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On (m, n) –upper Q-fuzzy soft subgroups

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ABSTRACT
In this Paper, we introduce the notions of (m, n)-Upper Q- fuzzy subgroups, studied some properties of them and discussed the product of them.

Introduction

Fuzzy sets was first introduced by Zadeh [20] and then the fuzzy sets have been used in the reconsideration of classical mathematics. Yuan et al. [17] introduced the concept of fuzzy subgroup with thresholds. A fuzzy subgroup with thresholds l and μ is also called a (λ, μ)-fuzzy subgroup. Yao continued to research (λ, μ)-fuzzy normal subgroups, (λ, μ)-fuzzy quotient subgroups and (λ, μ)-fuzzy sub rings in [18,19]. Shen researched anti-fuzzy subgroups in [11] and Dong [6] studied the product of anti-fuzzy subgroups. Molodtsov [7] introduced the concept of soft sets that can be seen as a new mathematical theory for dealing with uncertainty. Molodtsov applied this theory to several directions [7], and then formulated the notion of soft number, soft derivative, soft integral, etc in [8] , the soft set theory has been applied to many different fields with great success. Maji et.al [8] worked on theoretical study of soft sets in detail presented an application of soft set in the decision making problem using the reduction of soft sets.

Majiet.al [9] presented the concept of fuzzy soft sets by embedding the ideas of fuzzy sets. By using this definition of fuzzy soft sets many interesting applications of soft set theory have been expanded by some Researchers. Roy and Maji [9] gave some applications of fuzzy soft sets. A.Solairaju and R.Nagarajan introduced the concept of Structures of Q- fuzzy groups [12]. A.Solairaju and R.Nagarajan studied some structure properties of upper Q-fuzzy index order with upper Q- fuzzy subgroups[13].In this paper, We analyze the notions of (m, n)-Upper Q- fuzzy subgroups, studied some properties for upper Q-fuzzy subgroups, some propositions and theorem for this subject and discussed the product of them.

Preliminaries

This section, briefly reviews the basic characteristics of fuzzy set and fuzzy soft sets.

Definition 2.1:[19] By a fuzzy subset of a nonempty set X we mean a mapping from X to the unit interval [0,1].

Definition: [11]If A is a fuzzy subset of X, then we denote $A(a) = \{x \varepsilon X | A(x) \leq \alpha\}$ for all $\alpha \varepsilon [0,1]$.

Throughout this article, we will always assume that $0 \leq m < n \leq 1$.

Let G, G_1, and G_2 always denote groups in the following. I, I_1, and I_2 are identities of G, G_1, and G_2, respectively.

Definition-2.2:[7] Let U be an initial universe, P (U) be the power set of U, E be the set of all parameters and A⊆ E. A soft set (f_A, E) on the universe U is defined by the set of order pairs (f_A, E) = {(e, f_A (e)): e∈ E, f_A∈ P (U)} where $f_A : E \rightarrow P(U)$ such that f_A (e) = ϕ if e ∉ A.

Here f_A is called an approximate function of the soft set.

Example: Let U = {u_1,u_2,u_3,u_4} be a set of four shirts and E = {white(e_1),red(e_2),blue (e_3)}

be a set of parameters. If A = {e_1 ,e_2} ⊆ E , f_A(e_1) = {u_1, u_2 ,u_3, u_4} and

f_A(e_2)= {u_1,u_2,u_3} , then we write the soft set (f_A , E)= {(e_1, {u_1, u_2 ,u_3, u_4}), (e_2,{u_1,u_2,u_3})} over U which describe the “colour of the shirts” which Mr. X is going to buy.

Definition-2.3:[9] Let U be an initial universe, E be the set of all parameters and A⊆ E. A pair (F, A) is called a fuzzy soft set over U where F: A → P (U) is a mapping from A into P (U) , where P (U) denotes the collection of all fuzzy subsets of U.

Example: Consider the above example, here we cannot express with only two real numbers 0 and 1, we can characterized it by a membership function instead of crisp number 0 and 1, which associate with each element a real number in the interval [0,1].Then

(f_A, E) = {{f_A(e_1) = {(u_1, 0.7), (u_2, 0.5), (u_3, 0.4), (u_4, 0.2)},

f_A (e_2) = {(u_1, 0.5), (u_2, 0.1), (u_3, 0.5)}} is the fuzzy soft set representing the “colour of the shirts” which Mr. X is going to buy.

Definition2.4; Afuzzy soft set A of a group G is called (m,n)-Upper Q-fuzzy soft subgroup of G if for all x,y z ε G,

[UFSG1] $A(xy,q) \cap m \leq \max\{A(x,q), A(y,q)\} \vee n$ and

[UFSG2] $A(x^{-1},q) \cap m \leq A(x,q) \vee n$ where x-1 is the inverse element of x.

Throughout this article, max {A(x,q), A(y,q)} is sometimes replaced as A(x,q) V A(y,q).

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Proposition-1: If A is (m,n) –upper Q-fuzzy soft group of G, then $A(1,q) \cap m \leq A(x,q)$ Vn for all x ε G, when 1 is the identity of G.

Proof: For all x εG and let x^{-1} be the inverse element of x. Then $A(1,q) \cap m = A(xx^{-1},q) \cap m = (A(xx^{-1},q) \cap m) \cap m \leq \max \{A(x,q), A(x^{-1},q)\}$ V n = max { $A(x,q) \cap m$, $A(x^{-1},q) \cap m$}V n(nUm) = A(x,q) Un.

Main results

Theorem-3.1: Let A be upper Q-fuzzy soft subset of a group G.Then A is (m,n)-upper Q-fuzzy soft group of G if and only if $A(x^{-1}y,q) \cap m \leq \max\{A(x,q), A(y,q)\}$Vn for all x,y ε G.

Proof: Let A is (m,n)-upper Q-fuzzy soft group of G.Then

$A(x^{-1}y,q) \cap m = A(x^{-1}y,q) \cap m \cap m \leq (\max\{A(x^{-1},q),A(y,q)\}Vn) \cap m$

$= (A(x^{-1},q) \cap m, A(y,q))V(n \cap m) \leq ((A(x,q)Vn)\ V\ (A(y,q)))$ Vn = max {A(x,q), A(y,q)}Vn.

Conversely, suppose $A(x^{-1}y,q) \cap m \leq \max\{A(x,q), A(y,q)\}$Vn for all x,y ε G, then $A(1,q) \cap m = A(x^{-1}x,q) \cap m \leq \max\{A(x,q), A(y,q)\}$Vn =A(x,q)Vn.

So $(A(x^{-1},q) \cap m = (A(x^{-1}.1,q) \cap m = (A(x^{-1},q) \cap m \cap m \leq \max\{A(x^{-1},q),VA(y,q)\}Vn) \cap m$

$=(A(x^{-1},q) \cap m)V(A(y,q)Vn) \cap m \leq \max\{(A(x,q)Vn), (A(y,q)\}Vn)\}$

= max{A(x,q), A(y,q)}Vn. So A is a (m,n)-upper Q-fuzzy soft group of G.

Theorem-3.2: Let A be a Q-fuzzy soft subset of a group. Then the following are equivalent

(i) A is (m,n)-upper Q-fuuzy soft subgroup of G.(ii)A(α) is a subgroup of G, for any αε [m,n], where A(α) ≠Φ.

Proof: "(i) implies (ii)"

Let A be [m,n]-upper Q-fuzzy soft subgroup of G.For any αε [m,n],such that A(α) ≠Φ,we need to show that $x^{-1}y$ε A(α),for all x,y ε A(α).Since A(x,q) ≤ α and A(y,q) ≤ α, then $(A(x^{-1},q) \cap m \leq \max\{(A(x,q)Vn), (A(y,q)\}Vn)\} \leq \max\{\alpha,\alpha\}$Vn =αVn=α. Note that α ≤ n, we obtain $(A(x^{-1}y,q) \leq \alpha$.so $x^{-1}y$ ε A(α).

"(ii) implies (i)"

Conversely,let A(α) is a subgroup of G. we need to prove that $(A(x_1,q) \cap m \leq \max\{(A(x,q)Vn)$,for all x ε G. If there exist x_o,y_o ε G such that $A(x_o^{-1}y_o,q) \cap m = \alpha \geq \max\{A(x_o,q), A(y_o,q)\}$Vn, then$A(x_o,q) \leq \alpha$, $A(y_o,q) \leq \alpha$ and αε [m, n].Thus x_oε A(α).

And y_oε A(α). But then$A(x_o^{-1}y_o,q)$) ≥α, that is $x_o^{-1}y_o$ does not belong to A(α).This is a contradiction with that A(α) is a subgroup of G. Hence $A(x^{-1}y,q) \cap m \leq \max\{(A(x,q)Vn), (A(y,q)\}Vn$ holds for any x,y εG.Therefore A is a (m,n) –upper Q-fuzzy soft subgroup of G. we getΦ = 1, where Φ is the empty set.

Theorem-3.3: Let f: $G_1 \rightarrow G_2$ be a homomorphism and let A be (m.n)-upper Q-fuzzy soft subgroup of G_1.Then f(A) is (m,n)-upper Q-fuzzy soft subgroup of G_2, where f(A)(δ,q) = inf {A(x,q) ⎪f(x) = δ}, for all δεG_2 and x εG.

Proof: If $f^{-1}(y_1)$= Φ or $f^{-1}(y_1)$= Φfor any y_1,y_2 ε G_2, then $(f(A)(\delta_1^{-1}\delta_2,q) \cap m \leq 1 = \max\{f(A)(\delta_1,q), f(A)(\delta_2,q)\}$Vn.

Suppose that $f^{-1}(y_1)$= Φ or $f^{-1}(y_1)$= Φfor any y_1,y_2 ε G_2. Then For any y_1,y_2 ε G_2, we have $(f(A)(\delta_1^{-1}\delta_2,q) \cap m = \inf\{A(t,q)\ ⎪ f(t) = \delta_1^{-1}\delta_2\} \cap m$ as tεG.

$= \inf\{A(t,q)\ ⎪ f(t) = \delta_1^{-1}\delta_2\} \cap m$ as tεG1≤ $\inf\{A(x_1^{-1}x_2,q) \cap m ⎪ f(x_1) = \delta_1, f(x_2) = \delta_2\}$ as x_1,x_2 ε G_1.

≤ inf {max{A(x,q), A(y,q)}Vn ⎪ $f(x_1) = \delta_1$, $f(x_2) = \delta_2$} as x_1,x_2 ε G_1.

≤ inf {max $A(x_1,q)$ ⎪$f(x_1) = \delta_1$}, inf {max $A(x_2,q)$ ⎪$f(x_2) = \delta_2$}} = max $\{f(A)(\delta_1,q), f(A)(\delta_2,q)\}$Vn

So f(A) is (m,n)-upper Q-fuzzy soft subgroup of G_2.

Theorem-3.4: Let f: $G_1 \rightarrow G_2$ be a homomorphism and let B be (m,n)-upper Q-fuzzy soft subgroup of G_2. Then $f^{-1}(B)$ is (m,n)-upper Q-fuzzy soft subgroup of G1, where $f^{-1}(B)(x,q)$ = B(f(x,q)), for all x εG_1.

Proof: For any x_1,x_2 ε G_1. $f^{-1}(x_1^{-1}x_2,q) \cap m = B(f(x_{1-1}x_2,q)) \cap m$ $= B((f(x_1,q))^{-1} f(x_2,q)) \cap m \leq \max\{B(f(x_1,q), B(f(x_2,q))Vn = \max \{f^{-1}(B)(x_1,q), f^{-1}(B)(x_2,q)\}$Vn

So, $f^{-1}(B)$ is (m,n)-upper Q-fuzzy subgroup of G_1.

Let G_1 be a group with the identity I_1 and G_2 be a group with the identity I_2, then $G_1 \times G_2$ is a group with the identity (I_1,I_2) if we define $(x_1,y_1)(x_2,y_2) = (x_1x_2, y_1y_2)$ for all (x_1,y_1), (x_2,y_2) ε $G_1 \times G_2$. Moreover, the inverse element of any (x,a) ε $G_1 \times G_2$ is (y,b) ε $G_1 \times G_2$ if and only if y is the inverse element of x in G_1 and b is the inverse element of a in G_2.

Theorem-3.5: Let A and B be two (m,n)-upper Q-fuzzy soft subgroups of groups G_1 and G_2 respectively. The Product of A and B denoted by A×B is (m,n)- upper Q-fuzzy soft subgroup of the group $_{G1xG2}$,where (A×B)(x,y)q = max {A(x,q), A(y,q)}, for all (x,y) ε $G_1 \times G_2$.

Proof: Let (x^{-1},a^{-1}) be the inverse element of (x,q) in $G_1 \times G_2$.Then x^{-1} is the inverse element of x in G_1 and a^{-1} is the inverse element of a in G_2.Hence $A(x^{-1},q) \cap m \leq A(x,q)$ Vn and $B(a^{-1},q) \cap m \leq B(a,q)$ Vn.

For all (y,b) ε $G_1 \times G_2$.we have

$((A\times B)((x,a)^{-1}(y,b)) \cap m = ((A\times B)((x^{-1},a^{-1})(y,b))q) \cap m$

$= \max\{A(x^{-1}y,q), B(a^{-1}b,q)\} \cap m$

$= \max\{A(x^{-1}y,q) \cap m, B(a^{-1}b,q) \cap m\}$

≤max {max (A(x,q), A(y,q)}Vn, max (B(a,q), B(b,q)}Vn}

= max {max (A(x,q), B(a,q)}, max (A(y,q), B(b,q)}}Vn

= max { A×B (x,a)q , A×B (y,a)q}) Vn

Hence A×B is (m,n)-upper Q-fuzzy soft subgroup of $G_1 \times G_2$.

Theorem-3.6: Let A and B be two (m,n)-upper Q-fuzzy soft subsets of groups G_1 and G_2, respectively.If A×B is (m,n)- upper Q-fuzzy soft subgroup of the group $G_1 \times G_2$,then atleast one of the following statements must hold.

(i) $A(I_1,q) \cap m \leq B(a,q)$ V n, for all a ε G_2 and

(ii) $B(I_2,q) \cap m \leq A(x,q)$ V n, for all x ε G_1.

Proof: Let A×B is (m,n)- upper Q-fuzzy soft subgroup of the group $G_1 \times G_2$.

By composition, suppose that none of the statements hold. Then we can find x εG_1 and a ε G_2 such that A(x,q) V n ≤ $B(I_2,q) \cap m$ and B(a,q)V n ≤ $A(I_1,q) \cap m$. Now

((A×B)(x,a))q V n = max { A (x,q) , B (y,q}} Vn

= max { A (x,q)V n , B (y,q) Vn}

≤ max { $A(I_1,q) \cap m$, $B(I_2,q) \cap m$} = $(A\times B)(I_1I_2)q \cap m$.

Thus A×B is (m,n)- upper Q-fuzzy soft subgroup of the group $G_1 \times G_2$ satisfying ((A×B)(x,a))qV n ≤ $(A\times B)(I_1,I_2) \cap m$. This is a contradict with that (I_1,I_2) is the identity of $G_1 \times G_2$.

Theorem-3.7: Let A and B be Q-fuzzy subsets of groups G_1 and G_2 respectively, such that $B(I_2,q) \cap m \leq A(x,q)$ U n for all a εG_1.If A×B is a (m,n)-upper Q-fuzzy soft subgroup of $G_1 \times G_2$,then A is a (m,n)-upper Q-fuzzy soft subgroup of G1.

Proof: From $B(I_2,q) \cap m \leq A(x,q)$ U n we obtain that m ≤ A(x,q) U n or $B(I_2,q) \cap m \leq A(x,q)$ U n for all a εG_1.

Let x,y ε G_1. Then (x,I_2), (y,I_2) ε $G_1 \times G_2$.

Two cases are possible.

(1) If m ≤ A(x, q) V n for all a εG_1, then $A(xy, q) \cap m \leq m \leq$ A(x, q) V n ≤ max { A(x, q, A(y, q} V n and $A(I_1,q) \cap m \leq m$ ≤ A(x, q) V n.

(2) If $B(I_2,q) \cap m \leq A(x, q)$ V n for all x ε G_1. Then

$A(xy, q) \cap m \leq (A(xy, q)\ V\ B(I_2I_2,q)) \cap m = (A\times B)(xy,I_2I_2)q) \cap m$

$=(A\times B)((x,I_2)(y,I_2)q)) \cap m \leq \max\{(A\times B)((x,I_2)q, (A\times B)((y,I_2)q\}$ V n.

$\leq$ max {(A (x,I_2), B (I_2,q) , (A ((y,I_2), B(I_2,q)} V n. = max{A (x, q) , A (y, q)} V n.
And A(I,q) ∩m ≤ (A (I1,q) U B (I_2,q)} ∩m = ((A×B)(I_1,I_2)q) ∩m
≤ ((A×B)(x,I_2)q) V n = max { A (x, q) , B (I_2,q)} Vn. = A (x, q) V n. Hence A is (m, n)-upper Q-fuzzy soft subgroup of G_1.
Analogously, we have

Theorem-3.8: Let A and B be Q-fuzzy subsets of groups G_1 and G_2 respectively, such that A(I_1,q) ∩m ≤ B(a, q) V n for all a εG_2.If A×B is (m, n)-upper Q-fuzzy soft subgroup of G_1×G_2,then B is a (m ,n)-upper Q-fuzzy soft subgroup of G_2.

From the Previous theorem, we have the following corollary.

Corollary 3.9: Let A and B be Q-fuzzy subsets of groups G_1 and G_2 respectively, such that A(I_1,q) ∩m ≤ B(a, q) U n for all a εG_2.If A×B is (m, n)-upper Q-fuzzy soft subgroup of G_1×G_2,then either A is (m, n)-upper Q-fuzzy soft subgroup of G_1 or (m, n)-upper Q-fuzzy soft subgroup of G_2.

Conclusion: We have studied in this paper the definition of the (m,n)-upper Q-fuzzy subgroup over an arbitrary group. Some proposition, theorems and examples given for it and this proposition and corollory are generalization for some properties in group theory.

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