



Signal model for the prediction of wind speed in Nigeria

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ABSTRACT

Rapid development of wind energy as an alternative source of power is providing rich environment for wind energy related research. Several mathematical models have been used to study wind data and the models are mainly physical and statistical models. In this study, a signal Modeling approach is developed to predict wind speed data in Nigeria. The signal modeling approach is based on the Markov property, which implies that given the present wind speed state, the future of the system is independent of its past. A Markov process is in a sense the probabilistic analog of causality and can be specified by defining the conditional distribution of the random process.

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Introduction

Wind is important natural resources that has been used for centuries for navigation and agriculture, but of recent has been receiving a lot of attention due to the focus on renewable energies. Recently, there has been growing interest in wind energy as an alternative source of power. Rapid development of wind energy provides rich environment for wind energy related research as well as present new research challenges (Song et al., 2011). Among these challenges, understanding the wind speed and knowing how it behaves is extremely important since power output depends on wind speed. The consideration of wind speed and direction along with power output can give further insight in wind farm dynamics than using only power output (Mur-Amada and Bayod-Rujula, 2007).

The knowledge of wind speed and direction is vital not only for its meteorological significance but also is invaluable in the design of structures, shelter belts, airports and their runways as well as in studies of physiologic comfort and the control of air pollution and pest such as locusts (Ayoade, 2006). Wind speed prediction is necessary because of the intermittent, fluctuating and nonlinear nature of wind (Sheela and Deepa, 2012). Several mathematical models have been used to study wind data (Azami et al, 2009; Dorvlo, 2001; Gracia, 1998; Hennessey, 1997) and the models are mainly physical and statistical models (Gnana and Deepa, 2011, Sheela and Deepa, 2012). The physical model considers the physical reasoning to get the best results while the statistical model considers online measurements of data. Among statistical approaches, Markov chain is a popular tool to model, forecast and simulate the wind speed in a discrete and statistical way (Song et al., 2011). Wind speed is typically judged as the velocity of wind. Wind velocity is a vector quantity involving both direction and magnitude.

Nigeria is fortunate because high wind velocities of hurricane or gale force are relatively uncommon. Wind velocity generally produces observable outputs which can be characterized as signals. The signals can be corrupted by either signal sources, for example noise or by transmission distortion. The fundamental problem of interest is on how to characterize such wind velocities in terms of signal models. In Rabiner (1989), Signal models are important in that they often work

extremely well in practice and enable one to realize important practical systems such as prediction systems, recognition systems, and identification systems in a very efficient manner. Statistically, signal model is the set of statistical models in which one tries to characterize only the statistical properties of the signal. Examples of such statistical models include Gaussian processes, Poisson processes, Markov processes, and Hidden Markov processes among others (Rabiner, 1989).

The underlying assumption of the statistical models is that the signal can be well characterized as a parametric random process, and that the parameters of the stochastic processes can be estimated in a precise well defined manner. In this study, a signal modelling approach is developed basically to predict wind speed data in Nigeria. The signal modelling approach is based on the Markov property, which implies that given the present state, the future of a system is independent of its past. A Markov process is in a sense the probabilistic analog of causality and can be specified by defining the conditional distribution of the random process. Markov processes are good models for many situations, since they are analytically tractable and widely used in many applied fields. The Markov models are often the most useful models for analysing the uncertainty in many physical or real-world systems that evolve dynamically in time. The basic concepts of a Markov process are those of a state and of a state transition.

Wind speed data and Markov Chains

Let X_t denote wind speed data observations of each month t . The collection of random variables $X = \{X_t, t = 0, 1, \dots\}$ is a random process in discrete time, while the monthly wind speed $\{X_t, t \geq 0\}$ have a continuous range and continuous state space. To enumerate all possible system state, an appropriate state description on the wind speed data is to classify the distribution as a Gaussian random walk. The random walk in a discrete case takes only 2 or 3 values for example ± 1 or $-1, 0, +1$. They constitute the basis of binomial or trinomial trees which can be used to construct discrete random processes in computers.

The defining condition of the random walk in this study is

$$S_n = S_0 + \sum_{j=1}^n X_j \quad (1)$$

Where S_0 and X_j are independent and identically distributed random variables each taking either the value -1 with probability q or the value $+1$ with probability p or the value 0 with probability r . S_n in (1) is the distribution of sums of a sufficient number of random variables and the main interest is determining the distribution of S_n for finite n given the assumptions about the distribution of the X_j . If $S_0 = 0$, then (1) becomes

$$S_n = \sum_{j=1}^n X_j \quad (2)$$

The sequence of values X_j is defined as a trinomial process with

$$X_j = \begin{cases} +1 & \text{with probability } p \\ -1 & \text{with probability } q \\ 0 & \text{with probability } r \end{cases}$$

The system can be in one of the enumerable sequence of states conventionally denoted as S_0, S_1, \dots, S_n , where the sequence $(S_n, n \in \mathbb{N})$ is called a random walk (Tomasz et al, 1999; Berger, 1993). If S_n take different values for different states, then the conditional distribution of S_n given $S_{n-1}, S_{n-2}, \dots, S_0$ can be written as

$$P\{S_n = j \mid S_{n-1} = i, \dots, S_0 = 0\} \quad (3)$$

And if the Markovian property is satisfied is always equal to

$$P\{S_n = j \mid S_{n-1} = i\} = p_{ij} \quad (4)$$

for all $i, j \in S$ and $n \geq 0$

The matrix $P = \{p_{ij}, i, j \in S\}$ is the transition probability matrix of the Markov chain. The matrix P is called a homogeneous transition or stochastic matrix and satisfy

$$p_{ij} \geq 0, i, j \in S \quad \text{and} \quad (5)$$

$$\sum_{j \in I} p_{ij} = 1 \quad i \in S \quad (6)$$

Transient analysis

The fundamental relationship specifying the n step transition probabilities of lower order for all $n, m = 0, 1, \dots$ as

$$p_{ij}^{(n+m)} = \sum_{k \in T} p_{ik}^{(n)} p_{kj}^{(m)} \quad i, j \in S \quad (7)$$

Equation (7) is called the Chapman-Kolmogorov equations.

Equation (7) shows that the probability of going from i to j in $n + m$ steps is obtained by summing the probabilities of the mutually exclusive events of going first from state i to some state k in n steps and then going from state k to state j in m steps. In particular, we have for any $n = 1, 2, \dots$

$$p_{ij}^{(n+1)} = \sum_{k \in T} p_{ik}^{(n)} p_{kj} \quad i, j \in S. \quad (8)$$

A good deal of the practical importance of Markov chain theory attaches to the fact that the states can be classified in a very distinctive manner according to certain basic properties of the system (Bailey, 1984). Markov chains are classified in a manner that describes their sample path behaviour and limiting probabilities. If every state in a Markov chain can be reached from every other state, the chain is said to be irreducible. All the states of an irreducible chain must form a closed set and no other subset can be closed. The closed set C must also satisfy all the conditions of a Markov chain and hence may be studied

independently. Hence the n step transition probabilities $p_{ij}^{(n)}$ can be recursively computed from the one-step transition

probabilities p_{ij} . The relation can be defined as

$$p_{ij}^{(n)} = \sum_k p_{ik}^{(n-1)} p_{kj} \quad (9)$$

Absolute and transition probabilities

The value assumed by S_n is called the state of the process at time n . The absolute probability of the outcome at the n th trial is defined as

$$P\{S_n = j\} = p_j^{(n)} \quad (10)$$

So that the initial distribution is given by $p_i^{(0)}$ where

$$p_i^{(0)} = P\{S_0 = i\} \quad (11)$$

But

$$\begin{aligned} p_j^{(n)} &= P\{S_n = j\} \\ &= \sum_k P\{S_n = j \mid S_0 = i\} P\{S_0 = i\} \\ &= \sum_k p_{ij}^{(n)} p_i^{(0)} \end{aligned}$$

Therefore

$$p_j^{(n)} = \sum_k p_i^{(0)} p_{ij}^{(n)} \quad (12)$$

Equation (12) is an ideal procedure in studying the behaviour of the system when it is at equilibrium. It shows that the behaviour of the system can be studied over a short period of

time by the repeated application of transition matrix P_{ij} to any initial distribution vector.

Stationary analysis

After a sufficient long period of time the system settles down to a condition of statistical equilibrium in which the state occupation probabilities are independent of the initial conditions.

The properties of stationarity mean that the strong law of large numbers and central limit theorems are available in an asymptotic analysis, whereas working with deterministic changes will produce a non-stationary process. Ergodicity defines the limiting values or steady state probabilities such that the actual limiting distribution, if it exists, can of course be determined quite easily. The equation for the steady state distribution can be obtained as

$$\lim_{n \rightarrow \infty} P\{S_n = j | S_0 = i\} = \lim_{n \rightarrow \infty} P\{S_n = j\} = \pi_j \quad (13)$$

If a Markov chain is ergodic, then the limiting distribution is stationary, and this is the only stationary distribution, and it is obtained by solving (13). The steady state distribution in (13) represents many replications of the same set of equations and can be written as

$$\pi \rho = \pi \quad (14)$$

It turns out that this set of linear equation in (14) is a dependent set and possesses an infinite number of solutions. To qualify as

a probability distribution, it requires that the π_i sum to 1. Such that

$$\sum_{all i} \pi_i = 1 \quad (15)$$

(15) is called the normalizing equation. The usual practice is first to obtain an un-normalized solution by manipulating (15) to

express all of the π_i in terms of one of them; then to use the normalizing equation to fix the value of the last one, and finally to substitute the value into the expressions for the others.

The existence of ergodicity implied, that all states are finite, a periodic and irreducible. Using (14), the ρ is the unique probability vector satisfying

$$\pi(\rho - \lambda I) = 0 \quad (16)$$

The eigen values of ρ , that is, the roots of the characteristics equation

$$(\rho - \lambda I) = 0 \quad (17)$$

are all distinct.

Results

The monthly wind speed data from 1980-2008 for Abeokuta was abstracted from the Ministry of Aviation, Metrological Department, Federal Secretariat Abeokuta, South-Western , Nigeria is used in this study. Our data set resulted from the classification of the wind speed data as a trinomial distribution and is as shown in Figure 1.

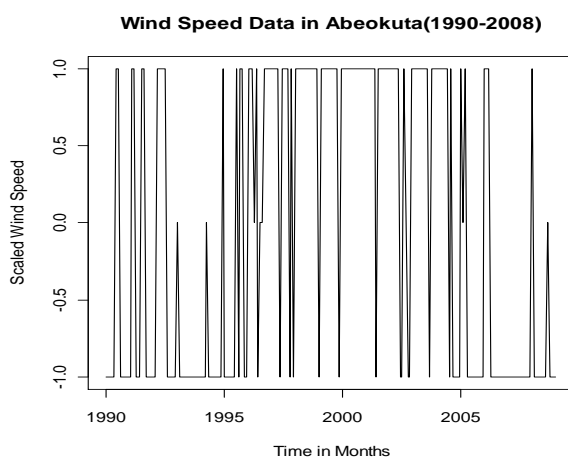


Fig 1: Realization of the Abeokuta Wind Speed Data

The initial state distribution is as shown in Table 1,

Table 1: The Initial State Distribution

$$P^{(0)} = [0.43, 0.34, 0.23]$$

While the transition probabilities derived from the use of Chapman-Kolmogorov equations are in Table 2

Table 2: Transition Matrix

$$p_{ij} = \begin{bmatrix} 0.70 & 0.06 & 0.24 \\ 0.14 & 0.72 & 0.14 \\ 0.30 & 0.30 & 0.40 \end{bmatrix}$$

Each row of this matrix P is a probability vector and is a precise statement of probability for change in the wind speed data. Since P is irreducible and acyclic, there exists some positive integer n such that every element of $P^{(n)} \geq 0$, that is, $P_{ij}^{(n)} > 0$ for all $i, j \in I$.

The absolute states probabilities for time n , is given by the vector of initial state probabilities and the n^{th} order homogeneity and is as shown in Table 3.

Table 3: Absolute Probability Distribution

$$P^{(0)} = [0.43 \ 0.34 \ 0.23]$$

N	$p_1^{(n)}$	$p_2^{(n)}$	$p_3^{(n)}$
1	0.42	0.34	0.24
2	0.41	0.35	0.24
3	0.41	0.35	0.24
4	0.41	0.35	0.24
5	0.41	0.35	0.24
6	0.41	0.35	0.24
7	0.41	0.35	0.24
8	0.41	0.35	0.24
9	0.41	0.35	0.24
10	0.41	0.35	0.24

Then by Perron-Frobenius Theorem, there exists a unique vector $\pi > 0$, with the stationary or equilibrium distribution as in Table 4.

Table 4: Limiting Distribution

0.41	0.35	0.24
0.41	0.35	0.24
0.41	0.35	0.24

Discussions

The system is ergodic and the ergodicity is arrived at, when the state transition probability is equal to the initial probability. The real power of the ergodicity is that it enables us to estimate the probability vector π (the stationary/equilibrium, and also limiting probability vector), and expectations with respect to π , from time averages over a single realization of the chain. The theoretical contribution of these results is that an interesting class of stationary processes is identified for the wind speed data in Abeokuta. Instead of writing the time -dependent equations, then proving the existence of limits, and finally solving the limiting equations, it is easier to consider the steady state directly and use its properties in the analysis.

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