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Numerical solution of navier stokes equations using staggered grid for two dimensional domain

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ABSTRACT
The Navier–Stokes equations, named after Claude-Louis Navier and George Gabriel Stokes, describe the motion of fluid substances. These equations arise from applying Newton's second law to fluid motion, together with the assumption that the stress in the fluid is the sum of a diffusing viscous term (proportional to the gradient of velocity) and a pressure term. For solving a free surface flow problem, the basic governing equations are Navier Stokes equations (N-S equations). Hence the study of methods, used for solving N-S equations is important. There are various methods to solve the N-S equations. The equations are useful as they describe the physics of many things of scientific and engineering interest. The Navier–Stokes equations are also of great interest in a purely mathematical sense.

Introduction

The Navier-Stokes equations are the fundamental partial differentials equations that describe the flow of incompressible fluids. The Navier–Stokes equations in their full and simplified forms help with the design of aircraft and cars, the study of blood flow, the design of power stations, the analysis of pollution, and many other things. Coupled with Maxwell's equations they can be used to model and study magneto hydrodynamics. When combined with the continuity equation of fluid flow, the Navier-Stokes equations yield four equations in four unknowns (namely the scalar and vector u). However, except in degenerate cases in very simple geometries (such as Stokes flow, these equations cannot be solved exactly, so approximations are commonly made to allow the equations to be solved approximately. Changes in properties of a moving fluid can be measured in two different ways. One can measure a given property by either carrying out the measurement on a fixed point in space as particles of the fluid pass by, or by following a parcel of fluid along its streamline. As it must, the Navier-Stokes equations satisfy conservation of mass, momentum, and energy. Mass conservation is included implicitly through the continuity equation [1].

Basic Governing Equations

Conservation of mass (Continuity):

$$\frac{\partial}{\partial t}(\rho)+\frac{\partial}{\partial x}(\rho u)+\frac{\partial}{\partial y}(\rho v)=0$$

Conservation of X- momentum:

$$\frac{\partial}{\partial t}(\rho u)+\frac{\partial}{\partial x}(\rho uu)+\frac{\partial}{\partial y}(\rho uv)=-\frac{\partial p}{\partial x}+\frac{\partial}{\partial x}\left(\mu\frac{\partial u}{\partial x}\right)+\frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right)$$

Conservation of Y-momentum

$$\frac{\partial}{\partial t}(\rho u)+\frac{\partial}{\partial x}(\rho uv)+\frac{\partial}{\partial y}(\rho vv)=-\frac{\partial p}{\partial y}+\frac{\partial}{\partial x}\left(\mu\frac{\partial v}{\partial x}\right)+\frac{\partial}{\partial y}\left(\mu\frac{\partial v}{\partial y}\right)$$

Grid

There are two types of grids

1. Collocated Grid: In this type of system all the variables are defines at cell centers. The X–momentum, Y–momentum and continuity cell is the same cell.

2. Staggered Grid: In this type of system, the vector quantities are defined at the cell faces, while scalar quantities are defined at cell centers. The continuity cell is the cell with density at the centre. X–momentum and Y–momentum cells are the imaginary cells with

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respectively u and v velocities at centre respectively. We have chosen to work with staggered grid. The staggered grid with continuity, X– momentum and Y–momentum cells.

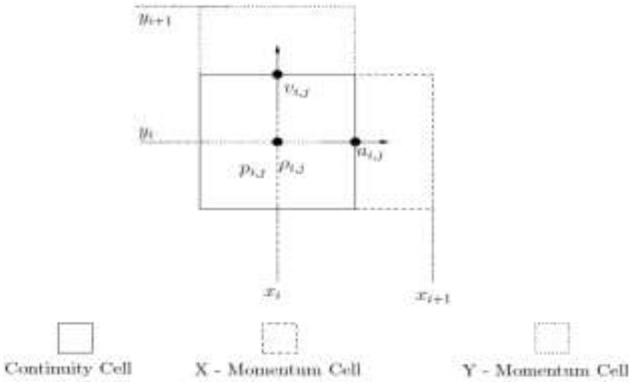

Figure 1: Grid to solve Navier stokes equation [2]

Descretization (Control Volume approach)

In fluid mechanics the conservation equations (for instance, the Navier-Stokes equations) are in integral form. They therefore apply on volumes. Finding forms of the equation that are independent of the control volumes allows simplification of the integral signs. In fluid mechanics and thermodynamics, a control volume is a mathematical abstraction employed in the process of creating mathematical models of physical processes. In descretizing the Navier Stokes equations, we have made certain assumptions.

1. Fluid is incompressible which implies that ρ is a constant.
2. Grid spacing is uniform.
3. There can be a variation of property or a variable along the cell face. The mean value of the property or variable over a face is assumed to be the value at centre of the face.
4. Properties of the fluid are constant. For solving the problem, the central difference scheme for convection term is selected.

For solving the implicit pressure equation Guess–Siedel Method is used [3, 4].The descretized forms of the basic governing equations are:

Continuity equation:

$$\frac{u_{i,j}-u_{i-1,j}}{\Delta x}+\frac{v_{i,j}-v_{i,j-1}}{\Delta y}=0$$

X–momentum equation:

$$u_{i,j}^{n+1}=u_{i,j}*\left\{1-\Delta t*\left[\left(\frac{u_{i-1,j}-u_{i+1,j}}{4\Delta x}\right)+\left(\frac{v_{i,j-1}+v_{i+1,j-1}-v_{i,j}-v_{i+1,j}}{4\Delta y}\right)+\frac{2\mu}{\rho}+\left(\frac{1}{\Delta x^2}+\frac{1}{\Delta y^2}\right)\right]\right\}+u_{i+1,j}*\frac{\Delta t}{\Delta x}*\left(\frac{\frac{\mu}{\rho}}{\Delta x}-\left(\frac{u_{i,j}+u_{i+1,j}}{4}\right)\right)+u_{i-1,j}*\frac{\Delta t}{\Delta x}*\left(\left(\frac{u_{i,j}+u_{i-1,j}}{4}\right)+\frac{\frac{\mu}{\rho}}{\Delta x}\right)+u_{i,j+1}*\frac{\Delta t}{\Delta y}*\left(\frac{\frac{\mu}{\rho}}{\Delta y}-\left(\frac{v_{i,j}+v_{i+1,j}}{4}\right)\right)+u_{i,j-1}*\frac{\Delta t}{\Delta y}*\left(\frac{\frac{\mu}{\rho}}{\Delta y}+\left(\frac{v_{i,j}+v_{i-1,j}}{4}\right)\right)+\left(p_{i,j}-p_{i+1,j}\right)*\frac{\Delta t}{\rho\Delta x}$$

Y–momentum equation:

$$v_{i,j}^{n+1} = v_{i,j} * \left\{1 - \Delta t * \left[\left(\frac{u_{i-1,j} + u_{i-1,j+1} - u_{i,j} - u_{i,j+1}}{4\Delta x}\right) + \left(\frac{v_{i,j-1} - v_{i,j+1}}{4\Delta y}\right) + \frac{2\mu}{\rho} * \left(\frac{1}{\Delta x^2} + \frac{1}{\Delta y^2}\right)\right]\right\} + v_{i+1,j} * \frac{\Delta t}{\Delta x} * \left(\frac{\frac{\mu}{\rho}}{\Delta x} - \left(\frac{u_{i,j} + u_{i,j+1}}{4}\right)\right) + v_{i-1,j} * \frac{\Delta t}{\Delta x} * \left(\frac{\frac{\mu}{\rho}}{\Delta x} - \left(\frac{u_{i,j} + u_{i-1,j+1}}{4}\right)\right) + v_{i,j-1} * \frac{\Delta t}{\Delta y} * \left(\frac{\frac{\mu}{\rho}}{\Delta y} - \left(\frac{v_{i,j} + v_{i,j-1}}{4}\right)\right) + v_{i,j+1} * \frac{\Delta t}{\Delta y} * \left(\frac{\frac{\mu}{\rho}}{\Delta y} - \left(\frac{u_{i,j} + u_{i,j+1}}{4}\right)\right) + \left(p_{i,j} - p_{i,j+1}\right) * \frac{\Delta t}{\rho \Delta y}$$

Stability

Observing the coefficients of every term indicates that, there is a stability criterion as follows. One condition is similar to the Fourier condition. For each time step Fo ≤ 0.5 Consider the coefficient of ui,j

$$\left\{1 - * \Delta t \left[\left(\frac{u_{i-1,j} - u_{i+1,j}}{4\Delta x}\right) + \left(\frac{v_{i,j-1} + v_{i+1,j-1} - v_{i,j} - v_{i+1,j}}{4\Delta y}\right) + \frac{2\mu}{\rho} + \left(\frac{1}{\Delta x^2} + \frac{1}{\Delta y^2}\right)\right]\right\}$$

For the coefficient to be non negative, we have

$$\Delta t_{max} = \frac{1}{\left\{1 - * \Delta t \left[\left(\frac{u_{i-1,j} - u_{i+1,j}}{4\Delta x}\right) + \left(\frac{v_{i,j-1} + v_{i+1,j-1} - v_{i,j} - v_{i+1,j}}{4\Delta y}\right) + \frac{2\mu}{\rho}\left(\frac{1}{\Delta x^2} + \frac{1}{\Delta y^2}\right)\right]\right\}}$$

Hence for each time step

$\Delta t < \Delta t_{max}$

Solution Procedure

1. The complete velocity field is known at the beginning of cycle either as a result of previous cycle of calculation or from prescribed initial condition. It is assumed that initial velocity field obeys mass conservation.
2. The time step Δt is calculated so as to proceed in time. The Δt is chosen such that it does not violate stability conditions.
3. Calculate the initial guess for velocity by using previous pressure field as,

$$u\hat{}_{i,j} = u^{n}{}_{i,j} + \Delta t * \left[\left\{\frac{1}{\rho \Delta x}\right\} * \left(p^{n}{}_{i,j} - p^{n}{}_{i+1,j}\right) + \dots\right]$$

$$v\hat{}_{i,j} = v^{n}{}_{i,j} + \Delta t * \left[\left\{\frac{1}{\rho \Delta y}\right\} * \left(p^{n}{}_{i,j} - p^{n}{}_{i,j+1}\right) + \dots\right]$$

Required velocity is given by

$$u^{n+1}{}_{i,j} = u^{n}{}_{i,j} + \Delta t * \left[\left\{\frac{1}{\rho \Delta x}\right\} * \left(p^{n+1}{}_{i,j} - p^{n}{}_{i+1,j}\right) + \dots\right]$$

$$v^{n+1}{}_{i,j} = v^{n}{}_{i,j} + \Delta t * \left[\left\{\frac{1}{\rho \Delta y}\right\} * \left(p^{n+1}{}_{i,j} - p^{n}{}_{i,j+1}\right) + \dots\right]$$

Using guessed velocities continuity equation can be written as

$$\frac{u\hat{}_{i,j} - u\hat{}_{i-1,j}}{\Delta x} + \frac{v\hat{}_{i,j} - v\hat{}_{i,j-1}}{\Delta y} = s_{i,j}$$

Where $s_{i,j}$ is mass source. With modified velocities, the continuity equation can be written as,

$$\frac{u^{n+1}{}_{i,j} - u^{n+1}{}_{i-1,j}}{\Delta x} + \frac{v^{n+1}{}_{i,j} - v^{n+1}{}_{i,j-1}}{\Delta y} = s_{i,j}$$

$$u'_{i,j} = \frac{\Delta t}{\rho \Delta x} * \left(p'_{i,j} - p'_{i+1,j}\right)$$

$$v'_{i,j} = \frac{\Delta t}{\rho \Delta y} * \left(p'_{i,j} - p'_{i,j+1}\right)$$

Get values of guessed pressure by solving pressure correction equation and calculate velocity for next time step as guessed velocity plus corrected velocity.

4. Repeat the procedure until steady state is reached [2].

Results and comparison

For a driven cavity problem, plots of u and v velocity versus dimension of the cavity are plotted and results are compared with Ghia and Ghia [5]. Results by Ghia and Ghia are treated as benchmark.

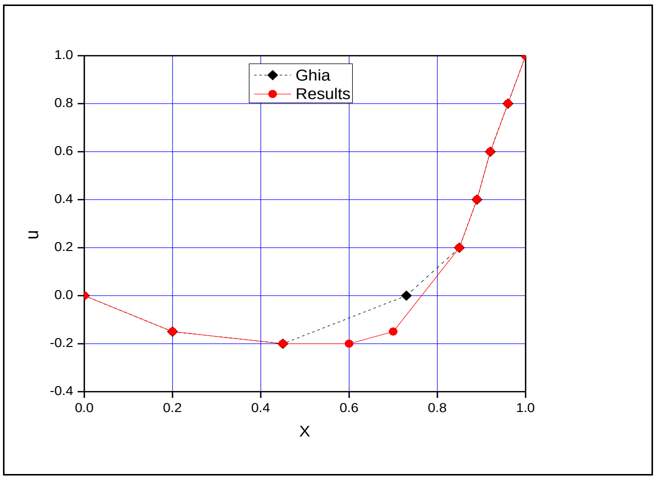

Figure 2. Comparison of u velocity with Ghia's result

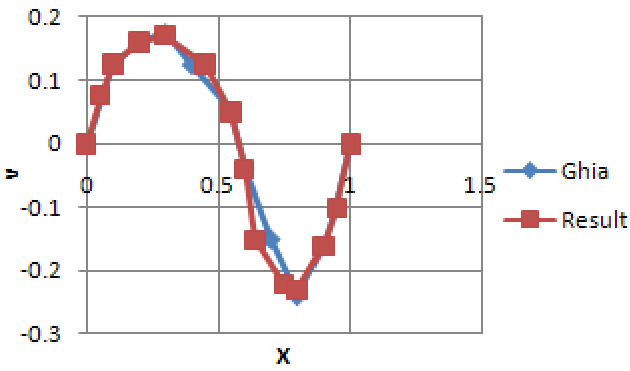

Figure 3. Comparison of v velocity with Ghia's result

Conclusions

For determining velocity field of fluid domain N-S equations are used. Here velocity field of two dimensional square domains is calculated and results are compared with Ghia and Ghia. The results are found consistent.

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Nomenclature

p Pressure

t Time

μ Dynamic viscosity

υ Kinematic viscosity

ρ Density

u Horizontal velocity component

v Vertical velocity component

û Guessed horizontal velocity component

Subscripts

i X-direction index

J Y-direction index

Superscripts

n Old time step

n+1 New time step

x In X direction

y In Y direction

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