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Linear equality constraint conditions for the second half of the variable coefficient model parameter estimation

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ABSTRACT

Half a variable coefficient model is a variable coefficient model, this paper mainly discusses the useful promotion in linear equality constraints in the second half of $A\beta = b$ variable coefficient model parameter estimation problem, this paper presents parameters in part β equality constraints under $A\beta = b$ double parameters of the conjugate gradient method and the parameters of $\alpha(\cdot)$ part estimate, on the double parameters conjugate gradient method of the decline of the projection and convergence is proved by an example, the algorithm of the optimal inspection benign.

Introduction

Consider the following half variable coefficient model

$$y_i = x_i^T\beta + z_i^T\alpha(u_i) + \varepsilon_i,\ \ A\beta = b \qquad (1\text{-}1)$$

Among them y_i is a responsive variable, (x_i, z_i, u_i) is fixed the random design sequence, $x_i = (x_{i1}, \cdots x_{ip})^T \in R$, $z_i = (z_{i1}, \cdots z_{iq})^T \in R$ is association variables, $\beta = (\beta_1, \cdots \beta_p)^T$ is p dimension of the unknown parameters vector, $\alpha(\cdot) = (\alpha_1(\cdot), \cdots \alpha_q(\cdot))^T$ is q dimension unknown function vector, β and $\alpha(u_i)$ is with estimated parameters, $\alpha_j(u_i)$ is defined in the unknown function $[0,1]$, $\{\varepsilon_i, i = 1, \cdots n\}$ is zero mean with the independent distribution variables sequence. β linear equation of parameters constraints is $A\beta = b$, Among them A is $m \times p$ order line full rank array, $b \in R^m$.

Half a variable coefficient model is made from Fan and Zhang [1]since 2002 first proposed, now has caused many scholars' attention, the discussion focused on the part of the main parameters of the unknown function is β and $\alpha(u_i)$ estimate, to avoid "dimension curse", often assume that u is one-dimensional, this model is variable coefficient model, the promotion of useful comprehensive partly linear model and variable coefficient model advantages. Rosen projection gradient method is the steepest descent method in the extension of the constraints, but this method cannot do it fast convergence speed, to get better convergence need to modify or add other conditions, and no constraint of the conjugate gradient method is solving unconstrained optimization problem of $\min_{x \in R^n} f(x)$ kind of very effective method, this article will projection gradient method and conjugate gradient method, construct a new algorithm, linear equality constraint optimization problem of double parameters of the conjugate gradient method projection.

In the actual problem, people of the model of the parameters of the more or less some understand, but there is always some prior information or assume that, such as in the temperature and the problem of the relationship between the bacterial growth, people often assume that temperature function satisfy certain PH value, namely the parameters such as is often carry a constraint. In the discussion of the optimization theory of constraints, often including equality constraint and inequality constraints. This paper mainly discusses the linear equality constraints in the second half of $A\beta = b$ variable coefficient model parameter estimation.

The estimated parameters

The unknown parameters of β estimate

In the model (1-1), Set $g(z_i, u_i) = z_i^T\alpha(u_i)$, because there are $E(\varepsilon_i) = 0$, so has $E(y_i - x_i^T\beta) = g(z_i, u_i) + E(\varepsilon_i) = g(z_i, u_i)$, so using nuclear function, the method to estimate $g(z_i, u_i)$, so $g(z_i, u_i)$'s estimates for the $g(z_i, u_i, \beta) = \sum_{i=1}^{n} \omega_{ni}(y_i - x_i^T\beta)$, among them $\omega_{ni} = w(u, u_1, \cdots u_n)$ is a probability weight function. Therefore, the model (1-1) define parameters of β squared residuals for

$$f(\beta) = \sum_{i=1}^{n} \varepsilon_i^2 = \sum_{i=1}^{n} \left(y_i - x_i^T\beta - g(z_i, u_i, \beta)\right)^2$$

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constraint conditions for $A\beta = b$.

From the point of view of the optimization theory, $f(\beta) = \sum_{i=1}^{n}\varepsilon_i^2 = \sum_{i=1}^{n}\left(y_i - x_i^T\beta - g(z_i, u_i, \beta)\right)^2$ is the objective function, constraint condition for $A\beta = b$, the problem is β estimated parameters of $f(\beta)$ in the constraint condition for optimization (minimum) problem is one of problems (P) :

$$\min_{\beta\in R^m} f(\beta) = \sum_{i=1}^{n}\varepsilon_i^2 = \sum_{i=1}^{n}\left(y_i - x_i^T\beta - g(z_i, u_i, \beta)\right)^2$$
$$s.t \quad A\beta = b$$

In this paper the problem of β valuation of a kind of improved optimization double parameters conjugate gradient method for the estimation projection.

Define the feasible region for $D = \{\beta \in R^P \mid A\beta = b\}$, $f(\beta)$ is continuously differentiable function, A is $m\times p$ order of full rank matrix, the problem (P) K-T conditions for $\begin{cases}\nabla f(\beta) + \lambda A^T = 0 \\ A\beta = b\end{cases}$, λ is *Lagrange* multiplier. Definition gradient function $g(\beta) = \nabla f(\beta)$, $g_k = \nabla f\left(\beta^{(k)}\right)$, projection matrix $P = I - A\left(A^T A\right)^{-1} A^T$,and P meet $P^2 = P \,\&\, P^T = P$, so to point β feasible, if $P\nabla f(\beta) = 0$, then β to meet problem (P) K-T point, if $f(\beta)$ is continuously differentiable of convex function, then β for problems (P) optimal solution.

Solving problem (P) a form of iterative algorithm for: $\beta^{(k+1)} = \beta^{(k)} + \lambda_k d_k$, among them $d_k = \begin{cases} -Pg_k & k = 1 \\ -Pg_k + \tilde{\beta}_k d_{k-1} & k \ge 2\end{cases}$,

d_k is for the search direction; λ_k for step, $\tilde{\beta}_k$ for a parameter, $\tilde{\beta}_k$ use the following mean:

$$\tilde{\beta}_k = \begin{cases} \dfrac{\|Pg_k\|^2 - \mu_1\left|(Pg_k)^T(Pg_{k-1})\right|}{\mu_2 d_{k-1}^T(g_k - g_{k-1})} & if\, 0 < (Pg_k)^T(Pg_{k-1}) < \min\left(2, \dfrac{1}{\sigma}\right)\|Pg_k\|^2 \\ 0 & else\end{cases}$$

among them $\mu_2 > 0$,when $0 < \sigma < 1/2$, has $0 < \mu_1 < 1/2$;when $1/2 < \sigma < 1$, has $0 < \mu_1 < \sigma$.

Algorithm as follows (A) :

Step 1. The initial points are given $\beta^{(1)} = \left(\beta_1^{(1)}, \text{L L } \beta_p^{(1)}\right)$, make it satisfies $A\beta^{(1)} = b$, makes $k := 1$, $\beta^{(0)} = 0$, $d_0 = 0$;

Step 2 . Calculation Pg_k, if $\|Pg_k\| = 0$, stop calculation, the problem (P) get the K-T point for $\beta^{(k)}$;Otherwise calculation direction $d_k = -Pg_k + \tilde{\beta}_k d_{k-1}$;

Step 3. Conduct Wolfe line search $f(\beta^{(k)} + \lambda_k d_k) \le f(\beta^{(k)}) + \rho\lambda_k g_k^T d_k$, $d_k^T g(x_k + \alpha_k d_k) \ge \sigma d_k^T g_k$, among them $0 < \rho < \sigma < 1$, Produce step λ_k, and make $\beta^{(k+1)} = \beta^{(k)} + \lambda_k d_k$, $f_{k+1} = f\left(\beta^{(k+1)}\right)$, Return to step 2.

2.2 The unknown function coefficients $\alpha(\cdot)$ estimate

Because $\varepsilon_i = \varphi\varepsilon_{i-1} + e_i - \theta e_{i-1}$, $E(\varepsilon_i) = 0$, consider model $y_i - x_i^T\hat{\beta} = z_i^T\alpha(u_i) + \varepsilon_i$, this model is the typical variable coefficient model, practical [3] in the estimation of parameters can be $\alpha_j(\cdot)$ part of the estimate,is $\overline{\alpha}(u_0) = \left(\tilde{Z}^T R\tilde{Z}\right)^{-1}\tilde{Z}^T R\tilde{Y}^*$, among them $Y^* = y_i - x_i^T\hat{\beta}$.

3. The conclusion and theorem

We first presents the assumption that conditions and pass lemma, assumptions are as follows:

(A1) f in level set $L = \left\{\beta \in R^p : f(\beta) < f\left(\beta^{(1)}\right)\right\}$ has lower bound, including $\beta^{(1)}$ for initial point;

(A2) In the level set L a neighborhood in U, f continuously differentiable, the guide g function that meet the *Lipschitz* conditions, there is $L > 0$ constant to: $\|g(\beta) - g(\gamma)\| \le L\|\beta - \gamma\|$, $\forall\beta \in U$, $\forall\gamma \in U$;

Lemma 1 [8] : For general function $f(\beta)$,if has $Ad_1 = 0$, then the direction of $d_k = -Pg_k + \tilde{\beta}_k d_{k-1}$ type produces sequence $\{d_k\}$ satisfy $Ad_k = 0$, $(Pg_k)^T d_k = g_k{}^T d_k$, $(Pg_{k+1})^T d_k = g_{k+1}{}^T d_k$, $\forall k \ge 1$.

Lemma 2 [9]:Set the objective function to meet conditions (A1,A2),consideration $\beta^{(k+1)} = \beta^{(k)} + \lambda_k d_k$, among them $d_k^T g_k < 0$, λ_k from strong Wolfe line search calculated, so has $\sum_{k\ge1, d_k\ne0}\dfrac{\left(d_k^T g_k\right)^2}{\|d_k\|^2} < +\infty$, this type is called *Zoutendijk* conditions.

The lemma 1 and 2 is very easy to get the following lemma.

Lemma 3: Set the objective function to satisfy (A1-A2), $\beta^{(k+1)} = \beta^{(k)} + \lambda_k d_k$, Among them $(Pg_k)^T d_k < 0$, λ_k from strong Wolfe line search calculated, so has $\sum_{k\ge1, d_k\ne0}\dfrac{\left((Pg_k)^T d_k\right)^2}{\|d_k\|^2} < +\infty$, This type is called promotion *Zoutendijk* conditions.

Lemma 4: If $Pg_k \ne 0$, The algorithm (A) the direction for fall produce d_k direction, namely

$$(Pg_k)^T d_k = g_k^T d_k < 0, \quad \forall k \ge 1$$

Proof: By induction proof: When $n=1$,has $d_1=-Pg_1$, $g_1^T d_1=-g_1^T Pg_1=-g_1^T P^T Pg_1=-(Pg_1)^T Pg_1=-\|Pg_1\|^2<0$, hypothesis when $n=k-1$,has $(Pg_{k-1})^T d_{k-1}=g_{k-1}^T d_{k-1}<0$ established.

The divide kind of case proof when $n=k$ has $(Pg_k)^T d_k=g_k^T d_k<0$ established.

① If $\alpha_k=0$, by

$$(Pg_k)^T d_k=(Pg_k)^T(-Pg_k+\alpha_k d_{k-1})$$
$$=-\|Pg_k\|^2+\alpha_k(Pg_k)^T d_{k-1}=-\|Pg_k\|^2<0$$, Namely card.

② If $\alpha_k=\dfrac{\|Pg_k\|^2-\mu_1\left|(Pg_k)^T(Pg_{k-1})\right|}{\mu_2 d_{k-1}^T(g_k-g_{k-1})}$, so has

$$0<(Pg_k)^T(Pg_{k-1})<\min\left(2,\frac{1}{\sigma}\right)\|Pg_k\|^2$$

Wolfe by search standards have

$$\mu_2 d_{k-1}^T(g_k-g_{k-1})=\mu_2 d_{k-1}^T g_k-\mu_2 d_{k-1}^T g_{k-1}$$
$$\geq\mu_2\sigma d_{k-1}^T g_{k-1}-\mu_2 d_{k-1}^T g_{k-1}=\mu_2(\sigma-1)d_{k-1}^T g_{k-1}>0$$

Breathing on both sides by $d_k=-Pg_k+\alpha_k d_{k-1}$ with $(Pg_k)^T$ and $(Pg_k)^T(Pg_{k-1})>0$ can take

$$(Pg_k)^T d_k=(Pg_k)^T(-Pg_k+\alpha_k d_{k-1})=-\|Pg_k\|^2+\alpha_k(Pg_k)^T d_{k-1}$$
$$=-\|Pg_k\|^2+\frac{\|Pg_k\|^2-\mu_1\left|(Pg_k)^T(Pg_{k-1})\right|}{\mu_2 d_{k-1}^T(g_k-g_{k-1})}(Pg_k)^T d_{k-1}$$
$$=\frac{-\|Pg_k\|^2\mu_2 d_{k-1}^T(g_k-g_{k-1})+\|Pg_k\|^2(Pg_k)^T d_{k-1}}{\mu_2 d_{k-1}^T(g_k-g_{k-1})}$$
$$-\frac{\mu_1\left|(Pg_k)^T(Pg_{k-1})\right|(Pg_k)^T d_{k-1}}{\mu_2 d_{k-1}^T(g_k-g_{k-1})}$$
$$=\frac{-\|Pg_k\|^2 d_{k-1}^T g_k(\mu_2-1)+\|Pg_k\|^2\mu_2 d_{k-1}^T g_{k-1}}{\mu_2 d_{k-1}^T(g_k-g_{k-1})}]$$
$$-\frac{\mu_1\sigma\left|(Pg_k)^T(Pg_{k-1})\right|g_{k-1}^T d_{k-1}}{\mu_2 d_{k-1}^T(g_k-g_{k-1})}$$
$$\leq\frac{-\|Pg_k\|^2(\mu_2-1)\sigma d_{k-1}^T g_{k-1}+\|Pg_k\|^2\mu_2 d_{k-1}^T g_{k-1}}{\mu_2 d_{k-1}^T(g_k-g_{k-1})}$$
$$-\frac{\mu_1\sigma\left|(Pg_k)^T(Pg_{k-1})\right|g_{k-1}^T d_{k-1}}{\mu_2 d_{k-1}^T(g_k-g_{k-1})}$$
$$=\frac{-\|Pg_k\|^2\left[(\mu_2-1)\sigma-\mu_2\right]d_{k-1}^T g_{k-1}-\mu_1\sigma\left|(Pg_k)^T(Pg_{k-1})\right|g_{k-1}^T d_{k-1}}{\mu_2 d_{k-1}^T(g_k-g_{k-1})}$$

When $0<\sigma<\frac{1}{2}$, has $0<(Pg_k)^T(Pg_{k-1})<2\|Pg_k\|^2$,so has

$$(Pg_k)^T d_k\leq\frac{-\|Pg_k\|^2\left[(\mu_2-1)\sigma-\mu_2\right]d_{k-1}^T g_{k-1}-2\mu_1\sigma\|Pg_k\|^2 g_{k-1}^T d_{k-1}}{\mu_2 d_{k-1}^T(g_k-g_{k-1})}$$
$$=\frac{-\|Pg_k\|^2 d_{k-1}^T g_{k-1}(\mu_2\sigma-\sigma-\mu_2+2\mu_1\sigma)}{\mu_2 d_{k-1}^T(g_k-g_{k-1})}$$
$$=\frac{-\|Pg_k\|^2 d_{k-1}^T g_{k-1}\left[\mu_2(\sigma-1)-\sigma(1-2\mu_1)\right]}{\mu_2 d_{k-1}^T(g_k-g_{k-1})}$$

because $0<\sigma<\frac{1}{2}$, $0<\mu_1<\frac{1}{2}$, $\mu_2>0$, so $\mu_2(\sigma-1)-\sigma(1-2\mu_1)<0$, And because $\mu_2 d_{k-1}^T(g_k-g_{k-1})>0$,so has $(Pg_k)^T d_k<0$.

When $\frac{1}{2}<\sigma<1$, has $0<(Pg_k)^T(Pg_{k-1})<\frac{1}{\sigma}\|Pg_k\|^2$,so has

$$(Pg_k)^T d_k\leq\frac{-\|Pg_k\|^2\left[(\mu_2-1)\sigma-\mu_2\right]d_{k-1}^T g_{k-1}-\mu_1\|Pg_k\|^2 g_{k-1}^T d_{k-1}}{\mu_2 d_{k-1}^T(g_k-g_{k-1})}$$
$$=\frac{-\|Pg_k\|^2 d_{k-1}^T g_{k-1}(\mu_2\sigma-\sigma-\mu_2+\mu_1)}{\mu_2 d_{k-1}^T(g_k-g_{k-1})}$$
$$=\frac{-\|Pg_k\|^2 d_{k-1}^T g_{k-1}\left[\mu_2(\sigma-1)-(\sigma-\mu_1)\right]}{\mu_2 d_{k-1}^T(g_k-g_{k-1})}$$

because $\frac{1}{2}<\sigma<1$, $0<\mu_1<\sigma$, $\mu_2>0$, so has $\mu_2(\sigma-1)-(\sigma-\mu_1)<0$, and because $\mu_2 d_{k-1}^T(g_k-g_{k-1})>0$, so has $(Pg_k)^T d_k<0$. As proof to have to drop sex was established.

Theorem 1: a problem (P) meet the assumptions (A1-A2), consider algorithm, Step λ_k by Wolfe search criteria to determine, The algorithm generated iteration points listed $\{\beta_k\}$ Or to some k has $Pg_k=0$ Or have the type is founded

$$\liminf_{k\to\infty}\|Pg_k\|=0$$

Proof: if theorem was not, so has $C>0$, makes $\|Pg_k\|^2>C$, $k=1,2\text{L L}$

For $d_k=-Pg_k+\alpha_k d_{k-1}$ take on both ends of the breathing modulus square to get

$$\|d_k\|^2=(\alpha_k)^2\|d_{k-1}\|^2-2(Pg_k)^T d_k-\|Pg_k\|^2 \qquad (3\text{-}1)$$

by lemma 4 available, when $0<\sigma<\frac{1}{2}$,has

$$\alpha_k<\frac{\|Pg_k\|^2}{\mu_2 d_{k-1}^T(g_k-g_{k-1})}\leq-\frac{(Pg_k)^T d_k}{\left[\mu_2(\sigma-1)-\sigma(1-2\mu_1)\right]g_{k-1}^T d_{k-1}}$$

(3-2)

Will (3-2) generation into type (3-1), and in (3-1) type with both sides divided by $\left((Pg_k)^T d_k\right)^2$

$$\frac{\|d_k\|^2}{\left((Pg_k)^T d_k\right)^2}\leq\frac{\|d_{k-1}\|^2}{\left[\left(\mu_2(\sigma-1)-\sigma(1-2\mu_1)\right)g_{k-1}^T d_{k-1}\right]^2}-\frac{2}{(Pg_k)^T d_k}-\frac{\|Pg_k\|^2}{\left((Pg_k)^T d_k\right)^2}$$
$$=\frac{\|d_{k-1}\|^2}{\left[\left(\mu_2(\sigma-1)-\sigma(1-2\mu_1)\right)g_{k-1}^T d_{k-1}\right]^2}-\left(\frac{1}{\|Pg_k\|}+\frac{\|Pg_k\|}{\left((Pg_k)^T d_k\right)}\right)^2+\frac{1}{\|Pg_k\|^2}$$

$$\le \frac{\|d_{k-1}\|^2}{\left[\left(\mu_2(\sigma-1)-\sigma(1-2\mu_1)\right)g_{k-1}^T d_{k-1}\right]^2}+\frac{1}{\|Pg_k\|^2}$$

(3-3)

$$=\frac{\|d_{k-1}\|^2}{\left[\left(\mu_2(\sigma-1)-\sigma(1-2\mu_1)\right)\right]^2\left(\left(Pg_{k-1}\right)^T d_{k-1}\right)^2}+\frac{1}{\|Pg_k\|^2}$$

because $\frac{\|d_1\|^2}{\left(\left(Pg_1\right)^T d_1\right)^2}=\frac{1}{\|Pg_1\|^2}$, So the mean (3-3) recursion

$$\frac{\|d_k\|^2}{\left(\left(Pg_k\right)^T d_k\right)^2}\le\frac{1}{\left[\left(\mu_2(\sigma-1)-\sigma(1-2\mu_1)\right)\right]^2}\frac{1}{\|Pg_1\|^2}+\sum_{i=2}^{k}\frac{1}{\|Pg_i\|^2}$$

$$\le\sum_{i=1}^{k}\frac{1}{\|Pg_i\|^2}\le\frac{K}{C}$$

Namely have $\frac{\left(\left(Pg_k\right)^T d_k\right)^2}{\|d_k\|^2}\ge\frac{C}{K}$,so $\sum_{k=1}^{\infty}\frac{\left(\left(Pg_k\right)^T d_k\right)^2}{\|d_k\|^2}=+\infty$, the lemma 3 and the conclusions of the contradiction, that was not, so has $\liminf_{k\to\infty}\|Pg_k\|=0$.

When $\frac{1}{2}<\sigma<1$, has

$$\alpha_k<\frac{\|Pg_k\|^2}{\mu_2 d_{k-1}^T\left(g_k-g_{k-1}\right)}\le-\frac{\left(Pg_k\right)^T d_k}{\left[\mu_2(\sigma-1)-(\sigma-\mu_1)\right]g_{k-1}^T d_{k-1}},$$

Proof of the above process similar to prove, that also can attain $\liminf_{k\to\infty}\|Pg_k\|=0$. As can $\liminf_{k\to\infty}\|Pg_k\|=0$.

Example analysis

In this section we give numerical example is used to evaluate the presented model in double parameters conjugate gradient method effectiveness of the projection. [10] is the measurement of 9 sample point (its numerical in given in table 1) in equality constraint conditions using linear model,is $y_i=x_1\beta_1+x_2\beta_2+x_3\beta_3+x_4\beta_4+x_5\beta_5+x_6\beta_6+\varepsilon_i$, And through the least squares method solution was β estimator, parameters, is $\beta=(0.038,0.353,0.121,0.286,-0.064,0.273)^T$, Now to the 9 sample point with this half of the variable coefficient model, in order to avoid dimension curse, gets $\alpha(u_i)=\sin(2\pi u_i)$, so the model is $y_i=x_1\beta_1+x_2\beta_2+x_3\beta_3+x_4\beta_4+x_5\beta_5+\sin(2\pi u_i)x_6+\varepsilon_i$, Among them u_i subject to uniform distribution $U[0,1]$, ε_i standard to obey the normal distribution $N[0,1]$, Take kernel function as $Epanechnikov$ function, namely

$K(u)=\frac{1}{\beta(1/2,2)}(1-u^2)=0.75(1-u^2)\ (-1\le u\le1)$, Window wide for $h=0.05$, Using the proposed double parameters conjugate gradient method, through the projection Matlab software the iterative calculation, given parameters estimator for the β''s $\beta'=(0.036,0.356,0.11,0.284,-0.061)^T$, Equality constraint conditions for $A\beta=b$, Among them $A=\begin{pmatrix}a_1\\a_2\end{pmatrix}=\begin{pmatrix}2&6&7&4&1&3\\1&2&4&7&8&9\end{pmatrix}$, $b=\begin{pmatrix}b_1\\b_2\end{pmatrix}=\begin{pmatrix}4.12\\3.21\end{pmatrix}$。

To compare the two methods about target function for a minimum of residual for $RSS_{\min}$, namely $RSS_{\beta}=23.39>RSS_{\beta'}=22.21$, The paper half variable coefficient model is a more ideal model, and prove the double parameters conjugate gradient method projection effectiveness.

Conclusion

Conjugate gradient method is solving large-scale unconstrained optimization is one of the effective methods, and projection gradient algorithm is a constraint optimization problems to solve one of the ways, This paper proposed α_k new double parameters conjugate gradient method, and it projected A contains two parameters, so that they can adjust the parameters to ensure $\alpha_k>0$, thus ensuring the iterative algorithm is more accurate results. In this paper, we prove that the algorithm is full of decline, and proves that the Wolfe in line search conditions are global convergence. After part of main gives the function of $\alpha(\cdot)$ local about coefficient linear estimator.

References:

[1]Zhang W, Lee S Y, Sonf X. Local Polynomial Fitting in Semivarying-coefficient odel[J].Mutival Anal,2002,82:166-188.
[2]Zhang Riquan, Lu Yiqiang. Variable coefficient model [M]. Beijing: science press, 2004:30-32.
[3]Fan J, Zhang W. Statistical Estimation in Varying Coefficient Models[J].Ann Statist,1999,(27):1491-1518.
[4]Gao Jiti, Hong Shengyan, Liang Hua. Part of the linear model estimation of the convergence rate of the [J]. Journal of mathematics, 1995, (5).
[5]Li Zehua, Wu Xiaohua. Under the constraint condition of variable coefficient model parameter estimation of [J]. J, 2010, statistics and decision making.
[6]Fan, J.and Zhang J.T. Two-step estimation of functional linear models with applications to longitudinal data .Journal of the Royal Statistical Society (SeriesB) 2000 (62): 303-322.
[7]Gao Huaiming. Linear constraints of projection of gradient method optimization problem [J]. 2007.
[8]Liang Yumei, Jian Huibao.The optimization of linear constraints a conjugate gradient method projection | J]. The strategy and management. 2003,12 (2):31-35.
[9]G,Zoutendijk , Nolinear programming, computational Methods, In: Integer and No-linear programming,Abedie,ed,North-Holland,Amsterdam,1970,37-86.
[10]Xu Wenyuan,Wang Dongqian. With linear inequality constraints least square system [J]. Science and math. 1984, (1).

Table 1

No	x_1	x_2	x_3	x_4	x_5	x_6	y_i
1	100	100	100	100	100	100	100
2	10.15	72.25	100	100	100	100	85
3	0	0	87.5	98.4	94.7	100	60
4	0	0	32.8	65.55	88.4	100	45
5	0	0	27.3	52.45	86.8	100	37
6	0	0	12.0	15.05	78.8	95.5	29
7	0	0	4.7	0.95	5.2	77.9	22
8	0	0	4.2	0.55	2.4	71.9	19
9	0	0	1.5	0.20	0.4	56.4	15