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Solution of inverse heat conduction problem by a boundary integral method

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ABSTRACT

The presented paper treats as one dimensional inverse heat conduction problem using boundary integral method. Here we try to give an algorithm for the inverse heat conduction problem by using basic solution. To verify our algorithm, we consider a numerical problem and then solve it.

Introduction

In this article we will consider a direct problem and then write its inverse which is treated as inverse heat conduction problem mostly the inverse heat conduction problem is related to heat manufacturing process. Over the years, a number of related computational works have employed various techniques in the analysis of inverse heat conduction problem, for example a boundary element method [3], a fundamental solution method [7-8], a boundary element method[9], Fourier and wavelet method[10] and some other methods [11, 12]. In this paper, we use boundary integral method (BIM) to solve the problem. In Boundary integral method, we use initial and boundary data with the basic solution of given differential equation in some domain on boundary of the domain.

Ammari and Kang [13] also solve inverse heat conduction problem by using boundary integral method. In [13] both single layer potential and double layer potential are used. But our work is based on the use of only single layer potential. In this paper our boundary integral method is based on the output of [14]. In [14] a formula for that heat conduction problem with Neumann boundary condition is discussed. Here the equation is assumed to be homogeneous.

Mathematical formulation of the problem:

Boundary integral method includes the initial and boundary conditions together with the basic solution of a given differential equation defined in some domain say Ω. Further we construct an integral equation on this boundary. By boundary integral method we try to obtain the unknown kernel and after getting it the solution of given problem is obtained by integrating the product of basic solution and unknown kernel over the boundary.

Now we consider the following direct problem.

$$U_t - K^2 U_{xx} = 0 \qquad 0 \le x \le 1, 0 \le t \le T \qquad (2.1)$$

$$U(x,0) = U_0(x), \qquad 0 \le x \le 1 \qquad (2.2)$$

$$\frac{\partial u}{\partial x}(0,t) = \Psi(t) \qquad 0 \le t \le T \qquad (2.3)$$

$$\frac{\partial u}{\partial x}(1,t) = \Phi(t) \qquad 0 \le t \le T \qquad (2.4)$$

When k>0 is a constant and U(x, t) is the temperature distribution function.

The above problem is known as direct problem when it is solved with given ψ(t) and ϕ(t).

Let the inverse problem of above direct problem is

$$U_t - K^2 U_{xx} = 0 \qquad 0 \le x \le 1, 0 \le t \le T \qquad (2.5)$$

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U (x,0) $=U_0$ (x) $0 \leq x \leq 1$ (2.6)

$\frac{\partial u}{\partial x}$ (1,t) = Φ (t) $0 \leq t \leq T$ (2.7)

$\frac{\partial u}{\partial x}$(1,t) = η (t) $0 \leq t \leq T$ (2.8)

where y ε (0,1) is a fixed observation point.

Boundary Integral Method:

Method for Initial data: -

During the study of [14], we observed that the given solution is explained by the boundary integral equation. Here the case of homogeneous initial data is first discussed i.e. we suppose that U_0(x) = 0.

Let G (x,t) denote the basic solution of heat conduction problem then we have

G(x,t) = $\frac{H(t)}{2a\sqrt{\pi t}} \; e^{(-\frac{x^2}{4k^2 t})}$

where H(t) is the Heaviside function.

In [14] only single layer potential is considered and for the direct problem 2.1 to 2.4 following formula is used.

$$U(x,t) = \int_0^t G(x, t-\gamma)\psi_1(\tau)d\tau + \int_0^t G(x-1, t-\tau)\psi_2(\tau)d\tau \quad (3.1)$$

Substituting (3.1) into (2.3) and (2.4) we get

$$\frac{1}{2}\psi_1(t) - \int_0^t \frac{\partial G}{\partial x}(-1, t-\tau)\psi_2(\tau)d\tau = \psi(t) \quad (3.2)$$

$$\frac{1}{2}\psi_2(t) + \int_0^t \frac{\partial G}{\partial x}(1, t-\tau)\psi_1(\tau)d\tau = \phi(t) \quad (3.3)$$

To find the solution of direct problem (2.1) – (2.4) i.e. to find the value of u(x,t) , equation (3.2) and (3.3) is solve with respect to ψ_1(t) and ψ_2(t)

Method for non homogeneous initial data:

Now we discuss the method having non homogeneous initial data. Let us consider the function given by

$$U(x,t) = \int_0^1 G(x-x_0, t)U_0(x_0)dx_0$$

satisfying

$$U_t - k^2 V_{xx} = 0$$

and $\lim_{t\to 0+} V(x,t) = U_0(x)$

Here, we assume that

$$\bar{U} = U - V$$

So that, we get

$\bar{U}_t$ $-a^2\bar{U}_{xx}$ = 0 (x,t) ϵ (0,1) x (0,t)

$\bar{U}$ (x ,0) = 0 x ϵ (0,1)

$\frac{\partial \bar{U}}{\partial x}$ (0,t) = f (t) - $\frac{\partial U}{\partial x}$ (0,t) $0 \leq t \leq T$

$\frac{\partial \bar{U}}{\partial x}$ (1,t) = g (t) - $\frac{\partial U}{\partial x}$ (1,t) $0 \leq t \leq T$

Since we have $\bar{U}$ is the solution of (2.1) – (2.4) with homogeneous initial data, it can be solved by (3.1) – (3.3). Therefore the problem can be solved by superposition of v and solution of homogeneous initial data.

Discretization Method

In this method we partition the time internal t for n elements where n is an integer. Let t_0= 0, t_i= t_0+ ih, i=1,2,3,……..,n where n is an integer and h = $\frac{T}{n}$.

Now we discretize (3.2) and (3.3) as

$$\frac{1}{2}\psi_1(t_i) + h\sum_{J=1}^{i} \frac{\partial G}{\partial X}(-1, t_i - t_j)\psi_2(t_j) = \psi(t_i)$$

i=1,2,3,……,n

$$\frac{1}{2}\psi_2(t_i) + h\sum_{J=1}^{i} \frac{\partial G}{\partial X}(1, t_i - t_j)\psi_1(t_j) = \phi(t_i)$$

i=1,2,3,……,n

In this way, the integral equation (3.2) and (3.3) is changed to linear equations

$$(\tfrac{1}{2} I + G)\, x = \psi \qquad (3.4)$$

where

$$G = \begin{pmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{pmatrix}$$

With $G_{11} = G_{22} = 0$

$$G_{12}(i,J) = h\frac{\partial G}{\partial x}(-1, t_i - t_J)$$

$$G_{21}(i,J) = h\frac{\partial G}{\partial x}(1, t_i - t_J) \qquad \text{if } J \le i$$

$$G_{12}(i,J) = G_{21}(i,J) = 0 \qquad \text{if } J \ge i+1$$

$\Psi = (\psi_1, \psi_2)^T$

$\psi_1(i) = \psi(t_i)$ i= 1,2,3,………,n

$\psi_2(i) = \phi(t_i)$

T denotes the transpose of a vector.

On solving the above linear equation we find the value of x and also we get

ψ_1(i) = x (i) i=1,2,3,……..,n

ψ_2(i) = x (i+ n) i=1,2,3,……..,n

Therefore we obtain the discrete values of ψ_1(t) and ψ_2(t)

Numerical Example:

Let f(t) be a piecewise continuous function and consider the direct problem.

$u_t - k^2 u_{xx} = 0$ 0≤x≤1, 0≤t≤T

U(x,0) = U_0 (x) 0≤x≤1

$\frac{\partial u}{\partial x}$ (0,t) = Ψ (t) 0≤t≤T

$\frac{\partial u}{\partial x}$(1,t) = Φ (t) 0≤t≤T

In this problem we choose

Φ (t) =1 and we define the piecewise continuous function f(t)

$$f(t) = \begin{cases} 1 & 0 \le t \le 5 \\ -1 & 6 \le t \le 10 \end{cases}$$

To find the solution of the direct problem we use the boundary integral method described in the presented paper. After obtaining the numerical values from above, we use it as measured data for the inverse problem. Here we choose y = 0.5, M =50, N =55.

Therefore the condition number of the coefficient matrix G is 50.682

Let f^*(t) denote the numerical result for the inverse problem.

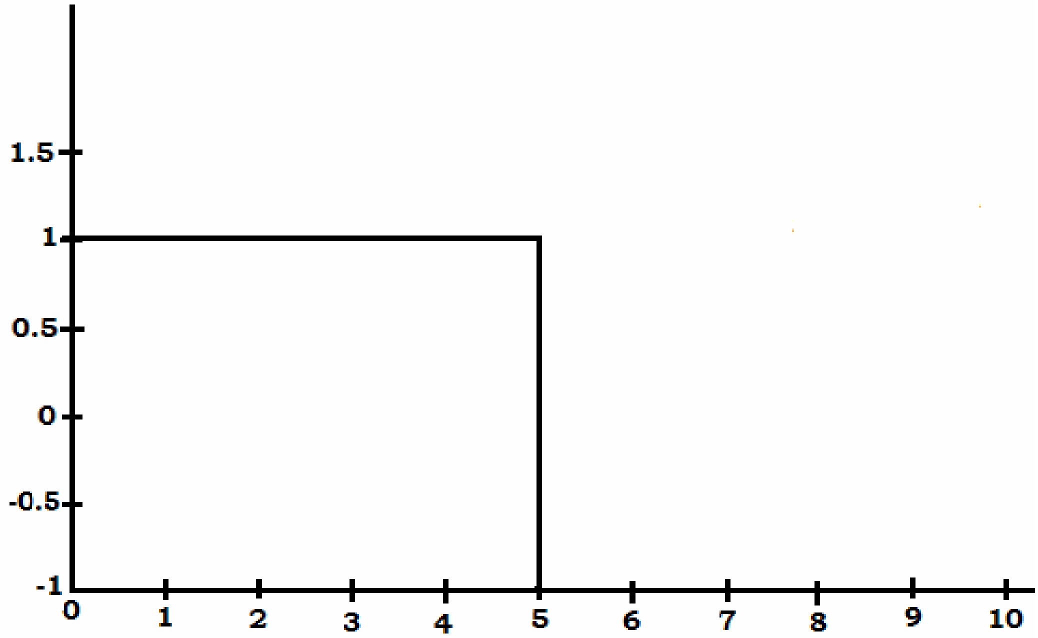

Fig.1

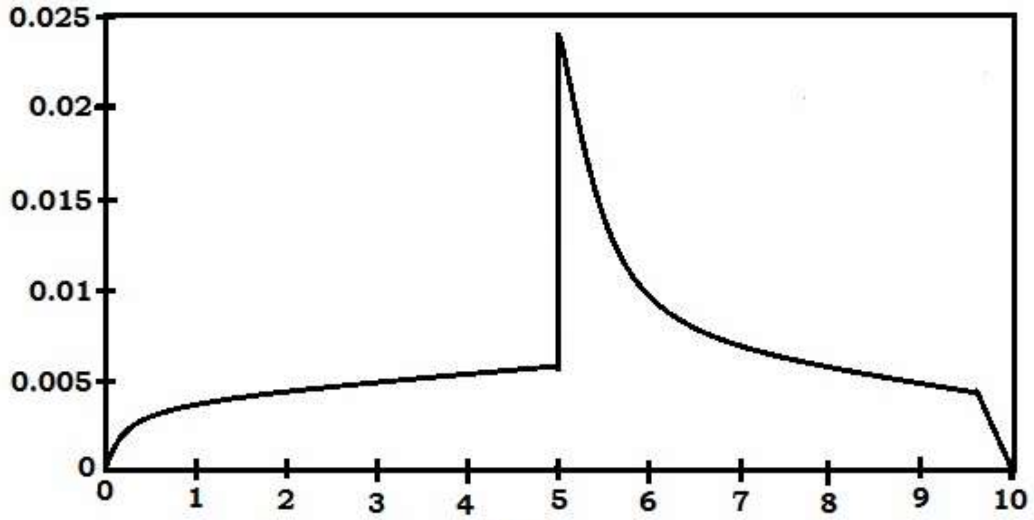

Fig.2

The numerical result for f(t) is shown in figure 5a and the error between exact f(t) and numerical result $f^*(t)$ is shown in figure 5b.

Conclusion:

In this paper we consider the one dimensional inverse heat conduction problem. Here we try to solve the inverse heat conduction problem using boundary integral method. In presented paper we discussed the boundary integral method for the one dimensional inverse heat conduction problem. Also we applied it on a numerical example. The numerical result shows that the method is effective. In a similar way the method is applied for two and three dimensional cases.

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