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Remarks on Weak compactness via Grills

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ABSTRACT
In this paper we define sets G-weakly compact relative to a topological space through θ-open sets. Further we investigate the relationships between these sets and G θ-weakly compact subspaces.

Introduction

Cammaroto and Lo Faro [3] introduced and characterized the notion of weakly compact spaces which is strictly weaker than the notion of almost-compactness due to Sin- gal and Singal [11]. Choquet [5] introduced an attractive theory of Grills; a collection satisfies some condition in Topological space. The topology equipped with the grill collection is called Grill topology. This topic has an excellent potential for application in other branches of mathematics like compactifications, Proximity spaces, different types of extension problems etc. This subject was continued to study by general topologists Roy and Mukherjee [7], [8] in recent years. They further introduced different types of compactness properties termed as G -compactness, G -paracompactness and so on. The purpose of this paper is to define G -weakly compact sets through θ-open sets and investigate their basic properties.

Preliminaries

Definition 2.1: [7]

A collection G of nonempty subsets of a set X is called a grill if

1. $A \in G$ and $A \subseteq B \subseteq X$ implies that $B \subseteq G$, and
2. $A \cup B \in G$ $(A, B \subseteq X)$ implies that $A \in G$ or $B \in G$.

Definition 2.2:[7]

A grill G on a set X is said to be a σ-grill if for any countable collection $\{A \ : n \in N\}$ of subsets of X, $A \notin G$ whenever A $\notin G$ for each $n \in N$.

Definition 2.3: [11]

A subset of a space is said to be regularly open, it is the interior of some closed set or equivalently, if it is the interior of its own closure. A set is said to be regularly closed if it is the closure of some open set or equivalently, if it is the closure of its own interior.

Definition 2.4 [1]

A topological space with the grill G is said to be almost G-Compact if for every open cover $\nu = \{U_\alpha \ : \alpha \in \Lambda\}$ there exists a finite sub collection $\{U_i \ : i \in I\}$ such that $X \setminus \cup_{i=1} clU_i \notin G$.

Definition 2.5: [4]

A space X is said to be almost-regular if for each point $x \in X$ and each neighbourhood U of x, there exists a neighbourhood V of p such that $V \subset intclU$.

Weak compactness by θ-open sets

Definition 3.1

A θ-open cover $\{V_\alpha \ : \alpha \in \Lambda\}$ of a space X is said to be θ-regular if for each $\alpha \in \Lambda$ there exists a nonempty regular closed set F_α in X such that $F_\alpha \subset V_\alpha$ and $X = \cup\{int(F_\alpha) : \alpha \in \Lambda\}$.

Definition 3.2

A space (X, τ, G) is said to be G -θ weakly compact if every θ-regular cover of X has a finite subfamily such that, $X \setminus \cup^n_{i=1} clV_i \notin G$.

Definition 3.3

A subset S of a space (X, τ, G) is said to be G -θ weakly compact if S is G -θ weakly compact as the subspace of X.

Definition 3.4

A subset S of a space (X, τ, G) is said to be G -θ weakly compact relative to X if for each θ-regular cover $\{V_\alpha \ : \alpha \in \lambda\}$ of S by θ-open sets of X there exists a finite subset Λ_0 of Λ such that $S \setminus \cup \{cl(V_\alpha); \alpha \in \lambda_0\}$.

Definition 3.5

A space (X, τ, G) is said to be G θ-almost compact if every θ-open cover of X has finite subfamily such that , $X \setminus \cup^n_{i=1} clV_i \notin G$.

Remark 3.6

Every G -θ almost compact space is G -θ weakly compact.

Theorem 3.7

If A is a θ-open set as well as G -θ weakly compact subspace of a space (X, τ, G), then A is G- θ weakly compact relative to X where G is a σ-grill.

Proof

Let $\{V_\alpha : \alpha \in \Lambda\}$ be a θ-regular cover of A by θ-open sets of X. Then there exists a nonempty regular closed set F_α such that $F_\alpha \subset V_\alpha$ and $A \subset \cup\{int(F_\alpha) : \alpha \in \Lambda\}$. For each $\alpha \in \Lambda$, int $(F_\alpha) \cap A$ is θ-open cover. Also $F_\alpha \cap A$ is closed in A. But $cl_A(int(F_\alpha) \cap A) \subset F_\alpha \cap A \subset V_\alpha \cap A$. Moreover $A = \cup\{int(F_\alpha) \cap A\}$ and $int(F_\alpha) \cap A \subset int_A(cl_A(int(F_\alpha) \cap A))$. But $cl_A(Int(F_\alpha) \cap A)$ is a regular closed in A and $\{V_\alpha \cap A : \alpha \in \Lambda\}$ is a θ-regular cover of the subspace A. Since A is a G- θ weakly compact as a subspace there exists a finite subset Λ_0 of Λ such that , $A \setminus \cup_{\alpha\in\Lambda_0}\{cl_A(V_\alpha \cap A)\} \notin G$. Also $cl_A(V_\alpha \cap A) \subset cl(V_\alpha)$ for each $\alpha \in \Lambda_0$ and so we can obtain a finite subcollection such that , $A \setminus \cup_{\alpha\in\Lambda_0} \{cl(V_\alpha)\} \notin G$. Hence the proof.

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Theorem 3.8

If every proper regular closed subset of a space (X, τ, G) is G -θ weakly compact relative to X, then X is G-θ weakly compact, where G is a σ-grill.

Proof

Let $\{V_\alpha : \alpha \in \Lambda\}$ be a θ regular cover of X. Then, for every $\alpha \in \Lambda$ there exists a nonempty regular closed set F_α in X such that $F_\alpha \subset V_\alpha$ and $X = \cup\{int(F_\alpha) : \alpha \in \Lambda\}$. Choose α_0 where $\alpha_0 \in \Lambda$, let $K = X - int(F_{\alpha 0})$, then K is a regular closed set in X and $K \subset \cup\{int(F_\alpha) : \alpha \in \Lambda - \alpha_0\}$. Therefore $\{V_\alpha : \alpha \in \Lambda - \alpha_0\}$ is a cover of K by θ-open sets of X satisfying θ-regular cover property. and hence for some finite subset Λ_0 of Λ we have $K \setminus \cup \{cl(V_\alpha); \alpha \in \Lambda_0\} \notin G$. But we know that $X = K \cup (X - K)$. Also $X = K \cup int(F_{\alpha 0})$.

That is $X = K \cup V_{\alpha 0}$. Thus we obtain $X \setminus \cup \{cl(V_\alpha) : \alpha \in \Lambda_0 \cup \{\alpha_0\}\} \notin G$, hence X is G -θ weak compact.

Corollary 3.9

If every proper regular closed subset of a space X is G- θ weakly compact, then X is G- θ weak compact.

Theorem 3.10

Let (X, τ, G) be a G- θ weakly compact space. If A is a θ-closed subset of X, then A is G- θ weakly compact relative to X where G is a σ-grill.

Proof:

Let $\{V_\alpha : \alpha \in \Lambda\}$ be a θ-regular cover of A by θ-open sets. Suppose that $X - A \neq \varphi$. Since A is θ-closed set in X, $X - A$ is a θ-open in X. Therefore, the family $\{V_\alpha : \alpha \in \Lambda\} \cup \{X - A\}$ is a θ- regular cover of X by θ-open sets. Since X is G-θ weakly compact there exists a finite subset Λ_0 of Λ such that $X \setminus [\cup_{\alpha\in\Lambda 0} cl(V_\alpha) \cup cl(X - A)] \notin G$. But we know that $A \not\subseteq (X - A)$, Then $A \setminus [\cup_{\alpha\in\Lambda 0} cl(V_\alpha)] \notin G$. Hence the proof.

Definition 3.11

A function $f : (X, \tau_1, G_1) \rightarrow (Y, \tau_2, G_2)$ is said to be almost θ-open if and only if $f^{-1}(cl(V)) \subset cl(f^{-1}(V))$ for every θ-open set V of Y.

Theorem 3.12

Let $f : (X, \tau_1, G_1) \rightarrow (Y, \tau_2, G_2)$ be an almost θ-open perfect function. If K is G_2-weakly compact relative to Y, then $f^{-1}(K)$ is G_1-weakly compact relative to X where G is a σ-grill.

Proof

Let K be G_2-weakly compact relative to Y and $\{U_\alpha : \alpha \in \Lambda\}$ any θ-open regular cover of $f^{-1}(K)$. For each $\alpha \in \Lambda$, there exists a regular closed set F_α of X such that $F_\alpha \subset U_\alpha$ and $f^{-1}(K) \subset \cup\{int(F_\alpha) : \alpha \in \Lambda\}$. Since f is a perfect function for each $y \in K$, $\{f^{-1}(y)\}$ is compact and there exists a finite subset $\Lambda(y)$ of Λ such that $\{f^{-1}(y)\} \subset \cup\{int(F_\alpha) : \alpha \in \Lambda(y)\}$. Therefore we have, $\{f^{-1}(y)\} \subset \cup_{\alpha \in\Lambda(y)} int(F_\alpha) \subset \cup_{\alpha\in\Lambda(y)}F_\alpha \subset \cup_{\alpha\in\Lambda(y)}U_\alpha$. Now, we put $G_y = Y - f(X - \cup\{int(F_\alpha) : \alpha \in \Lambda(y)\})$. $H_y = Y - int(cl(f(X - \cup\{F_\alpha : \alpha \in \Lambda(y)\})))$ and $V_y = Y - f(X - (\cup\{U_\alpha : \alpha \in \Lambda(y)\}))$.

The closeness of f implies that V_y are θ-open in Y. Since f is almost θ-open, H_y is regular closed in Y and $y \in G_y \subset H_y \subset V_y$. Therefore the family $\{V_y : y \in K\}$ is a regular cover by θ-open sets. Since K is G_2 θ weakly compact relative to Y, there exists a finite number of points of K, say $y_1, y_2, ...y_n$ such that $K \setminus \cup\{cl(V_{yi}) : i = 1, 2, ...n\} \notin G$.

Therefore we obtain that $f^{-1}(K) \setminus \cup\{cl(U_\alpha) : \alpha \in \Lambda(_{yi}), i = 1, 2, 3, ...n\} \notin G_1$. This shows that $f -1(K)$ is G_1 -θ weakly compact relative to X.

Theorem 3.12

Let $f : (X, \tau_1, G_1) \rightarrow (Y, \tau_2, G_2)$ be an almost θ-open perfect function. If K is G_2-weakly compact relative to Y, then $f^{-1}(K)$ is G_1-weakly compact relative to X where G is a σ-grill.

Proof

Let K be G_2-weakly compact relative to Y and $\{U_\alpha : \alpha \in \Lambda\}$ any θ-open regular cover of $f^{-1}(K)$. For each $\alpha \in \Lambda$, there exists a regular closed set F_α of X such that $F_\alpha \subset U_\alpha$ and $f^{-1}(K) \subset \cup\{int(F_\alpha) : \alpha \in \Lambda\}$. Since f is a perfect function for each $y \in K$, $\{f^{-1}(y)\}$ is compact and there exists a finite subset $\Lambda(y)$ of Λ such that $\{f^{-1}(y)\} \subset \cup\{int(F_\alpha) : \alpha \in \Lambda(y)\}$. Therefore we have, $\{f^{-1}(y)\} \subset \cup_{\alpha \in\Lambda(y)} int(F\alpha) \subset \cup_{\alpha\in\Lambda(y)}F\alpha \subset \cup_{\alpha\in\Lambda(y)}U_\alpha$. Now, we put $G_y = Y - f(X - \cup\{int(F_\alpha) : \alpha \in \Lambda(y)\})$. $H_y = Y - int(cl(f(X - \cup\{F_\alpha : \alpha \in \Lambda(y)\})))$ and $V_y = Y - f(X - (\cup\{U_\alpha : \alpha \in \Lambda(y)\}))$.

The closeness of f implies that V_y are θ-open in Y. Since f is almost θ-open, H_y is regular closed in Y and $y \in G_y \subset H_y \subset V_y$. Therefore the family $\{V_y : y \in K\}$ is a regular cover by θ-open sets. Since K is G_2- θ weakly compact relative to Y, there exists a finite number of points of K, say $y_1, y_2, ...y_n$ such that $K \setminus \cup\{cl(V_{yi}) : i = 1, 2, ...n\} \notin G$.

Therefore we obtain that $f^{-1}(K) \setminus \cup \{cl(U_\alpha) : \alpha \in \Lambda(yi), i = 1, 2, 3, ...n\} \notin G_1$ This shows that $f^{-1}(K)$ is G_1- θ weakly compact relative to X.

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