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# State-dependent model for the analysis of inflationary rates

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**ABSTRACT**

In this study, an extension of the class of state-dependent model (SDM) for which optimal forecasts may be computed using a recursive examination procedure referred to as the Kalman filter is developed for the analysis of Inflationary rates in Nigeria. The SDM formulation yields a practical means of estimation for the complex time varying dynamical process and provided a generic flexible framework for inflationary rate modelling and inference. A straight forward implementation was achieved in the study by the use of R software package.



## Introduction

Inflation is marked by an increase in the general level of prices or a decrease in the value of money. It is highly affected by interrelated economic, social, political and even psychological factors. These factors interact with each other in a complicated manner. It is generally very difficult to forecast movements of inflation, especially when the relationships among the variables are not the same in the forecast period as in the historical period due to structural change. Structural change can be allowed for by ad hoc changes in the estimate of the model, or by including an equation in the model that explicitly shows the change in the relationship.

Inflation rate has enormous consequences on the economy and by extension on the investors. Inflationary rates are inherently noisy, nonstationary and deterministically chaotic. These characteristics suggest that inflation rates are highly nonlinear and there is no complete information that could be obtained from past behaviours of inflationary rates to fully capture the dependency between the future rates and that of the past. Recently, there is a growing interest in non-linear models combined with greater computational facility for describing data where the variance changes through time.

A class of nonlinear time series models called state-dependent models (SDM) was developed by Priestley (1980, 1982). This broad class includes the linear autoregressive moving average (ARMA) models (Box and Jenkins, 1976), bilnear models (Granger and Andersen, 1978), exponential autoregressive models (Ozaki, 1980; Haggin and Ozaki, 1981; Ozaki, 1982), and threshold autoregressive models (Tong and Lim, 1980; Ozaki, 1981; Tong, 1982). Cartwright and Newbold (1983) have extended the state-dependent models developed by Priestley to deal with the problem of outlying observations. In Cartwright (1984), the SDM by Priestley was extended by permitting the residual variance to vary through time according to a moving average scheme. The state-dependent models as discussed in Priestley (1980, 1982) and Cartwright and Newbold (1993) are structural time series models.

A structural time series model, Harvey (1989) sets out to capture the salient features of a time series data and can be written as state space model. State space models, Durbin and Koopman (2000, 2001, and 2002) and Chatfield (2004) are a widely used tool in time series analysis to deal with processes which gradually change over time. The state space model represents a physical system as $n$ first order coupled differential equations and is a fundamental concept in modern control theory. Kalman (1960) estimated coefficient of a non-linear differential equation using an optimal sequential estimation techniques often referred to as Kalman filter.

Kalman's derivation took place within the context of state space models whose core is the recursive least squares estimation. Within the state space notation, the Kalman filter derivation rests on the assumption of normality of the initial state vector, and as well as the disturbances of the system. The state of a system is defined to be a minimum set of information from the present and past such that the future behaviour of the system can be completely described by the knowledge of the present state and the future input. The State space representation is based on the Markov property, which implies that given the present state, the future of a system is independent of its past.

In this article, we introduce a class of structural models in order to capture the salient feature of inflation in Nigeria. The structural model is reduced as an autoregressive moving average (ARMA) process. Akaike (1974) was the first to demonstrate that structural models can be reduced to ARMA (p,q) model. The relationship between the structural model and its reduced forms gives considerable insight into the potential effectiveness of the different ARMA models (Harvey, 1989; Chatfield, 2004). ARMA models, typically are parsimonious model (Box and Jenkins, 1976, Box et al, 1994) and is based on the premise that the autocorrelation function (ACF) and the related statistics can be accurately estimated and are stable over time. By adopting Box-Jenkins ARMA approach to time series analysis, model identification, parameter estimation and diagnostic checks are feasible for the analysis of Nigerian inflationary rates.

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**The Data**

The data are secondary data, on annual inflationary rate from 1961 to 2010 published by the Central Bank of Nigeria Statistical Bulletin.

**Materials and Methods**

The Nigerian inflationary rate was examined by using a basic structural time series modelling approach. The key to handling structural time series models is the state space form with the state of the system representing the various unobserved components such as trends and seasonal. The prime objective of state space model is to estimate the signal in the presence of noise. The state space approach to time series model focused attention on the state vector of a system. The measurement vector represents noisy observations that are related to the state vector. It is assumed that the noise contaminates the signal in an additive manner so that the actual observations are given by the following measurement equation

$$Z_t = F_t\theta_t + v_t \qquad (1)$$

$v_t \sim N(0,\sigma_v^2)$

where $Z_t\ (t = 1,2,..............N)$ is the observed noise corrupted time series, $F_t$ is assumed to be an ($(n\times 1)$ known column vector, $\theta_t$ and $\mathrm{v}_t$ are the time series representing an $(n\times 1)$ state vector and the observation noise respectively. The vector $\theta_t$ may not be directly observable. It is often assumed as a vector difference equation or state equation represented as

$$\theta_t = H_t\theta_{t-1} + w_t \qquad (2)$$

where the $(n\times n)$ matrix $H_t$ is assumed known, and $w_t$ denotes an $(n\times 1)$ vector of deviations such that $w_t^T = (w_{1,t}, w_{2,t}, ..........w_{n,t})$.

The pair of equations in (1) and (2) constitute the general form of the state space model. The errors in the measurement (or observation) equation in (1) and state (or transition) equation in (2) are generally assumed to be serially uncorrelated and also to be uncorrelated with each other at all time periods. Further, the measurement error $v_t$ is assumed as an independent random Gaussian process while $w_t$ is a white Gaussian noise with zero mean and variance matrix $\sigma_w^2$. Additionally $v_t$ and $w_t$ are assumed to be orthogonal at all pairs of time.

**Estimation of the Structural Parameters**

The estimation of the parameters and the state vectors efficiently can be calculated by the Kalman filter, which is an important general method of handling state-space models. Essentially Kalman filtering is a method of signal processing, which provides optimal estimates of the current state of a dynamic system (Chatfield, 2004). Kalman (1960) defined filtering as any mathematical operation which uses past data or measurements on a given dynamical system to make more accurate statement about present, future or past variables in that system. For the linear Gaussian estimation problem, the required probability density function (pdf) remains Gaussian at every iteration of the filter, and the Kalman filter, propagate and update the mean and covariance of the distribution (Chatfield, 2004).

The Kalman filter recursively evaluates the estimator of the state vector conditional on the past observations up to time $(t-1)$. By considering Equation (2), where $w_t$ is still unknown at time $t-1$, the obvious estimator for $\theta_t$ is given as

$$\hat{\theta}_{t/t-1} = H_t\hat{\theta}_{t-1} \qquad (3)$$

with variance covariance matrix

$$P_{t/t-1} = H_t P_{t-1} H_t^T + W_t \qquad (4)$$

Equations (3) and (4) are the prediction equations. Equation (4) follows from standard results on variance -covariance matrices for vector random variables (Chatfield, 2004; Stark and Woods, 1986). When new observation has been observed, the estimator for $\theta_t$ can be modified to take account of this extra information. At time $(t-1)$, the best forecast of $Z_t$ is given as $F_t\hat{\theta}_{t/t-1}$ so that the prediction error is given by

$$\varepsilon_t = Z_t - F_t\hat{\theta}_{t/t-1} \qquad (5)$$

$\varepsilon_t$ in (5) is called the prediction error. This quantity can be used to update the estimate of $\theta_t$ and of its variance-covariance matrix and the best way to do this is by means of the following equation

$$\hat{\theta}_t = \hat{\theta}_{t/t-1} + K_t\varepsilon_t \qquad (6)$$

and

$$P_t = P_{t/t-1} - K_t F_t P_{t/t-1} \qquad (7)$$

where

$$K_t = P_{t/t-1}F_t[F_t P_{t/t-1} F_t^T + \sigma_v^2]^{-1} \qquad (8)$$

$K_t$ in (8) is called the Kalman gain matrix and is a vector of size $(m\times 1)$. Equation (6) and (7) constitute the second updating stage of the Kalman filter and are called the updating equations.

**Results**

The study applied the model to annual inflationary rate from 1961 to 2010 published by the Central Bank of Nigeria Statistical Bulletin. The plot of the annual inflationary rate is as in Figure 1 and was achieved through the use of R software. The first step in state space modelling is to find an optimal autoregressive (AR) model that fits the data. The selection of a tentative model is frequently accomplished by matching estimated autocorrelations with the theoretical autocorrelation and partial autocorrelation functions. Table 1 is the ACF and the PACF of the annual inflation rate and the correlogram is as in Figures 2 and 3. Based on the ACF and PACF of the annual inflation rate in Table 1, one may suggest an AR (1). The R package use the Akaike Information Criterion (AIC) to provide an optimal or best fit for the autoregressive model. The value of the AIC for the annual inflationary rate is as in Table 2. The AIC is minimum at p=1. Hence the optimal AR order p is chosen to be one.

The Gauss Markov signal model generated from the annual inflationary data using ARMA models is

$$\theta_t = 0.55\theta_{t-1} + w_t \qquad t \geq 0$$

with mean equal to zero and $\sigma_w^2 = 0.188$ The Kalman gain $K_t$ as defined in (8) is $K_t = 0.302$. The prediction error variance as defined in (5) is $\varepsilon_t = 0.434$. The Kalman filter is asymptotically given by

$$\hat{\theta}_{t/t} = 0.384\theta_{t-1/t-1} + 0.434Z_t$$

**Discussion**

In this paper, inflationary rate in Nigeria for a period of 50 years was analysed using the State-Dependent model as proposed by Priestley (1980, 1982) and Cartwright and Newbold (1983). The SDM estimated coefficient of a non-linear differential equation using an optimal sequential estimation technique often referred to as Kalman filter. Kalman filter degenerates into simpler algorithm that is identical with the conventional time series method of forecasting. The importance of the Kalman algorithm is based on the fact that it constitutes the main procedure of estimating dynamic systems represented in state space form.

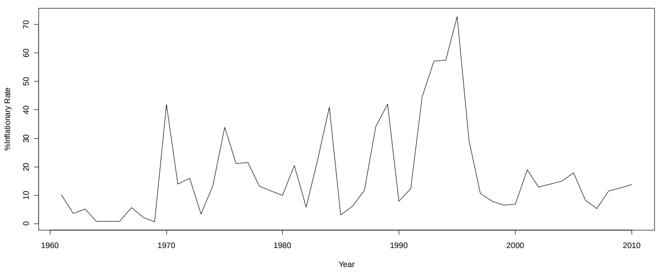


**Figure 1: Time Plot of the Inflationary rate**

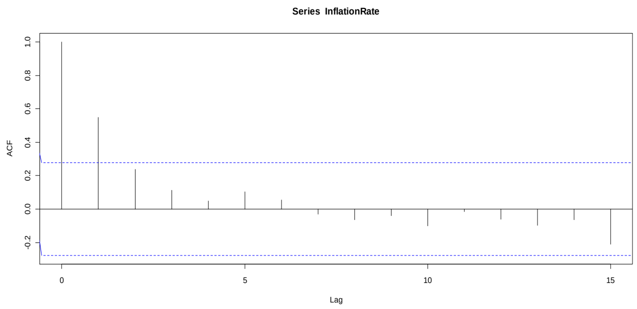


**Figure 2 : ACF for the Inflationary Rate**

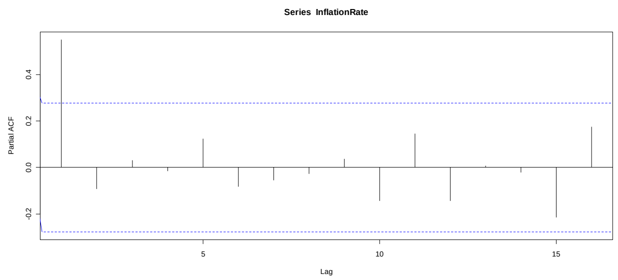


**Figure 3 : P ACF for the Inflationary Rate**

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