Available online at www.elixirpublishers.com (Elixir International Journal)

Network Engineering

Elixir Network Engg. 76 (2014) 28182-28188

ISSN: 2229-712X

The Unsteady State Mathematical Model for Chlorine Concentration Decay for Axisymmetric Flow in a Pipe Line Network

V.N Kala[1], Jaipal[2] and Rakesh Chandra Bhadula[3]

[1]Department of Applied Science, G.B Pant Engg. College Pauri Garhwal, India.

[2]Department of Mathematics , D.B.S.(P.G.) College Dehradun, India.

[3]Uttarakhand Technical University Dehradun, India.

ARTICLE INFO

Article history:

Received: 17 December 2013;

Received in revised form:

25 October 2014;

Accepted: 31 October 2014;

Keywords

Hankel transform,

Peclet number,

Unsteady state,

Axisymmetric flow.

ABSTRACT

The unsteady state mass conservation equation for chlorine concentration decay in drinking water in a pipe line network for axisymmetric flow is solved analytically with the help of Hankel transform. The numerical values and graphical representations are shown by using MATLAB for various values of chlorine consumption rate, diffusivity, time and fluid velocity etc.

Introduction

Everyone irrespective of their stage of development and their social and economical conditions, has the right to have access to an adequate supply of safe drinking water. One of the major function of WHO is to achieve such goals is the responsibility to propose regulations to make recommendations with respect to international health matters. One of the most popular disinfectant is chlorine which is used to kill pathogenic bacteria during to treatment of drinking water process all over the world. Chlorine also plays an important role in the maintenance of water quality in the distribution system through a residual. The distribution system consists a most important component that is pipeline network. The problem of finding a point of booster chlorination in drinking water pipe line network is of great importance. Clark et al. (1994) showed how chlorine residuals can vary throughout the day at different locations in the distributive systems. Clark et al. (1995) used first order kinetics and rate of chlorine decay in their model. They showed that the fluid velocity and pipe radius affect the propagation of chlorine residual levels, disinfection efficiency and the formation of disinfection by-products. Hoefel *et al* (2005) in micro trial resistant to chlorination has observed both of these in lab studies and in full scale chlorine disinfection Practice for water and Waste-water treatment. Biswass et al. (1993) considered a steady-state model for chlorine concentration decay in pipes. They determine the cup-mixing average chlorine concentration at any location decay in pipes. The transport of chlorine from the bulk flow to the pipe wall (due to concentration decay at wall) was not considered in their model. Wojcicka *et al* (2007) in previous studies have found that indigenous bacteria are related from different environment. Huang *et al* (2011) studied that the influence of chlorination on end toxin activities of secondary sewage effluent and Pure Cultured Gram-negative bacteria was instigated. David and Bryan (1996) developed an adjective transport model by neglecting the contribution of radials as well as axial diffusion terms. Munavali and Mohan (2005) presented a simulation-optimization model for water quality parameter estimation in the distribution system under dynamic state. Osman, and Metin (1999) solved two dimensional convection dispersive equation numerically for various boundary and initial conditions, considering the decay of chlorine in the bulk flow, but they did not consider the transfer of chlorine from bulk flow to the pipe wall. Jaipal and Bhadula (2012) presented two dimensional steady state mathematical model and unsteady state model (2013) that accounts for transport in the axial direction of diffusion and that incorporates chlorine decay within the bulk flow and transport of the chlorine from bulk flow to the pipe wall to predict the chlorine concentration in a drinking water distribution system. Eran *et al* (2011) studied the chlorination and ultraviolet (UV) irradiation of

Tele:

E-mail addresses: rakeshbhadula.14@gmail.com

rotating biological contractor in treating the light-grey water. They examined the ability of chlorine and UV to inactive indicator bacteria and specific Pathogens. Cherchi and Gu (2011) investigated the impact of the cell growth stage on chlorine disinfection efficiency and the impact of the growth stage on chlorination resistance by comparing the inactivation efficiencies of two indicator bacterial strains obtained from various growth phases.

Mathematical Modelling

The unsteady state mass conservation equation for dispersion of chlorine in drinking water in a pipe for axisymmetric flow considering transport of chlorine from bulk flow to the wall can be written as

$$\frac{\partial c}{\partial t}+U\frac{\partial c}{\partial x}=D\frac{\partial^2 c}{\partial x^2}+D(\frac{\partial^2 c}{\partial r^2}+\frac{1}{r}\frac{\partial c}{\partial r})-k_b c-\frac{k_f(c-c_w)}{r_h} \quad (1)$$

Where D is diffusion coefficient, U is initial velocity component along x axes, k_b and k_f are the chlorine decay rate constant for bulk flow(s^{-1}) and mass transfer coefficient (m/s) respectively c_w is the chlorine concentration at wall (kg/m^3) and r_h is the hydraulic radius of the pipe wall.

Assuming that the reaction of chlorine at the pipe wall is of first order with respect to the wall concentration c_w and that it proceeds at the same rate as chlorine is transported to the wall gives the following mass balance equation for the chlorine at the wall.

$$k_f(c-c_w)=k_w c_w \quad (2)$$

Substituting the value of c_w from equation (2) into equation (1).We get

$$\frac{\partial c}{\partial t}+U\frac{\partial c}{\partial x}=D\frac{\partial^2 c}{\partial x^2}+D(\frac{\partial^2 c}{\partial r^2}+\frac{1}{r}\frac{\partial c}{\partial r})-k_b c-\frac{k_f k_w c}{r_h(k_f+k_w)} \quad (3)$$

The initial and boundary conditions are

$$c(x,r,t=0)=0 \quad (4.i)$$

$$c(x=0,r,t)=c_0 \quad (4.ii)$$

Where c_0 is initial concentration

$$\frac{\partial c}{\partial x}=0 \quad as\ x\to\infty, t\geq 0 \quad (4.iii)$$

and wall condition is

$$\frac{\partial c}{\partial r}=0 \quad \text{at} \quad r=r_0 \quad (4.iv)$$

Introducing the following non dimensional quantities are defined by

$$c^*=\frac{c}{c_0},\quad x^*=\frac{x}{L},\quad r^*=\frac{r}{r_0},\quad t^*=\frac{Ut}{L} \quad (5)$$

Where r_0 is the pipe radius (m)

$$\frac{\partial c}{\partial t}+\frac{\partial c}{\partial x}=\frac{1}{p_e}\frac{\partial^2 c}{\partial x^2}+B_0(\frac{\partial^2 c}{\partial r^2}+\frac{1}{r}\frac{\partial c}{\partial r})-B_1 c \quad (6)$$

Where $B_1=\frac{L}{U}\left[k_b+\frac{k_w k_f}{r_h\left(k_w+k_f\right)}\right]$, $p_e=\frac{UL}{D_x}$ and $B_0=\frac{LD}{Ur_0}$

The initial and boundary condition become

$$c(x,r,t)=0, t=0 \quad (7.i)$$

$$c(x,r,t)=1, x=0 \quad (7.ii)$$

$$\frac{\partial c}{\partial r}=0, r=1 \quad (7.iii)$$

$$\frac{\partial c}{\partial x}=0, t\geq 0, x\to\infty \quad (7.iv)$$

Applying finite Hankel transformation (Sneddon,1972) on equation (6) and equation (7.i) to (7.iv),weget

$$\frac{\partial c_H}{\partial t}=\frac{1}{p_e}\frac{\partial^2 c_H}{\partial x^2}-\frac{\partial c_H}{\partial x}-B_0\lambda_n^2 c_H-B_1 c_H \quad (8)$$

$$\frac{\partial c_H}{\partial t}=\frac{1}{p_e}\frac{\partial^2 c_H}{\partial x^2}-\frac{\partial c_H}{\partial x}-\lambda c_H \quad (9)$$

Where $B_0\lambda_n^2+B_1$, λ_n is finite Hankel transformation parameter as determined by the transcendental equation $\frac{d(J_0(\lambda_n))}{dr}=0$, $J_0(\cdot)$ is the zero order Bessel function of the first kind, $c_H(x,\lambda_n,t)$ is the second kind finite Hankel transformation of $c(x,r,t)$ as defined by the following conjugate equation

$$c_H(x,\lambda_n,t)=\int_0^1 rc(x,r,t)J_0(\lambda_n r)dr \quad (10.i)$$

$$c(x,r,t)=2\sum_{n=0}^{\infty}c_H(x,\lambda_n,t)\frac{J_0(\lambda_n r)}{\left|J_0(\lambda_n)\right|^2} \quad (10.ii)$$

The initial and boundary conditions become

$$c_H(x,\lambda_n,t=0)=0 \quad (11.i)$$

$$c_H(x=0,\lambda_n,t)=\frac{J_1(\lambda_n)}{\lambda_n} when \lambda_n\neq 0 \quad (11.ii)$$

$$\frac{\partial c_H}{\partial x}=0, as\ x\to\infty, t\geq 0 \quad (11.ii)$$

Again introducing following transformation

$$c_H(x,\lambda_n,t)=P(x,t)\exp\left[\frac{p_e x}{2}-\left(\frac{p_e}{4}+\lambda\right)t\right] \quad (12)$$

Equation (9) reduced into

$$\frac{\partial P}{\partial t}=\frac{1}{p_e}\frac{\partial^2 P}{\partial X^2} \quad (13)$$

The initial and boundary condition (11.i) to (11.iii) become

$$P(x,t)=0, x\geq 0, t=0 \quad (14.i)$$

$$P(x,t)=\frac{J_1(\lambda_n)}{\lambda_n}\exp\left[\left(\frac{p_e}{4}+\lambda\right)t\right], x=0, t>0 when \lambda_n\neq 0 \quad (14.ii)$$

$$\frac{dP}{dx}=0, t\geq 0, x\to\infty \quad (14.i)$$

Solving equation (12) together with initial boundary conditions (13.i) to (13.iii) by Laplace transformations technique and then putting the value of $P(x,t)$ in equation (11), we get

$$c_H(x,\lambda_n,t)=\frac{1}{2}\frac{J_1(\lambda_n)}{\lambda_n}\left[\exp\left(\frac{p_e+\sqrt{p_e^2+4\lambda p_e}}{2}x\right)erfc\left(\frac{\sqrt{p_e}x+\sqrt{p_e+4\lambda t}}{2\sqrt{t}}\right)\right]$$
$$+\frac{1}{2}\frac{J_1(\lambda_n)}{\lambda_n}\left[\exp\left(\frac{p_e-\sqrt{p_e^2+4\lambda p_e}}{2}x\right)erfc\left(\frac{\sqrt{p_e}x-\sqrt{p_e+4\lambda t}}{2\sqrt{t}}\right)\right] \quad (15)$$

When $\lambda_n\neq 0$

Finally putting equations (15) in equation (10.ii), we get

$$c(x,r,t)=\sum_{n=0}^{\infty}\left[\exp\left(\frac{p_e+\sqrt{p_e^2+4\lambda p_e}}{2}x\right)erfc\left(\frac{\sqrt{p_e}x+\sqrt{p_e+4\lambda t}}{2\sqrt{t}}\right)\right]\frac{J_0(\lambda_n r)}{|J_0(\lambda_n)|^2}\frac{J_1(\lambda_n)}{\lambda_n}$$
$$+\sum_{n=0}^{\infty}\left[\exp\left(\frac{p_e-\sqrt{p_e^2+4\lambda p_e}}{2}x\right)erfc\left(\frac{\sqrt{p_e}x-\sqrt{p_e+4\lambda t}}{2\sqrt{t}}\right)\right]\frac{J_0(\lambda_n r)}{|J_0(\lambda_n)|^2}\frac{J_1(\lambda_n)}{\lambda_n} \quad (16)$$

When $\lambda_n\neq 0$

Results And Discussion

To observe the effect of diffusivity, fluid velocity and chlorine consumption rate on the chlorine concentration in the water fig.1 to fig.5 are plotted for various value of parameters.

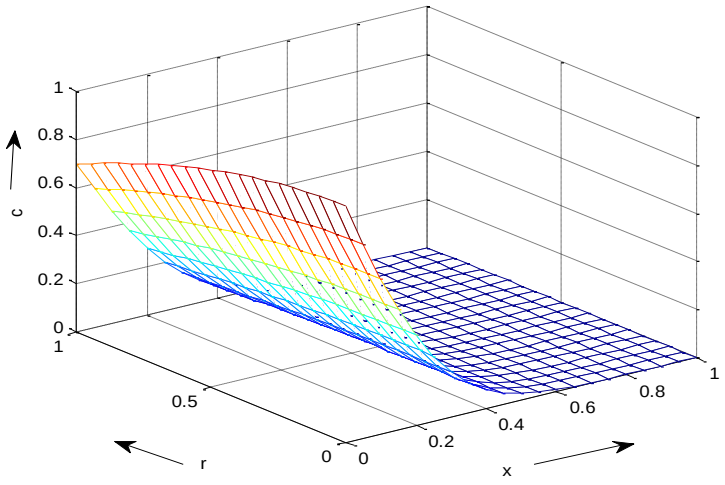

Fig.1 Variation of chlorine concentration with axial distance x and radial distance r (D =0.1, U =0.5, B_1 = 1.0, t =0.1, L =1.0)

It is clear from fig.1 that chlorine concentration decreases very fast along the axial distance while slowly along radial distance. Chlorine concentration decreases rapidly from $x=0$ to $x=0.6$ and after that it becomes constant. It appears that after $x=0.6$ concentration is zero. In fact it is not zero and $x=0.6$, $c=0.007125$ while at $x=0.8$, $c=0.000267$, and

$x = 1.0$, $c = 0.00000391$ (at $r = 0$) which are very near to zero. To maintain the safe limit for chlorine concentration we have to inject chlorine again after $x = 0.4$ and before $x = 0.6$. The variation of chlorine concentration along radial direction is very small and is difficult to observe from the figure. But we can see the change from the numerical value as at $x = 0.1$, $r = 0$, $c = 0.643566$ while at $x = 0.1$, $r = 1$ (i.e at the wall of pipe) $c = 0.469745$.

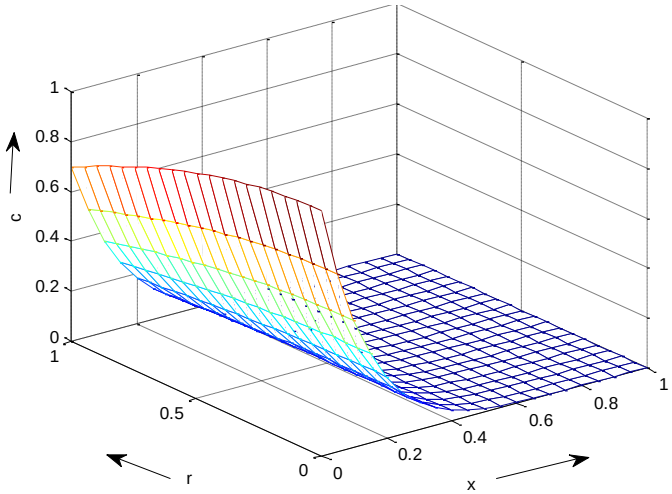

Fig.2 Variation of chlorine concentration with axial distance x and radial distance r (D = 0.1, U = 0.5, B_1 = 10.0, t = 0.1, L = 1.0)

The effect of chlorine consumption rate B_1 (which depends upon transport of chlorine from bulk flow to the wall, chlorine decay rate constant and mass transfer coefficient) can be observed by comparing fig.1 (for B_1=1) and fig.2 (for B_1=10). As the chlorine consumption rate B_1 increases from B_1=1 to B_1=10 the more chlorine is transported towards the pipe wall and less chlorine remains in the bulk flow. For B_1=1 (fig.1) the chlorine concentration approaches to zero after $x = 0.6$ while for K =10 (fig.2) chlorine concentration approaches to zero before $x = 0.6$.

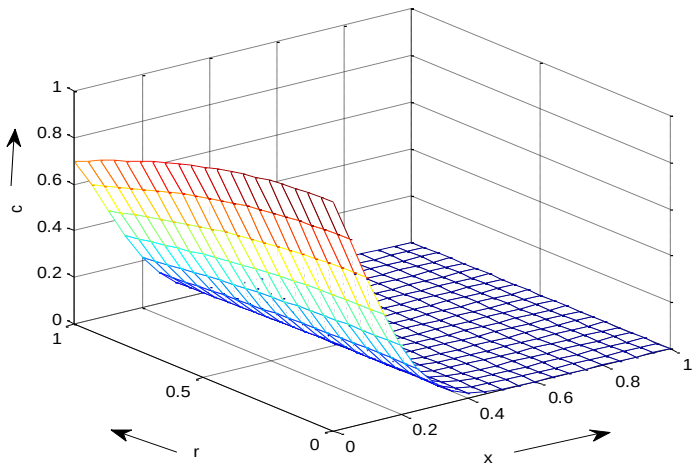

Fig.3 Variation of chlorine concentration with axial distance x and radial distance r (D =0.1, U =1.0, B_1=1.0, t = 0.1, L = 1.0)

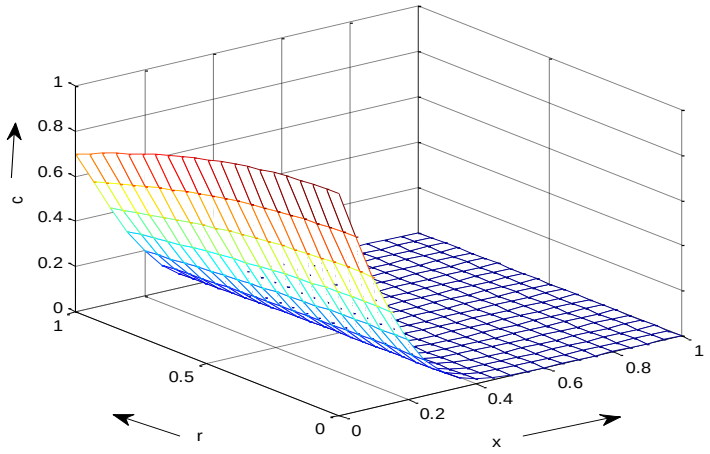

Fig.4 Variation of chlorine concentration with axial distance x and radial distance r (D =0.2, U = 1.0, B_1 = 1.0, t =0.1, L = 1.0)

The effect of diffusivity on chlorine concentration can be observed by comparing fig.3 (D =0.1) And fig.4 (D =0.2). The nature of chlorine concentration decay profile is very much similar in both the figures. But the difference is clear from the numerical values as $c = 0.003011$ for D =0.1 and $c = 0.002566$ for D =0.2 (at r=0 and x=0.5).Thus as diffusivity increases in r direction then chlorine concentration decreases at the same point in x direction. This is due to fact that when diffusivity increases then more mixing takes place and so chlorine concentration becomes constant at some earlier axial distance.

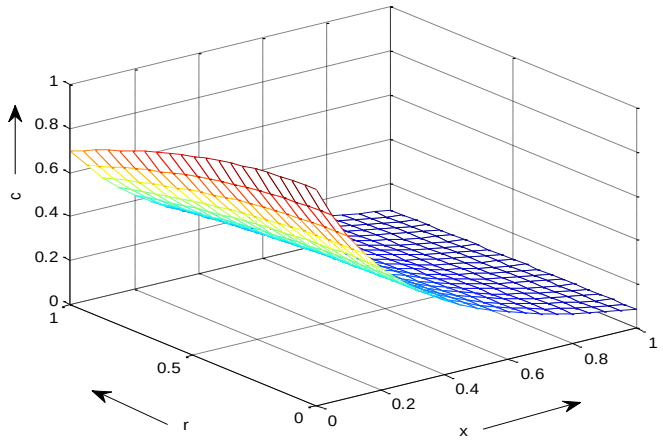

Fig.5 Variation of chlorine concentration with axial distance x and radial distance r (D = 0.1 , , U = 0.5, B_1 = 1.0, t = 0.8, L =1.0)

As this model is time dependent so to see the dependence of chlorine concentration on time we compare fig.1 and fig.5 At $t = 0.1, c = 0.00000391$ (numerical value taken from fig.1 at $x = 1.0 \, and \, r = 0$) while at $t = 0.8$, $c = 0.0857123$ (numerical value taken from fig.5 at $x = 1.0 \, and \, r = 0$) which is true fact since initially the chlorine injected at $x = 0 \, and \, r = 0$ and it takes some time to reach at $x = 1.0$ (i.e end of the pipe).

Conclusion

The unsteady state mathematical model for axisymetric flow in a pipe for low peclet number ,diffusion and transport of chlorine from the bulk flow to the pipe wall presented in the paper can be use effectively to locate the position for booster chlorination to maintain the safe limit of the drinking water. Our model can be used to locate booster chlorination in the system to optimize disinfection in the drinking water.

References

1. **Biswass, P., Lu, C. and Clark, R.M.** (1993), "Model for chlorine concentration decay in drinking water distribution pipes", *Water Research,* Vol. 27 (3), 1715-1724.

2. **Bhadula C. R., Jaipal and Kala N.V.** (2013) "unsteady state mathematical model for chlorine concentration decay in drinking water pipe line network"International Journal Of Scientific Research, Vol.2,Issue 5,397-399.

3. **Cherchi C. and GU, A.Z** (2011) "Effect of bacterial growth stage on resistance to chlorine disinfection" Water Science and Technology, 64.1 , 7-13.

4. **Clark, R.M. Grayman, W.M., Goodrich, J.A., Deininger, R.A. and Skow, K.** (1994), "Measuring and modelling of chlorine propagation in water distribution system", *J. Water Resour.Plang. and Mgmt., ASCE,* Vol. 120 (6), 803-820.

5. **Clark, R.M., Rossman, A.L and Wymer, J.L** (1995), "Modeling distribution system water quality" : Regulatory implications', *J. Water Resour. Plang.and Mgmt., ASCE,* Vol. 121(6), 423-428.

6. **Crank, J.** (1975), " The mathematics of diffusion', Second Edition Clarendon Press, Oxford."

7. **David, H.A. and Bryan, W.K.** (1996), "Modelling low velocity/high dispersion flow in water distribution systems", *J. Water Resour. Plang. and Mgmt.,* Vol. 122(3), 218-221.

8. **Edwards, D.K., Denny, V.E. and Mills, A.F.** (1979), "Transfer Processes", McGraw-Hill New York, N.Y.

9. **Friedler Eran, Yardeni Anat, Gilboa Yael and Alfiya yuval** (2011) " Disinfection of greywater effluent and regrowth Potential of selected bacteria" Water Science and Technology, 63.5, 931-940.

10. **Hoefel, D., Monis, P.T., Grooby, W.L. Andrews, S. and Saint, C.P**. (2005) " Culture – independent techniques for rapid detection of bacteria associated with loss of chlorine mine residual in a drinking water system". Applied and Environmental microbiology 71(11) 6479-6488.

11. **Huang [a] Huang , Wu[a] – Yuan Qian, Yang[a] Yang, Hu[a,b] Hong –Ying** (2011) "Effect of chlorination on endotoxin activities in secondary Sewage effluent and typical Gram-negative bacteria". Water Resarch, 45, 4751-4757.

12. **Jaipal, Bhadula C.R. and Kala.N.V**(2012)" Modelling of chlorine concentration decay in drinking water pipeline network". International transaction in applied science.april-june2012,volume 4, no.02,329-336.

13. **Mnavali G.R.Kumar Mohan M.S.**(2005)"Water quality parameter estimation in a distribution system under dynamic state". Water research,39,4287-4298.

14. **Ozdermir. N.Osman and Ger .Metin.**A (1999)"Unsteady 2-D chlorine transport in water supply pipes "Water Research Vol33,No.17,3637-3645.

15. **Senddon .HI.(1972)** "The use of integral transform. New York McGraw-Hill".

16. **Wojcicka, L., Holmann, R., Banter. C. Andrews, R.C. Auvray. I. and Liere, J** (2007) "Inactivation of environmental and reference strains of heterotrophic bacteria and Escherichia Coli 0157 : H7 by free chlorine and monochloramine". Journal of water supply Research and Technology – Aqua 56(2) 137-150.