T ${ }^{\text {I }}$.
End algorithm.
Then the final spanning tree will be:


Fig 4
The algorithm when its vertices are simple and graph:
For a graph shown in Fig.(5) which its vertices are simple and graph the algorithm will be:


Fig 5

## Input:

Connected graph with different vertices $\mathrm{V}_{\mathrm{n}}, \mathrm{V}_{\mathrm{nm}}$ such that $0 \leq \mathrm{n}$ $\leq \mathrm{k}, 0 \leq \mathrm{m} \leq \mathrm{k}, \mathrm{k}$ integer $\geq 1$.

## Algorithm body:

Crete a subgraph that visit $\mathrm{V}_{\mathrm{n}}, \mathrm{V}_{\mathrm{nm}}$ proceeding from vertex to vertex along internal spanning tree $\mathrm{T}\left(\mathrm{V}_{\mathrm{nm}}\right.$ not simple vertices) then along outer spanning tree $\mathrm{T}^{\prime}$.

1. Initialized $T, T^{\backslash}$ to have all vertices and no edge.
2. Visit $\mathrm{V}_{4}$ then

2a. Attach $\mathrm{e}_{4}$ to T.
Return to step 2.
Output T.
Else
3. Visit $\mathrm{V}_{1}$ then $\mathrm{V}^{0}$, then

4a. Visit internal vertices $\mathrm{V}^{01}$ to $\mathrm{V}^{03}$.
4b. Attach the internal edges $\left\{\mathrm{V}^{\mathrm{o1}}, \mathrm{~V}^{04}\right\},\left\{\mathrm{V}^{\mathrm{o0}}, \mathrm{~V}^{04}\right\},\left\{\mathrm{V}^{02}\right.$, $\left.\mathrm{V}^{04}\right\},\left\{\mathrm{V}^{03}, \mathrm{~V}^{04}\right\}$ to $\mathrm{T}^{\prime}$.
4c. Attach the outer edge $e^{1}$ to $T^{\prime \prime}$ and visit $V^{1}$.
Return to step 4.
Output $\mathrm{T}^{\dagger}, \mathrm{T}^{\prime \prime}$.
End
Output T, $\mathrm{T}^{\prime}, \mathrm{T}^{\prime \prime}$.
End algorithm
The spanning tree will be:


Fig 6
The algorithm when its vertices are knot and graph:
For a graph shown in Fig.(7) which its vertices are knot and graph the algorithm will be:


Fig 7

## Input:

Connected graph with different vertices $\mathrm{V}_{\mathrm{n}}, \mathrm{V}_{\mathrm{nm}}$ such that $0 \leq \mathrm{n}$ $\leq \mathrm{k}, 0 \leq \mathrm{m} \leq \mathrm{k}, \mathrm{k}$ integer $\geq 1$.

## Algorithm body:

Crete a subgraph that visit $\mathrm{V}_{\mathrm{n}}, \mathrm{V}_{\mathrm{nm}}$ proceeding from vertex to vertex along internal spanning tree $\mathrm{T}\left(\mathrm{V}_{\mathrm{nm}}\right.$ not simple vertices) then along outer spanning tree $\mathrm{T}^{\top}$.

1. Visit $V_{1}$ then $V^{0}$, then

4a. Visit internal vertices $\mathrm{V}^{01}$ to $\mathrm{V}^{03}$.
4b. Attach the internal edges $\left\{\mathrm{V}^{\mathrm{o} 1}, \mathrm{~V}^{04}\right\},\left\{\mathrm{V}^{\mathrm{oj}}, \mathrm{V}^{04}\right\},\left\{\mathrm{V}^{02}\right.$,
$\left.\mathrm{V}^{04}\right\},\left\{\mathrm{V}^{03}, \mathrm{~V}^{04}\right\}$ to T .
4c. Attach the outer edge $\mathrm{e}^{1}$ to $\mathrm{T}^{1}$ and visit $\mathrm{V}^{1}$.
Return to step 4.
OutOutput T, $\mathrm{T}^{\dagger}$.
Else

1. Visit $V_{3}$ then

3a. Attach the internal edges $\left\{\mathrm{e}_{31}, \mathrm{e}_{32} \mathrm{e}_{33}\right\}$.
3b. Attach the outer edges $\left\{\mathrm{e}_{31}, \mathrm{e}_{32}^{\prime}, \mathrm{e}_{33}\right\}$.
Return to step 3.
Output T", $\mathrm{T}^{\text {III. }}$
End
OutputT, $T^{\prime}, T^{\prime \prime}, T^{\prime \prime \prime}$.
End algorithm.
Then the spanning tree will be:


Fig 8

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