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Modeling and design of state feedback with integral controller for TRMS (Twin Rotor MIMO System)

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ABSTRACT

This paper presents the modeling and design of State Feedback with Integral controller on Twin Rotor MIMO System. In this two SISO systems are considered (i) main rotor (ii) tail rotor. Main Rotor is used to control the pitch axis and the Tail Rotor is used to control the yaw axis. By using the state feedback the both the axis are controlled as desire. The main aim of this paper is to compare the control performance between State Feedback and PID controller that is designed with Relay feedback method. The entire work has been carried out in MATLAB environment and the performance is compared in the terms of settling time, peak overshoot, offset and overall performance. The regulatory response of the system is also presented.

Introduction

The twin rotor multi-input multi-output system (TRMS) is an aero-dynamical system similar to a helicopter as shown in Figure 1. It consists of a beam pivoted on its base in such a way that it can rotate freely both in its horizontal and vertical planes. This TRMS system has two degrees of freedom (DOF). Either the horizontal or the vertical degree of freedom can be restricted to 1 degree of freedom using the screws. At both end of a beam, there are two propellers driven by DC motors. The aerodynamic force is controlled by varying the speed of the motors. Therefore, the control inputs are the supply voltages of the DC motors. The TRMS system has main and tail rotors for generating vertical and horizontal propeller thrust. The main rotor produces a lifting force allowing the beam to rise vertically making a rotation around the pitch axis. While, the tail rotor is used to make the beam turn left or right around the yaw axis [1].

The state of the beam is described by four process variables. Horizontal and vertical angles measured by optical encoders fitted at the pivot, and another two additional state variables are the angular velocities of the rotors, measured by tacho-generators coupled to the driving DC motors.

Since Astrom and Hagglund (1984) introduced the auto tuning method, which used the relay feedback test, many variations have been proposed for auto tuning of PID controllers (Astrom and Hagglund, 1995; Hang et al., 2002; Yu, 2006). Several methods such as a saturation relay (Yu, 2006), relay with a P control preload (Tan et al., 2006) and a two level relay (Sung et al., 1995) were introduced to obtain more accurate ultimate information of the process by suppressing the effects of the high order harmonic terms. To obtain a Nyquist point other than the critical point, a relay with hysteresis or a dynamic element such as time delay has been used (Astrom and Hagglund, 1995; Kim, 1995; Tan et al., 1996; Chiang and Yu, 1993). A biased relay has been used to obtain the process steady state gain as well as the ultimate information (Shen et al., 1996a) from only one relay test[2].

For a certain class of process plants, the so-called \auto tuning" procedure for the automatic tuning of PID controllers can be used. Such a procedure is based on the idea of using an on/off controller (called a relay controller) whose dynamic behavior resembles to that shown in Figure 2(a). Starting from its nominal bias value denoted as 0 in the Figure) the control action is increased by an amount denoted by h and later on decreased until a value denoted by –h[2][6][7][8].

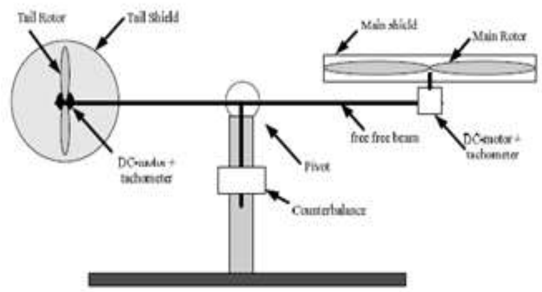

Fig.1 TRMS Model

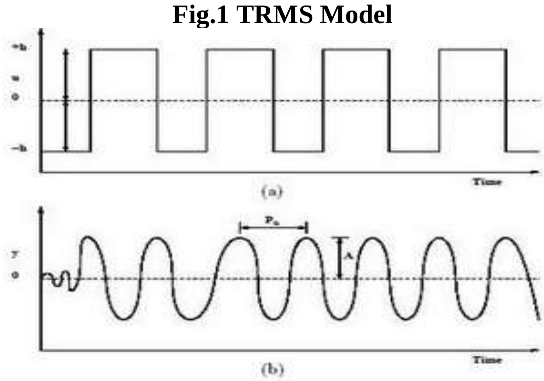

Fig.2 Waveforms of relay output in closed loop

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The closed-loop response of the plant, subject to the above described actions of the relay controller, will be similar to that depicted in Figure2 (b).Initially, the plant oscillates without a definite pattern around the nominal output value (denoted as 0 in the Figure) until a definite and repeated output response can be easily identified[3]. When we reach this closed-loop plant response pattern the oscillation period (Pu) and the amplitude (A) of the plant response can be measured and used for PID controller tuning. In fact, the ultimate gain can be computed as:

$$K(u) = (4 * a)/(\pi * M)$$

Having determined the ultimate gain K_{cu} and the oscillation period P_u the PID controller tuning parameters can be obtained from the following table:

Table I

Zeigler Nicholas Tuning Rules

	K_c	T_I	T_D
P	0.5 K_{CU}		
PI	0.45 K_{CU}	P_U/1.2	
PID	0.6 K_{CU}	P_U/2	P_U/8

System Description

The block diagram of Twin Rotor MIMO System (TRMS) can be shown in Fig. [3], it contains two main features:

a) Nonlinear, there are two non-linear inputs which are DC-motors.

b) Cross-coupling, Angular momentum and reaction turning moment are the two main effects from cross-coupling.

The transfer function of the main rotor and the tail rotor is defined below:

$$G_m(s) = \frac{0.01657s^2 + 0.4194s + 2.454}{s^3 + 1.487s^2 + 4.403s + 5.449}$$

$$G_t(s) = \frac{0.0009881s^2 + 0.03361s + 0.4065}{s^3 + 1.345s^2 + 0.4568s + 0.3828}$$

Where $G_m(s)$ represents the transfer function of main rotor and $G_t(s)$ represents the transfer function of tail rotor. These transfer functions will be utilized throughout this work.

Design of PID Controller

In order to evaluate the features of different algorithms, and to check the proposed extensions of the basic algorithm, a computer simulation was performed. It allows simple and quick testing of the algorithm behaviour for a wide class of model processes. The MATLAB/SIMULINK package was chosen as the programming environment for the computer simulation.

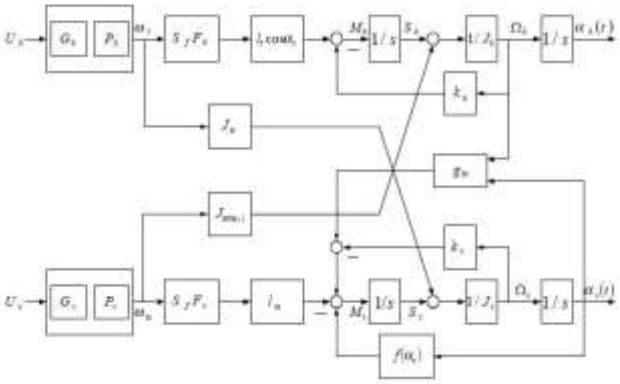

Figure 3.Block Diagram of TRMS System

Experimental Reading

The following section shows the simulation and experimental results for SISO system. The following table describes the PID parameters for SISO system using Zeigler Nicholas tuning rule.

Table II Classic Ziegler-Nicholas Tuning Rule

Tuning Rule	K_c	K_i	K_d
Ziegler-Nicholas	0.6K(u)	0.5T (u)	0.125T (u)
SISO pitch axis	1.2	1	5
SISO yaw axis	1.2	1.2	5

SISO Simulation Results

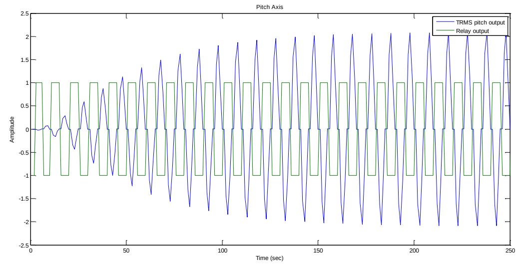

Figure 4: Response of relay feedback test simulation for pitch axis

Figure 4 shows the graph of horizontal (pitch) movement by applying relay to its input. The maximum amplitude of the relay is A= 1.8.The ultimate period is P_u=6 sec

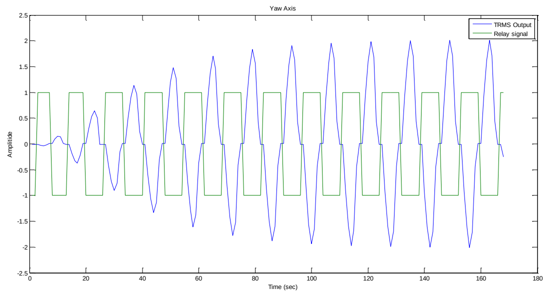

Figure 5: Response of relay feedback test simulation for yaw axis

Figure 5 shows the graph of horizontal (pitch) movement by applying relay to its input. The maximum amplitude of the relay is A= 2.The ultimate period is P_u=5 sec

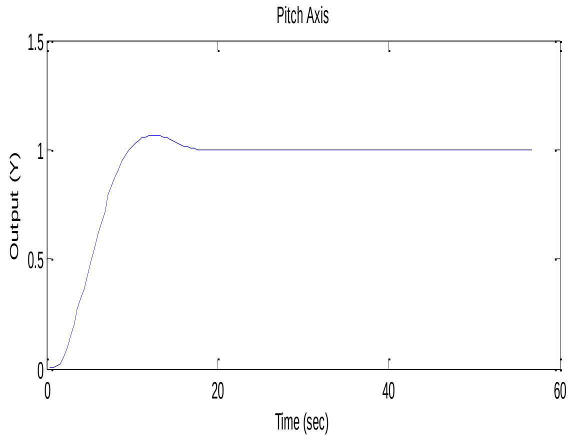

Figure 6: Simulation closed-loop response with PID controller tuned using Ziegler Nicholas tuning rule for pitch axis

Figure 6 represents the horizontal movement by applying directly multi-step input wit PID Tuning. The parameters of PID are $K_{P=}$1.2, Ki=1, Kd=5

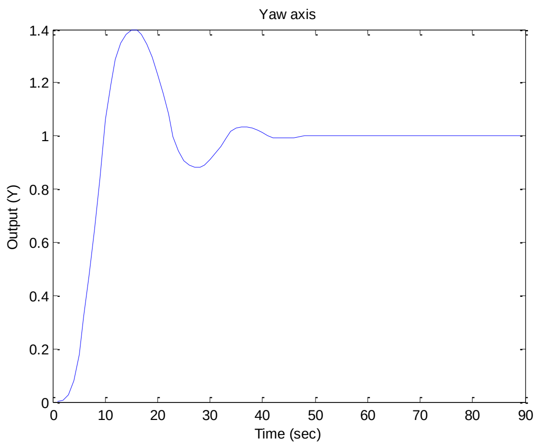

Figure 7: Simulation closed-loop response with PID controller tuned using Ziegler Nicholas tuning rule for yaw axis

Figure 7 represents the vertical movement by applying directly multi-step input with PID Tuning. The parameters of PID are K_p=1.2,K_i=1.2, Kd=5.

State Feedback with Integral Control

The concept of feed backing all the state variables back to the input of the system through a suitable feedback matrix in the control strategy is known as state variable feedback control technique as shown in fig. 3. Using this approach, the closed loop Eigen values of the system will be specified. Thus the aim is to design a feedback controller that will move some or all of the open loop poles of the measured system to the desired closed loop pole locations as specified. Hence this approach is also known as pole placement control design. The necessary and sufficient condition that the closed loop poles can be placed at any arbitrary location in s plane is that the system be completely state controllable [16].

In the state feedback two types of examples have been specified in Ogata.One is the Type 1 Servo System when the plant has Integrator and the second one is the Type 1 Servo System when the plant has no Integrator. In this we shall consider the second case. The diagram of it is shown below:

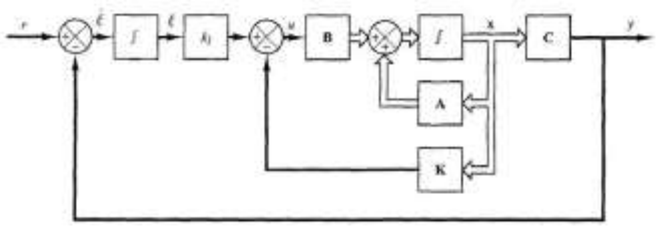

Figure 8 Type 1 Servo System

If the plant has no integrator, the basic principle of design of a type 1 servo system is to insert the Integrator in the feed forward path. From the figure we obtain:

$$\dot{x} = Ax + Bu \qquad (1)$$

$$y = Cx \qquad (2)$$

$$u = -Kx + k_1\xi \qquad (3)$$

$$\dot{\xi} = r - y = r - Cx \qquad (4)$$

Now since the system is completely controllable, if we apply the step input to the system then the dynamics would be given by:

$$\begin{bmatrix} \dot{x}(t) \\ \dot{\xi}(t) \end{bmatrix} = \begin{bmatrix} A & 0 \\ -C & 0 \end{bmatrix} \begin{bmatrix} x(t) \\ \xi(t) \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} u(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} r(t) \qquad (5)$$

Under steady state the equation is given by:

$$\begin{bmatrix} \dot{x}(\infty) \\ \dot{\xi}(\infty) \end{bmatrix} = \begin{bmatrix} A & 0 \\ -C & 0 \end{bmatrix} \begin{bmatrix} x(\infty) \\ \xi(\infty) \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} u(\infty) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} r(\infty) \qquad (6)$$

Now for t>0 we have r(∞) =r(t)=r and Subtracting the equations *(6)* from *(5)* we have :

$$\begin{bmatrix} \dot{x}(t) \\ \dot{\xi}(t) \end{bmatrix} - \begin{bmatrix} \dot{x}(\infty) \\ \dot{\xi}(\infty) \end{bmatrix} = \begin{bmatrix} A & 0 \\ -C & 0 \end{bmatrix} \begin{bmatrix} x(t) \\ \xi(t) \end{bmatrix} - \begin{bmatrix} x(\infty) \\ \xi(\infty) \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} u(t) - u(\infty) \qquad (7)$$

Thus the above equation can be written as

$$\begin{bmatrix} \dot{x}_e(t) \\ \dot{\xi}_e(t) \end{bmatrix} = \begin{bmatrix} A & 0 \\ -C & 0 \end{bmatrix} \begin{bmatrix} x_e(t) \\ \xi_e(t) \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} u_e(t) \qquad (8)$$

The above equation can be expressed as

$$\dot{e} = \tilde{A}e + \tilde{B}u_e \qquad (9)$$

Where

$$\tilde{A} = \begin{bmatrix} A & 0 \\ -C & 0 \end{bmatrix}; \quad \tilde{B} = \begin{bmatrix} B \\ 0 \end{bmatrix} \qquad (10)$$

$$u_e = -Kx_e + k_1\xi_e \qquad (11)$$

So,

$$u_e = -\tilde{K}e \qquad (12)$$

Where

$$\tilde{K} = [K| - k_i] \qquad (13)$$

By substituting the value of equation (12) in equation (9) we have:

$$\dot{e} = (\tilde{A} - \tilde{B}\tilde{K})\, e \qquad (14)$$

If the desired Eigen values are specified the state feedback matrix K and the integral constant k_i can be obtained.

Once the feedback gain matrix K and the integral gain constant k_i is determined the step response for this system can be obtained by substituting the equation (3) in (1) :

$$\begin{bmatrix} \dot{x}(t) \\ \dot{\xi}(t) \end{bmatrix} = \begin{bmatrix} A - BK & Bk_i \\ -C & 0 \end{bmatrix} \begin{bmatrix} x \\ \xi \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} r(t) \qquad (15)$$

Design of State Feedback with Integral Control Technique

The closed loop transfer function of the main rotor system (pitch axis) is given by:

$$G_m(s) = \frac{0.01657s^2 + 0.4194s + 2.454}{s^3 + 1.487s^2 + 4.403s + 5.449}$$

Assigning the state variable we get the completely controllable SISO linear time invariant system in the form of A, B, C, D matrix.

$$A = \begin{bmatrix} -1.4870 & -4.4035 & -5.449 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$ C= [0.0017 0.4149 2.454] D= [0]

As we see this system is same as the type 1 servo system in which the integrator was placed in the feed forward path.

Let the desired Eigen values for the system be μ=-2+j√3, -2-j√3,-3,-3.Then by making the use of Ackermann's formulae the value of $\tilde{K}$ will be given by:

$\tilde{K}$= [8.5 35.55 61.75|-25.67]

The value of K and k_i will be:

K= [8.5 35.55 61.75] and k_i=25.67

Thus substituting the values of K, ki, A, B and C in equation (15) we get the new variables as AA, BB, CC and DD. By applying step input to the system the following graph is obtained:

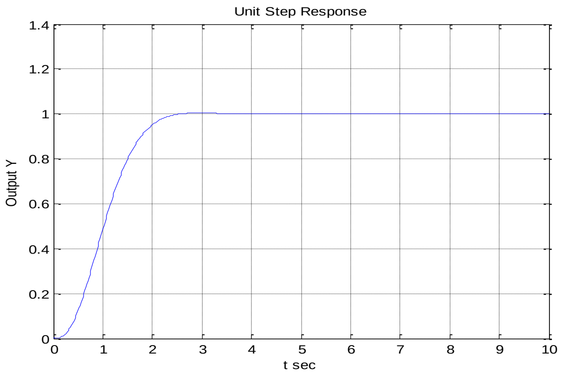

Figure 9: Simulation of closed-loop response with State feedback Integral controller for pitch axis.

As shown from the above graph the response of the system is smooth and fast than PID.Moreover there is no peak overshoot in the system and the settling time is 2.5 sec which is better than PID controller.

The same procedure will be done for the tail rotor system (yaw axis).

The closed loop transfer function of the main rotor system (pitch axis) is given by:

$$G_t(s) = \frac{0.0009881s^2 + 0.03361s + 0.4065}{s^3 + 1.345s^2 + 0.4568s + 0.3828}$$

Assigning the state variable we get the completely controllable SISO linear time invariant system in the form of A, B, C, D matrix.

$$A = \begin{bmatrix} -1.3450 & -0.4568 & -0.3828 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

$B = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ C= [0.0010 0.0336 0.4065] D= [0]

Let the desired Eigen values for the system be μ=-2+j√3, -2-j√3,-3,-3.Then by making the use of Ackermann's formulae the value of $\tilde{K}$ will be given by:

$\tilde{K}$= [8.6 39 72|-154]

The value of K and k_i will be:

K= [8.6 39 61.72] and k_i=154

Thus substituting the values of K, ki, A, B and C in equation (15) we get the new variables as AA, BB, CC and DD. By applying step input to the system the following graph is obtained:

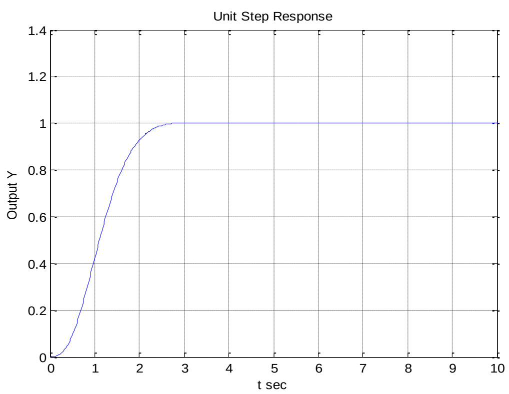

Figure 10: Simulation of closed-loop response with State feedback Integral controller for yaw axis.

As shown from the above graph the response of the system is smooth and fast than PID. Moreover there is no peak overshoot in the system and the settling time is 2.5 sec which is better than PID controller.

In this the graph for both the axis are approximately same but it is far better than PID controller response.

When the reference input is not applied to the system then that system is said to be regulatory system and the nature of the graph is shown below.

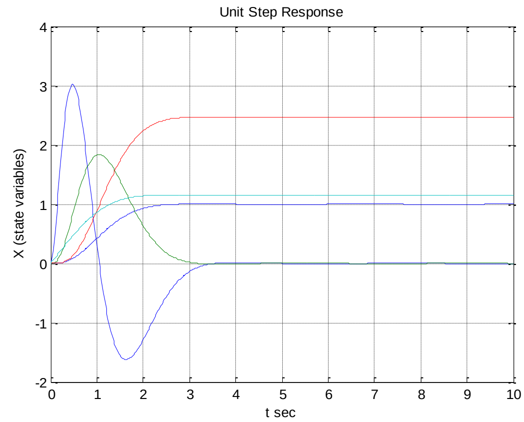

Figure 11: Regulatory Response for yaw axis.

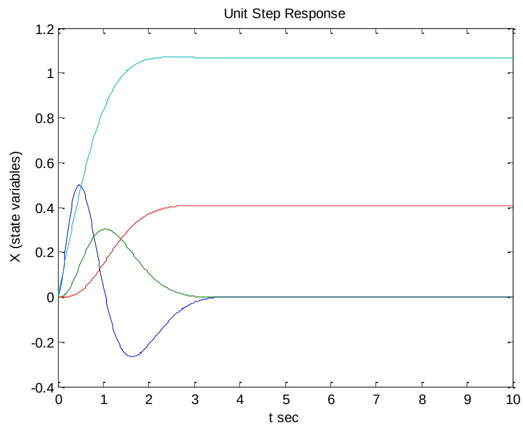

Figure 12: Regulatory Response for pitch axis.

The following table shows the comparison results of State Feedback Integral Control with that of PID controller using Relay feedback Method.

Table III: Performance comparison of different tuning rules

Performance Parameters	Classic Zeigler-Nicholas		State feedback with Integral	
	Pitch	Yaw	Pitch	Yaw
Settling Time (sec)	20	50	2.5	2.5
Peak Overshoot	1.06	1.4	NIL	NIL
Offset Error (%)	NIL	NIL	NIL	NIL
Overall performance	Poor	Poor	Excellent	Excellent

Conclusion

It has been shown that state feedback with Integral control can cop up with non linear characteristics at all operating points. Conversely, PID controller designed with Relay Feedback method using Z-N tuning rules are not able to settle at predefined time periods without overshoots. Hence the State Feedback with Integral control provides better response with less settling time and minimal overshoots.

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