Dust-acoustic shock waves with non-thermal ions in a dusty plasma

Louis E. Akpabio[1], Akpan N. Ikot[1] and Akpan B. Udoimuk[2]

[1]Theoretical Physics Group, Dept. of Physics, University of Uyo, Uyo – Nigeria.

[2]Department of Physics, University of Calabar, Calabar-Nigeria.

ABSTRACT

A dusty plasma system containing non-thermal ion distributions, Boltzmann-distributed electrons and mobile charge fluctuating positive dust has been considered. The nonlinear propagation of the dust-acoustic (DA) waves in such dusty plasma has been investigated by employing the reductive perturbation method to derive the Burgers equation. The effect of non-thermal ions on the height and thickness of DA shock waves are also studied.

Introduction

Dust and plasmas exist together in the universe and they make dusty plasmas. Dusty plasmas are found in cometary tails, asteroid zones, planetary ring, interstellar media, lower part of Earth's ionosphere and magnetosphere, etc. [Geortz, 1989; Mendis and Rosenberg, 1994, shukla, 2001; Horanyi and Mendis, 1986; Verheest, 2001; Horanyi, 1996]. Noticeable applications of dusty plasmas are also found in laboratory devices [Barkan et al, 1995; Merlino et al, 1998; Homann et al, 1997]. There has been a rapidly growing interest in the field of dusty plasmas because of its great variety of new phenomena associated with wave and instabilities [Verheest, 1992; Pieper and Gorce, 1996; Bliokh and Yaroshenko, 1985]. The consideration of charged dust grains in a plasma does not only modify the existing plasma wave spectra [de Angelis et al, 1988; Shukla and Stenflo, 1992]; but also introduces a number of new novel eigen modes such as dust acoustic (DA) wave [Rao et al, 1990, Barkan et al, 1996], dust ion acoustic (DIA) waves [Shukla and Silin, 1992, Barkan et al 1996] etc.

Very recent researchers are paying, more attention for investigating the non linear properties of DA waves by considering the charge fluctuation. Shock waves are generated when charge fluctuation are considered in dusty plasmas as discovered by the works of Mamun (2008) and Duha and Mamun (2009). It is well known that linear and non linear properties of plasma depend on the velocity distribution functions of the particle constituent of the plasma. The most commonly used distribution, is the Maxwellian velocity for collisionless plasmas, which is a distribution that is in thermal equilibrium. But from recent observations, it has been found that fast ions and electrons in space environments indicate that these particles have velocity distributions that deviate from Maxwellian behaviour. Many instances of fast particles have been observed in space, for example, non-thermal ions have been observed in the Earth's bow-shock [Asbridge, et al., 1968]. The loss of energetic ions from the upper ionosphere of Mars has been observed through satellite observations [Lundlin, et al., 1989]. Recently, a very large velocity proton near the earth in the vicinity of the Moon has been observed (Futaana et al., 2003].

In this paper, we consider an unmagnetized collisionless dusty plasma composed of non-thermal ions, Boltzmann-distributed electrons and charge fluctuating positively charged mobile dust. The reductive perturbation technique is applied to study the effect of non-thermal ions on dust-acoustic shock wave in dusty plasma.

Basic Equation

We assume for simplicity that all the grains have the same charge, equal to $q_d = +z_d e$, with z_d representing the charge state of the dust component. Hence, charge neutrality at equilibrium is given by $n_{e0} = n_{i0} + z_{d0} n_{d0}$, where n_{e0} (n_{i0}) is the equilibrium electron (ion) number density, n_{d0} is the dust density at equilibrium, z_{d0} represent equilibrium charge state of the dust component. All the dust grain is assumed to be spheres of radius r_d. The basic equations for one-dimensional DA waves for such a dusty plasma is given as

$$\frac{\partial n_d}{\partial t} + \frac{\partial}{\partial x}\left(n_d u_d\right) = 0, \qquad (1)$$

$$\frac{\partial u_d}{\partial t} + u_d \frac{\partial u_d}{\partial x} = -\frac{z_d e}{m_d}\frac{\partial \phi}{\partial x}, \qquad (2)$$

$$\frac{\partial^2 \phi}{\partial x^2} = 4\pi e\left(n_e - n_i - z_d n_d\right), \qquad (3)$$

Where ϕ is the electrostatic potential, n_d, n_e, n_i are respectively, the number density for the plasma species for dust, electrons and ions, u_d is the dust fluid speed. The non-thermal ion distribution is given as [Cairns et al, 1995]

$$n_i = n_{i0}\left(1 + \beta_i \frac{e\phi}{k_B T_i} + \beta_i \frac{e^2\phi^2}{k_B^2 T_i^2}\right)e^{-\frac{e\phi}{k_B T_i}}. \qquad (4)$$

Where

$$\beta_i = \frac{4\alpha_i}{1 + 3\alpha_i}, \qquad (5)$$

and the Boltzmann distributed electrons as

$$n_e = n_{e0} e^{\frac{e\phi}{k_B T_e}}. \qquad (6)$$

where α_i is a parameter determining the number of non-thermal ions present in our plasma model, k_B is the Boltzmann constant and $T_e(T_i)$ is the electron (ion) temperature. Neglecting all other charging processes, we assume that the dust is charged by photoemission current $\left(I_p^+\right)$, thermionic emission current $\left(I_t^+\right)$ and electron absorption current $\left(I_e^-\right)$ only. The charge state z_d of the dust component is not constant, but varies according to the following equation [Paul et al, 2009, Shukla and Mamun, 2002]:

$$\frac{\partial z_d}{\partial t} + u_d \frac{\partial z_d}{\partial x} = \frac{1}{e}\left(I_p^+ + I_t^+ + I_e^-\right) \qquad (7)$$

where

$$I_p^+ = \pi r_d^2 e J Y \exp\left(-\frac{z_d e^2}{k_B r_d T_{ph}}\right) \qquad (8)$$

$$I_t^+ = 2\pi r_d^2 e \left(\frac{2\pi n_e k_B T_p}{h^2}\right)^{3/2}\left(\frac{8k_B T_p}{\pi m_e}\right)^{1/2}$$

$$\times \left(1 + \frac{z_d e^2}{r_d k_B T_p}\right)\exp\left(-\frac{z_d e^2}{r_d k_B T_p} - \frac{w_e}{k_B T_p}\right) \qquad (9)$$

$$I_e^- = -\pi r_d^2 e n_{e0} \exp\left(\frac{e\phi}{k_B T_p}\right)\left(\frac{8k_B T_p}{\pi m_e}\right)^{1/2} \times \left(1 + \frac{z_d e^2}{r_d k_B T_p}\right) \qquad (10)$$

where h is the Planck's constant, T_{ph} is the photon temperature, w_e is the work function, J is the uv photon flux, Y is the yield of photons. The typical values of w_e, J and Y are given respectively as 8.2 eV, 5.0×10^4 photons /cm^2/s and 0.1. For convenience, we express the set of equations (1) to (7) in normalized form by introducing the following normalized variables:

$$N_d = n_d/n_{d0}, u_d = u_d/C_d, \phi = e\phi/k_B T_i, Z_d = z_d/z_{d0}, X = x/\lambda_{Dd}, T = t w_{pd},$$

$$\lambda_{Dd} = \left(k_B T_i/4\pi z_{d0}^2 n_{d0} e^2\right)^{1/2}, C_d \equiv \left(z_{d0} k_B T_i/m_d\right)^{1/2} \quad \text{and}$$

$$w_{pd} = \left(4\pi z_{d0}^2 n_{d0} e^2/m_d\right)^{1/2} \quad \text{to obtain the following}$$

equations

$$\frac{\partial N_d}{\partial T} + \frac{\partial}{\partial X}\left(N_d U_d\right) = 0 \qquad (11)$$

$$\frac{\partial U_d}{\partial T} + U_d \frac{\partial U_d}{\partial X} = -Z_d \frac{\partial \phi}{\partial X} \qquad (12)$$

$$\frac{\partial^2 \phi}{\partial X^2} = \left(1 + \mu_i\right)e^{\sigma\phi} - \mu_i\left(1 + \beta_i \phi + \beta_i \phi^2\right)e^{-\phi} - Z_d N_d \qquad (13)$$

$$\frac{\partial Z_d}{\partial T} + U_d \frac{\partial Z_d}{\partial X} = \mu\left[Pe^{-\alpha Z_d} + Q\left(1 + \beta Z_d\right)e^{-\beta Z_d} + \mathrm{Re}^\phi\left(1 + \beta Z_d\right)\right] \qquad (14)$$

where

$$\sigma = T_i/T_e; \mu_i = n_{i0}/z_{d0}n_{d0}; \mu_i = n_{i0}/z_{d0}n_{d0}; \mu = \pi r_d^2/z_{d0}w_{pd};$$

$$P = JY; Q = 2e^{-w_e/k_B T_e}\left(2\pi m_e k_B T_e\right)/h^{3/2}\left(8k_B T_e/\pi m_e\right)^{1/2}, R = n_{e0}\left(8k_B T_e/\pi m_e\right)^{1/2},$$

$$\alpha \equiv z_{d0} e^2/r_d k_B T_{ph}; \beta = z_{d0} e^2/r_d k_B T_e.$$

Nonlinear dust acoustic shock waves

To study the dynamics of nonlinear dust acoustic shock waves in the presence of non-thermal ions, Boltzmann distributed electrons and charge fluctuating positive dust grains; we employ the reductive perturbation technique [Washimi and Taniuti, 1966]. We introduce the stretched coordinates [Das et al, 1997] $\xi = \in\left(X - V_0 T\right)$ and $\tau = \in^2 T$, where $\in$ is a small parameter and V_0 is the DA shock waves velocity normalized by C_d. The variables N_d, U_d, Z_d and ϕ are expanded as

$$N_d = 1 + \in N_{d1} + \in^2 N_{d2} + \dots$$

$$U_d = \in U_{d1} + \in^2 U_{d2} + \dots,$$

$$Z_d = 1 + \in Z_{d1} + \in^2 Z_{d2} + \dots \qquad (15)$$

$$\phi = \in \phi_1 + \in^2 \phi_2 + \dots.$$

Now, substituting these expansion into equation (11-14) and collecting the terms of different powers of $\in$, in the lowest order, we obtain

$$U_{d1} = \phi_1/V_0 \qquad (16)$$

$$N_{d1} = \phi_1/V_0^2 \qquad (17)$$

$$Z_{d1} = \left[\sigma\left(1 + \mu_i\right) + \mu_i\left(1 - \beta_i\right) - 1/V_0^2\right]\phi_1 \qquad (18)$$

$$V_0^2 = \frac{\Gamma_i}{\Gamma_i\left[\sigma\left(1 + \mu_i\right) + \mu_i\left(1 - \beta_i\right)\right] + \mu R\left(1 + \beta\right)} \qquad (19)$$

Where

$$\Gamma_i = \mu P \alpha^2 - \mu P \alpha - \mu Q \beta^2 + \frac{3}{2}\mu Q \beta^3 + \mu R \beta.$$

The next order in $\in, O\left(\in^2\right)$ yields a system of equations that leads to Burgers equation as follows:

$$\frac{\partial N_{d1}}{\partial \tau} - V_0 \frac{\partial N_{d2}}{\partial \xi} + \frac{\partial U_{d2}}{\partial \xi} + \frac{\partial}{\partial \xi}\left(N_{d1}U_{d1}\right) = 0 \qquad (20)$$

$$\frac{\partial U_{d1}}{\partial \tau} - V_0 \frac{\partial U_{d2}}{\partial \xi} + U_{d1}\frac{\partial U_{d1}}{\partial \xi} = -\frac{\partial \phi_2}{\partial \xi} - Z_{d1}\frac{\partial \phi_2}{\partial \xi} \qquad (21)$$

$$\left[\sigma\left(1 + \mu_i\right) + \mu_i\left(1 - \beta_i\right)\right]\phi_2 + \left[\frac{\sigma^2\left(1 + \mu_i\right)}{2} - \frac{\beta_i \mu_i}{2} + \mu_i\left(\beta_i - 1\right)\right]\phi_1^2 = Z_{d2} + Z_{d1}N_{d1} + N_{d2} \qquad (22)$$

$$-V_0 \frac{\partial Z_{d1}}{\partial y} = \mu\left[-P\alpha + P\alpha^2 - Q\beta^2 + \frac{3Q\beta^3}{2} + RB\right]Z_{d2}$$

$$+ \mu\left[\frac{P\alpha^2}{2} - \frac{1}{2}Q\beta^2 + \frac{3}{2}\mu Q\beta^3\right]Z_{d1}^2$$

$$+ \mu R\left(1 + \beta\right)\phi_2 + \frac{1}{2}\mu R\left(1 + \beta\right)\phi_1^2 + \mu R \beta Z_{d1}\phi_1 \qquad (23)$$

Making using of equations (16) - (23) we eliminate N_{d2}, U_{d2}, Z_{d2} and ϕ_2 to obtain the following equation:

$$\frac{\partial \phi_1}{\partial \tau} + A\phi_1 \frac{\partial \phi_1}{\partial \xi} = B \frac{\partial^2 \phi_1}{\partial \xi^2} \qquad (24)$$

where the nonlinear coefficient A and the dissipation coefficient B are given by

$$A = V_0^3/2\left\{\frac{2}{V_0^4}\left[1 + \frac{V_0^2}{2}\left\{\sigma\left(1 + \mu_i\right) + \mu_i\left(1 - \beta_i\right)\right\}\right] - \frac{\mu}{\Gamma_i}\left(P\alpha^2 - Q\beta^2 + 3Q\beta^3\right)\left[\sigma\left(1 + \mu_i\right) + \mu_i\left(1 - \beta_i\right) - \frac{1}{V_0^2}\right]^2\right\}$$

$$-\frac{\mu R}{\Gamma_i}(1+\beta)-\frac{2\mu R\beta}{\Gamma_i}\left[\sigma(1+\beta_i)+\mu_i(1-\beta_i)-\frac{1}{V_0^2}\right]$$

$$-2\left[\frac{\sigma(1+\mu_i)}{2}+\mu\left(\frac{\beta_i}{2}-1\right)\right]\right\}\tag{25}$$

$$B=\frac{V_0^4}{2\Gamma_i}\left[\sigma(1+\mu_i)+\mu_i(1-\beta_i)-\frac{1}{V_0^2}\right].\tag{26}$$

The Burgers equation which describes the nonlinear propagation of the DA shock waves in the dusty plasma under consideration is given as equation (24). It can be observed that, the right hand side of equation (24) which represent the dissipative term is due to the presence of non thermal parameter (β_i), the ratio of ion and electron temperature (σ) and the charge fluctuating positive dust (μ_i).

Discussion and conclusion

Our expression for Z_{d1} as equation (18) agrees with what is obtained by Paul et al (2009) when non-thermal parameter β_i is set to zero and σ set to 1. We strongly feel that the last term in the denominator for equation (19) should be $\left[+\mu R(1+\beta)\right]$ as against what is obtained by Paul et al (2009) as $\left[-\mu R(1+\beta)\right]$. Likewise, the last term for f as $\left(-\mu R\beta\right)$ reported by Paul et al (2009) should be $\left(+\mu R\beta\right)$ as in our report for Γ_i. Equation (19) which gives the linear disperation relation for DA waves is greatly altered by the presence of the ion non-thermal parameter, ratio of ion and electron temperature, as well as the positive dust charge fluctuation. For stationary shock wave solution of equation (24), we set $\zeta=\xi-U_0\tau'$ and $\tau'=\tau$ to obtain the equation

$$-U_0\frac{\partial\phi_1}{\partial\zeta}+A\phi_1\frac{\partial\phi_1}{\partial\zeta}=B\frac{\partial^2\phi_1}{\partial\zeta^2}\tag{27}$$

The latter equation can be integrated, using the conduction that ϕ is bounded as $\zeta\rightarrow\pm\infty$ or by the application of Tanh method [Malfliet, 1992; Malfliet and Hereman, (a,b) 1996; Malfliet, 2004] to yield

$$\phi_1=\phi_0\left\{1-\tanh\left(\frac{\zeta}{\Delta_{sh}}\right)\right\}\tag{28}$$

where

$$\phi_0=\frac{U_0}{A}\tag{29}$$

and

$$\Delta_{sh}=\frac{2B}{U_0}.\tag{30}$$

Equation (28) represents a monotonic shock-like solution with the shock speed, the shock height, and the shock thickness given by U_0,ϕ_0 and Δ_{sh} respectively. It is obvious from equation (28) that, the presence of ion non- thermal parameter significantly modifies the shock wave amplitude and its width. As the width of the DA shock structures decreases; the non-thermal parameter increases, while the amplitude of the shock width structure varies as the charge fluctuating positive dust (μ_i) increases. This is due to the fact that dust charge fluctuation is a source of dissipation and lead to the development of DA shock waves in the dusty plasma

We have extended the recent work of Paul et al, 2009 to see under what conditions, the ion non-thermal effect and the charge fluctuating dust in dusty plasmas can be seen to modify the basic features of the non linear DA waves. In a subsequent paper; we shall numerically analyze the present dusty plasma model developed by us with a view to determine if, it will admit both positive and negative shock wave (potential) profiles. The findings in this paper are important in understanding nonlinear DA wave phenomena in space plasmas.

References

Asbridge, J. R; Bame, S. J. and Strong, I. B. (1968) *J. Geophys. Rev.* 73, 5777, DOI; 10.1029/JA073i017P05777.

Barkan, A.; Merlino, R. L. & D'Angelo, N. (1995). Laboratory Observation of the Dusty Acoustic Wave Mode, *Phys. Plasma* 2, 3563-3565.

Bliokh, P. V. & Yaroshenko, V. V. (1985). Electrostatic Waves in Saturn's Rings, *Sov.Astron.* (Eng. Transl.), 29, 330-336.

Barkan, A., D'Angelo, N. & Merlino, R. L. (1996). Experiments on Ion-acoustic Waves in Dusty Plasmas, *Planet. Space Sci* 44(3), 239-242.

Carins, R. A.; Mamun, A. A., Bingham, R.; Bostrom, R; O.; Nairn, C. M. C. & Shukla, P. K. (1995). Electrostatic Solitary Structures in Non-thermal Plasmas, *Geophys. Res. Lett.,* 22, 2709-2712.

deAngels, U; Formisano, V. and Giordano, M. (1988). Ion Plasma Waves in Dusting Plasmas. *J. Plasma Phys.* 40(3), 399-406.

Duha, S. S. and Mamun, A. A; (2009) Dust Ion Acoustic Waves Due to Dust Charge Fluctuation. *Physics Letters* A, 373(14), 1287-1287

Das, G. C.; Dwivedi, C. B.; Talukdar, M., and Sharma, J. (1997) Anew Mathematical Approach for Shock-Wave Solution in a Dusty Plasma *Phys. Plasma,* 4(12), 4236-4239.

Goertz, C. K.; (1989). Dusty Plasmas in the Solar System. *Rev. Geophys.* 27, 271-292.

Horanyi, M. & Mendis, D. A. (1986). The Dynamics of Charged Dust in the Tail of Comet Giacobini-Zinner. *J. Geophys. Res.* 91 (AI), 355-361.

Horanyi M., (1996). Charged Dust Dynamics in the Solar System. *Annu. Rev. Astron. Astrophys,* 34, 383-418.

Homann, A.; Melzer, A.; Peters, S. & Piel, A. (1997). Determination of the dust screening length by loser-excited lattice waves. *Phys. Rev. E. Stat. Phys. Plasmas Fluids Relate Intercrossing, Top.* 56(6), 7138 – 7141.

Futaana, Y; Machide, S; Saito, Y; Matsuoka, A. and Hayakawa, H. (2003) *J. Geophys. Res.* 108, 1025, D0I, 10.1029/2002JA009366.

Lundin, R; Zakharov, A; Pelinenn; R; Borg, H; Hultqvis, B; Pissarenko, N; Dubinin, E. M; Barabash, S. W; Liede, I. and Koshurian, H. (1989). Nature (London), 341,609.

Mendis, D. A. & Rosenberg, M. (1994). Cosmic dusty plasma, *Annu. Rev. Astron. Astrophys.* 32, 418-463.

Merlino, r. L.; Barkan, A; Thompson, C. and D. Angela, N. (1998). Laboratory Studies of wave and instabilities industry plasmas. *Phys. Plasmas.* 5(5), 1607-1614.

Ma. J. X. and Liu, J. (1997). Dust-acoustic soliton in a dusty plasma. *Phys. Plasmas.* 4(2), 252-255.

Malfliet W., (1992). Solitary Wave Solution wave solutions of nonlinear wave equations. *Am. J. Phys.* 60, 650-654.

Malfliet, W. & Hereman, W. (1996). The Tanh method: I. Exact solutions of nonlinear evaluation and wave equations. *Physic Scripta,* 54, 563-568.

Malflict, W. & Hereman, W. (1996). The Tanh Method: II. Perturbation technique for conservative systems. *Physica Scripta*, 54, 569-573.

Malflict, W. (2004). The tanh method: a tool for solving certain classes of nonlinear evolution and wave equations. *J. Comp. Appl. Math.* (164-165), 529-541.

Maharaj, S. K., Pillay, S. R; Bharuthram, R., Reddy, R. V. Singh, S. V. and Lakhina, G. S. (2006). Arbitrary amplitude dust acoustic double layers in a non-thermal plasma. *J. Plasma Physics*, 72(1), 43-58.

Mamun, A. A. (2008). Dust Electron-Acoustic Shock Waves Due to Dust Charge Fluctuation, *Physics Letters.* A, 372(25), 4610-4613.

Pieper, J. B. & Gorce, J. (1996). Dispersion of Plasma dust acoustic waves in the strong-coupling regime. *Phys Rev. Lett.* 77, 3937-3140.

Popel, S. I. & Yu, M. Y. (1995). Ion acoustic solution in impurity − containing plasmas. *Contrib. Plasma Phys:* 35(2), 103-108.

Paul, K. S.; Mandal G.; Manun, A. A. and Amin, M. R. (2009). Dust-Acoustic Shock waves in a dusty plasma with charge fluctuating positive dust. *IEEE Transactions on Plasma Science,* 37 (4), 627-631.

Rao, N. N.; Shukla, P. K. & Yu, M. Y. (1990). Dust-Acoustic waves in dusty plasmas *Planet. Space. Sci.* 38, 543-546/

Shukla, P. K. (2001). A survey of dusty plasma physics. *Phys Plasmas,* 8,1791-1803.

Shukla, P. K. & Stenflo, L. (1992). Stimulated Scattering of electromagnetic waves in dusty plasma. *Austrophys. Space Sci.,* 190(1), 23-32.

Shukla, P. K. & Silin, V. P. (1992). Dust ion acoustic wave. *Phys. Scr.*, 45, 508.

Shukla, P. K. and Mamun, A. A. (2002). Bristol. *Introduction to Dusty Plasma Physics* Iop publishing.

Verheest, F. (2001). Dordrecht. *Waves in Dusty Space Plasmas.* Kluwer Academic.

Verheest, F. (1992). Nonlinear dust-acoustic waves in multispecies dusty plasmas. *Planet. Space Sci.* 40, 1-6.

Washimi, H. & Taniuti, T. (1996). Propagation of ion-acoustic solitary wave of small amplitude. *Phys. Rev. Lett,* 17, 996-998.