Available online at www.elixirpublishers.com (Elixir International Journal)

Applied Mathematics

Elixir Appl. Math. 78 (2015) 29863-29871

ISSN: 2229-712X

New type of graph with different vertices

Mabrok EL- Ghoul[1], Habiba El-Zohny[2] and Hend El- Morsy[3]

[1]Department of Mathematics, Tanta University, Cairo, Egypt.

[2]Department of Mathematics, Alazhar University, Cairo, Egypt.

[3]Department of Mathematics, Umm-Alqura University, Kingdom of Saudi Arabia.

ARTICLE INFO

Article history:
Received: 26 April 2014;
Received in revised form:
15 January 2015;
Accepted: 20 January 2015;

Keywords
Graph,
Folding.

ABSTRACT

In this paper we will illustrate a new type of graph which its vertices are different and discuss each type of this graph and illustrate different example for each type. The representation of this graph by matrices will be obtained. Also we will discuss the folding of this new type.

Introduction

Definition of Graph: An (undirected) graph G is defined by two finite sets.a non-void set X of elements called vertices, a set E (which can be empty) of elements called edges, with for each edge e two associated vertices, x and y , distinct or not, called the end vertices of e [3].

Definition of simple graph: A "simple" graph is a graph with no loops or multiple edges [7].

Definition of Spanning subgraph: A subgraph H = (Y, F) of G is called a spanning subgraph if Y = X. It can be specified that it is a spanning subgraph induced by F. It isthe graph(X, F) and is denoted by G (F) [5].

Definition of Null graph: A graph which consists of a set of vertices and no edges is called a null graph[4].

Definition of multigraph: Let V be a finite nonempty set. We say that the pair (V,E) determine multigraph G with vertex set V and edge set E if for some $x,y \in V$, there are two or more edges in E of the form (a) (x,y) (for a directed multigraph),or (b) {x,y} (for an undirected multigraph) . In either case ,we write G = (V, E) to designate the multigraph , just as we did for graphs [8].

Definition of Spanning tree :Spanning tree for a graph G is a subgraph of G that contains every vertex of G and is a tree [6].

Definition of link-graph : knot-graph (link-graph) is a graph in which each vertex is a link[1].

Definition of graph with graph vertices: Consider the geometric graph G(V, E) where $V(G) = \{\{V^0_1, e^0_1\}, \{v^1_1, e^1_1\} \ldots\ldots \{v^n_1, e^n_1\}, \{V^0_2, e^0_2\}, \{V^1_2, e^1_2\} \ldots\ldots \{V^n_2, e^n_2\}, \{V^0_n, e^0_n\}, \{V^1_n, e^1_n\} \ldots\ldots \{V^n_m, e^n_m\}$ and $E(G) = \{e^1, e^2, e^3, \ldots\ldots e^n\}$. We are called this graph (graph with complex vertices) [2].

Definition of Graph folding: A graph map $f : G_1 \rightarrow G_2$, is called a graph folding if and only if f maps vertices to vertices and edges to edges ,i.e.,

(i) For each vertex $v \in V(G_1)$, f(v) is a vertex in $V(G_2)$,

(ii) For each $e \in E(G_1)$, f(e) is an edge in $E(G_2)$ [3].

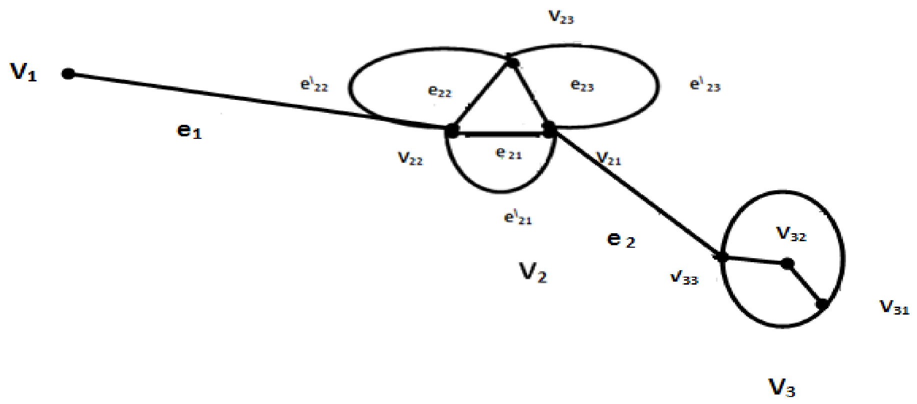

Fig 1

Main Results:

Definition : The graph with different vertices : is a graph which each vertex has different chap such as: link vertex , graph vertex , or simple vertex , whether this graph is simple or have multiedges (multigraph).

Types of graph with different vertices:

This graph may be one of two types , in each type we have two cases (Simple graph or multigraph).

Tele:
E-mail addresses: hendelmorsy@yahoo.com

Type 1: Graph which its vertices has all three chaps (simple, knot or graph) :

In this type the graph may be cycle or path or tree, then we can illustrate the adjacent and incident matrices for each case as follows:

Case 1: For simple graph:

For path graph with three different vertices:

For graph shown in Fig.(1) the adjacent and incident matrices are:

$$A(G) = \begin{bmatrix} 0 & 1_k & 0 \\ 1 & 0 & 1_g \\ 0 & 1_k & 0 \end{bmatrix}, \quad I(G) = \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 0 & 1 \end{bmatrix}$$

Where 1 represent the simple vertex , 1_k represent knot vertex , 1_g represent graph vertex.

For cycle graph with three different vertices:

For graph shown in Fig.(2) the adjacent and incident matrices are:

$$A(G) = \begin{bmatrix} 0 & 1_K & 1_g \\ 1 & 0 & 1_g \\ 1 & 1_K & 0 \end{bmatrix}, \quad I(G) = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

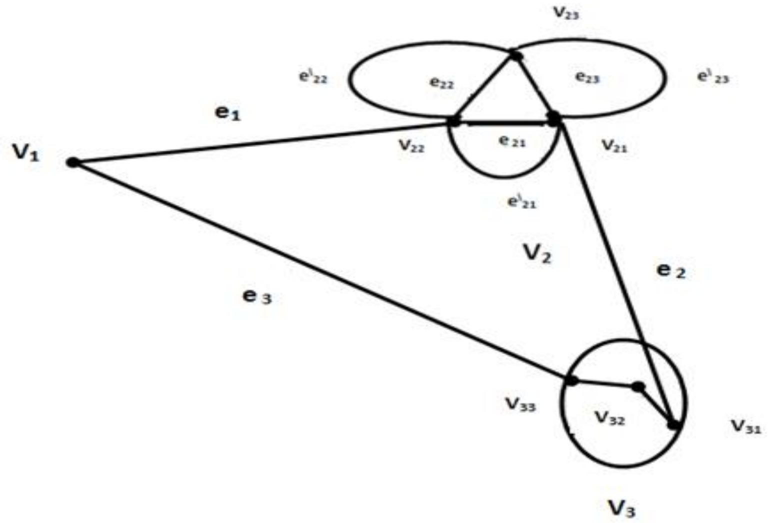

Fig 2

Case 2 : For multigraph:

For path graph with three different vertices:

For multigraph shown in Fig.(3) , we can compute adjacent and incident matrices as follows:

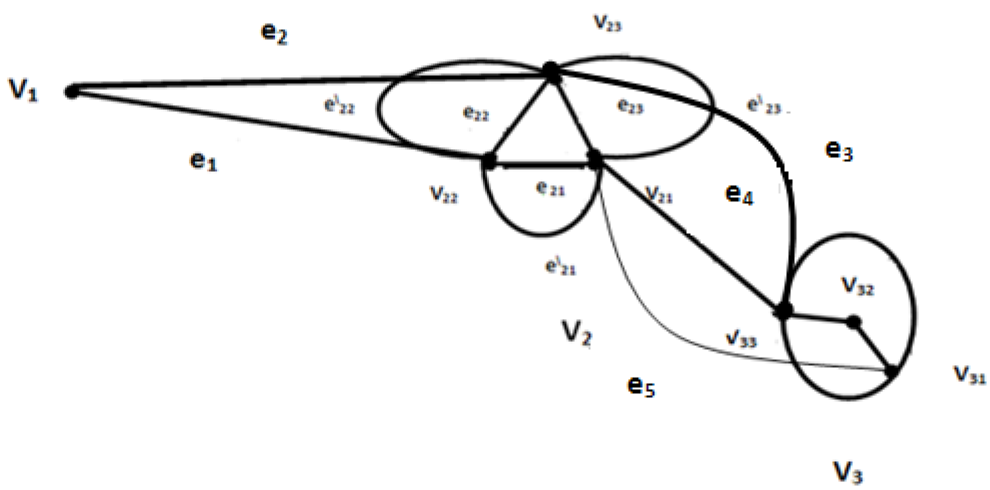

Fig 3

$$A(G) = \begin{bmatrix} 0 & 2_K & 0 \\ 2 & 0 & 3_{\varepsilon} \\ 0 & 3_K & 0 \end{bmatrix},\ I(G) = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 \end{bmatrix}$$

In the same way we can compute adjacent and incident matrices for graphs with more than 3 vertices and edges.

For cycle graph with three different vertices:

Consider a cycle multiple graph as shown in Fig.(5).

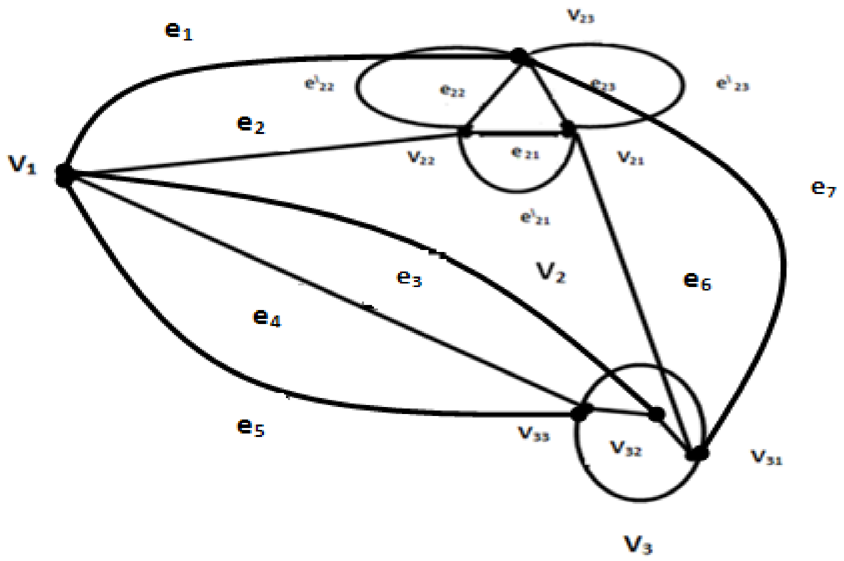

Fig 5

The adjacent and incident matrices are:

$$A(G) = \begin{bmatrix} 0 & 2_k & 3_{\varepsilon} \\ 2 & 0 & 2_{\varepsilon} \\ 3 & 2_k & 0 \end{bmatrix},\ I(G) = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

In the same way we can calculate adjacent and incident matrices for graph of more than 3 vertices in each case.

Type 2: Graph which its vertices has only two chaps:

In this type the graph have two chaps of vertices such as: simple and knot , simple and graph , knot and graph .

Case 1: For simple graph:

when its vertices are simple and knot:

As a simple example for this graph is shown in Fig.(6) , we can illustrate the adjacent and incident matrices as follows:

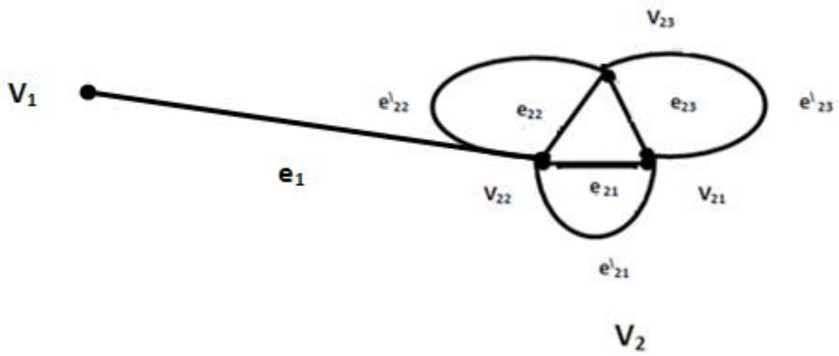

Fig 6

$$A(G) = \begin{bmatrix} 0 & 1_K \\ 1 & 0 \end{bmatrix},\ I(G) = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

when its vertices are simple and graph:

This type may have two or more vertices .As an example for this type is graph shown in Fig.(7),

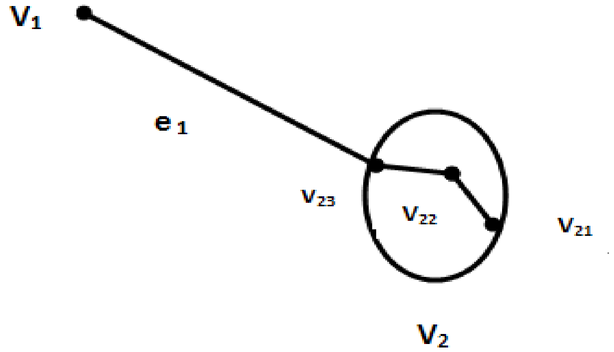

Fig 7

$$A(G) = \begin{vmatrix} 0 & 1_g \\ 1 & 0 \end{vmatrix}, \quad I(G) = \begin{vmatrix} 1 \\ 1 \end{vmatrix}$$

when its vertices are knot and graph:

This type may have two or more vertices .As an Example of this type is a graph shown in Fig.(8) ,

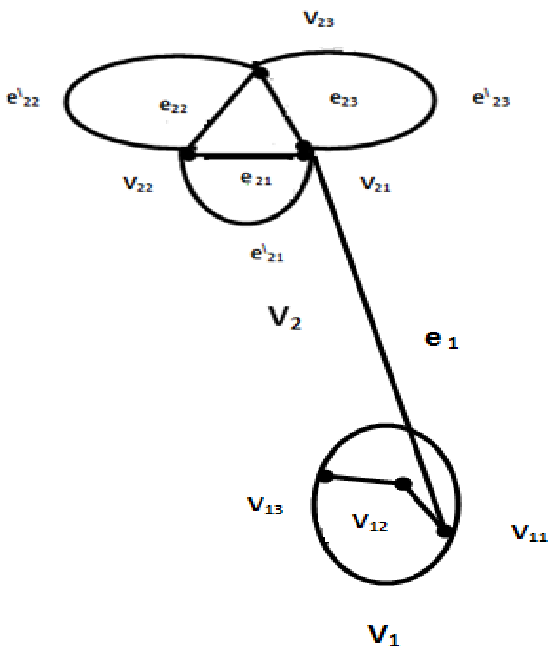

Fig 8

We can illustrate the adjacent and incident matrices as follows:

$$A(G) = \begin{vmatrix} 0 & 1_g \\ 1_K & 0 \end{vmatrix}, \quad I(G) = \begin{vmatrix} 1 \\ 1 \end{vmatrix}.$$

Case 2: For multigraph:

when its vertices are simple and knot:

For multigraph shown in Fig.(9) , we can compute adjacent and incident matrices as follows:

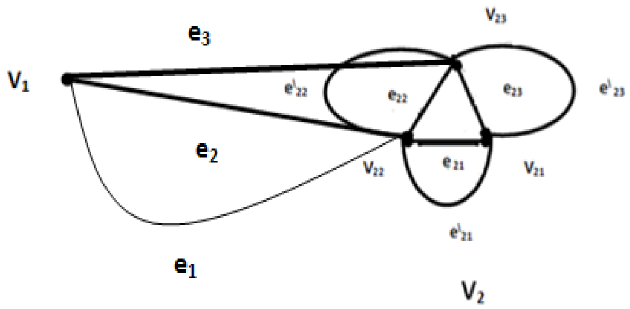

Fig 9

We can illustrate the adjacent and incident matrices as follows:

$$A(G) = \begin{vmatrix} 0 & 3_K \\ 3 & 0 \end{vmatrix},\ I(G) = \begin{vmatrix} 3 \\ 3 \end{vmatrix}$$

when its vertices are simple and graph:

For multiple graph shown in Fig.(10), we can compute adjacent and incident matrices as follows:

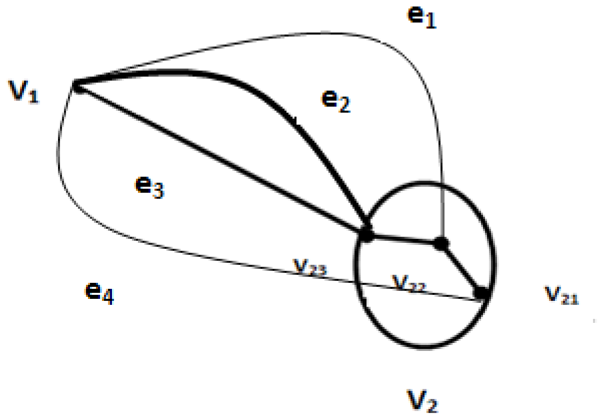

Fig 10

$$A(G) = \begin{vmatrix} 0 & 4_g \\ 4 & 0 \end{vmatrix},\ I(G) = \begin{vmatrix} 4 \\ 4 \end{vmatrix}.$$

when its vertices are knot and graph:

For multiple graph shown in Fig.(11), we can compute adjacent and incident matrices as follows:

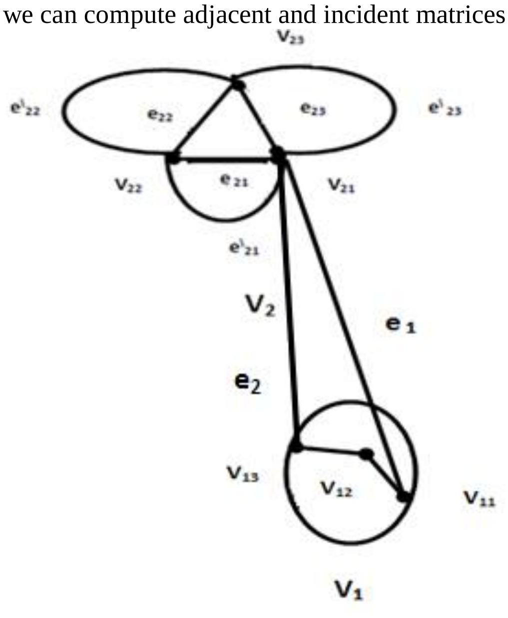

Fig 11

$$A(G) = \begin{vmatrix} 0 & 2_k \\ 2_g & 0 \end{vmatrix},\ I(G) = \begin{vmatrix} 2 \\ 2 \end{vmatrix}.$$

In the same way we can calculate adjacent and incident matrices for graph of more than 3 vertices in each case.

Loop of graph with different vertices:

Since this graph has different vertices then each vertex have special loop. We will illustrate loop of each vertex.

loop of each vertex:

For simple vertices:

The loop may take forms shown in Fig.(12),

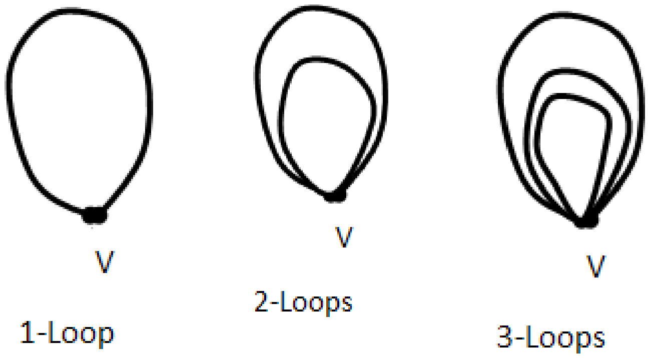

Fig.(12)

And so on.

For knot vertices:

Since knot vertices have inner and outer vertices , then we have inner and outer loops. We will illustrate these two forms. Inner loops may take the form shown in Fig.(13),

Or

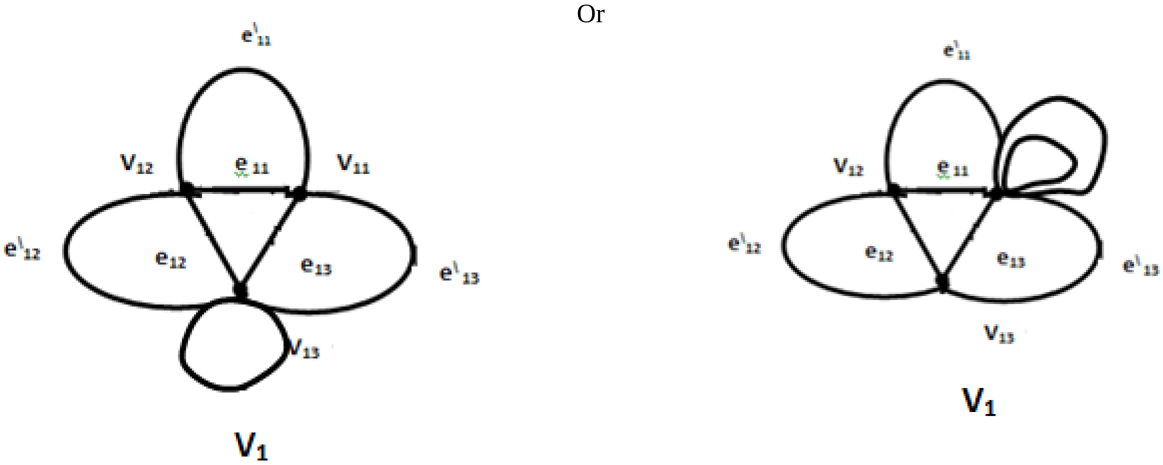

Fig 13

And the outer loops may be:

or

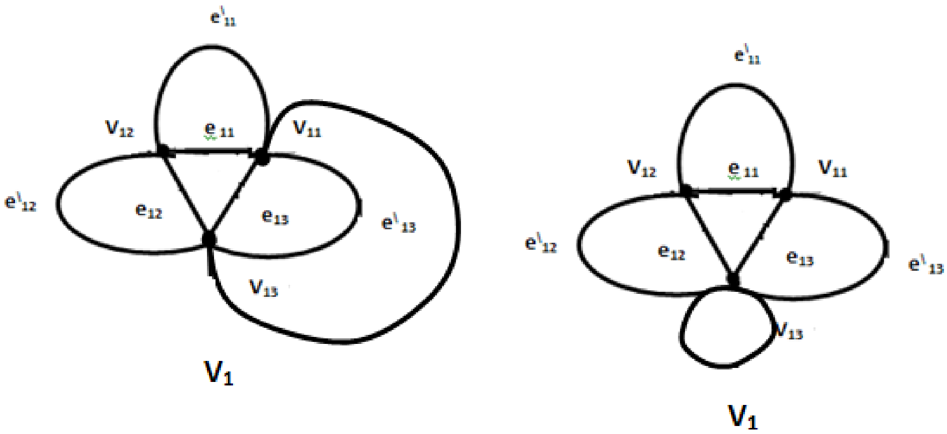

Fig 14

Then we find that loops of outer vertices may be away from one internal vertices or connect between two internal vertices.

For graph vertices:

In the same way as knot vertices i.e , we have two kinds of loops for graph vertices (internal and outer)loops.

The internal loop for internal vertices may be:

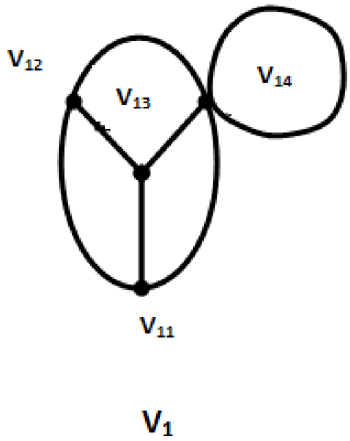

Fig 15

And the outer loop may be:

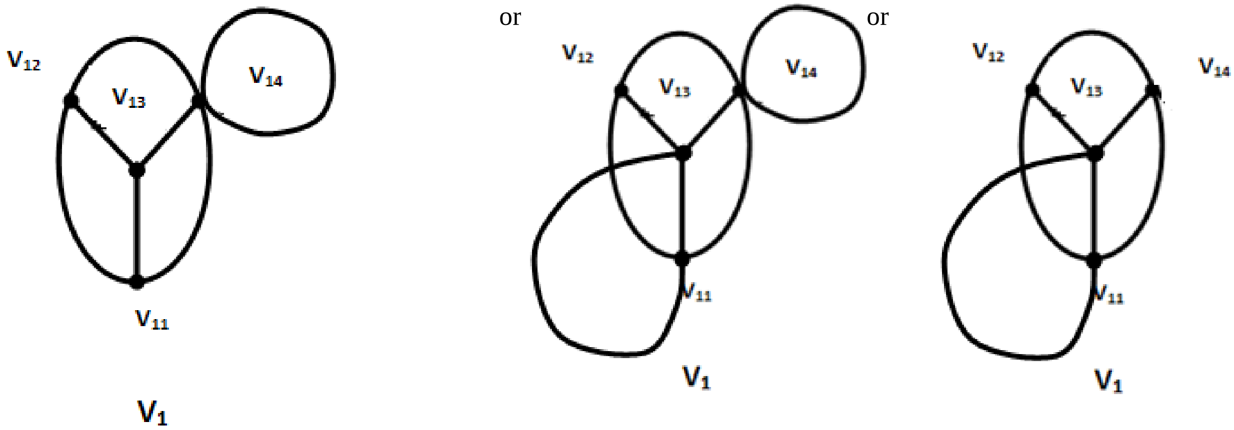

Fig 16

Folding of this type of graph:

Since this type of graph consists of different types of vertices and each type can be folded in different method , then we first will illustrate folding of each type solitary then illustrate folding of two type of vertices together.

Folding of each type solitary:

Folding of graph with graph vertices:

This type has two different types of folding (inner folding and outer folding) the folding of each type can be shown in Fig.(17) for inner folding and Fig.(18) for outer folding [2].

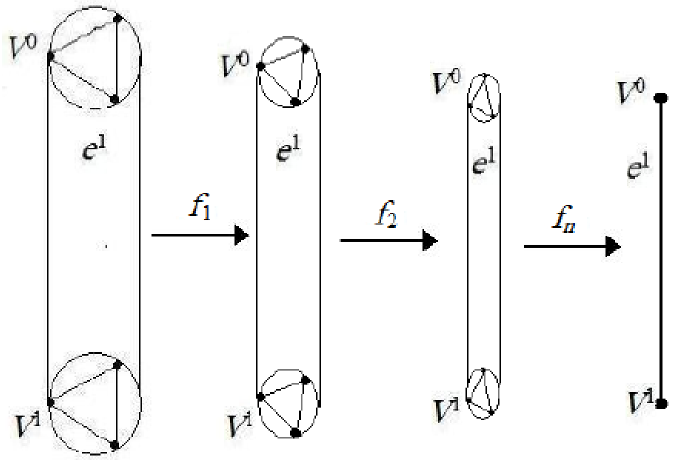

Fig 17

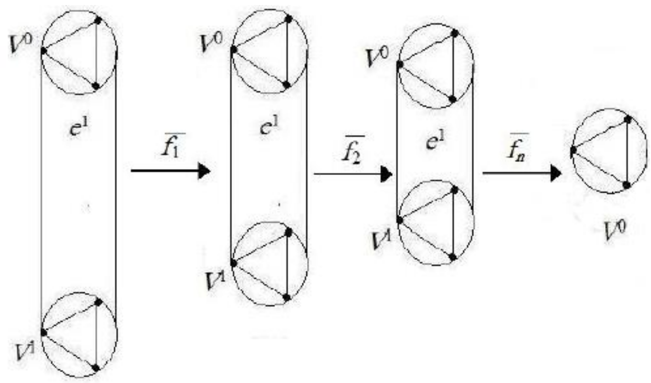

Fig 18

Folding of graph with knot vertices:

This type can be folded as shown in Fig.(19) [1].

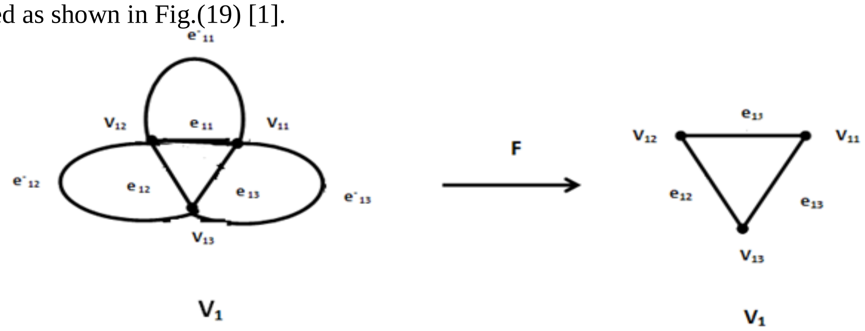

Fig 19

Folding of two types of vertices together:

Consider geometric graph G with different vertices, and let $f : G \rightarrow G$, such that : $f(V) < V$, then we can fold vertices with low dimension on vertices of high dimension, and simple vertices can be folded on knot vertex or graph vertex.

Example 1 : For graph consists of simple and knot vertices for which V_1 is simple vertex and V_2 is knot vertex if: $f : G \rightarrow G$ and $f(V_1) < V_1$, $f(V_2) < V_2$ then the result of folding is shown in Fig.(20).

k

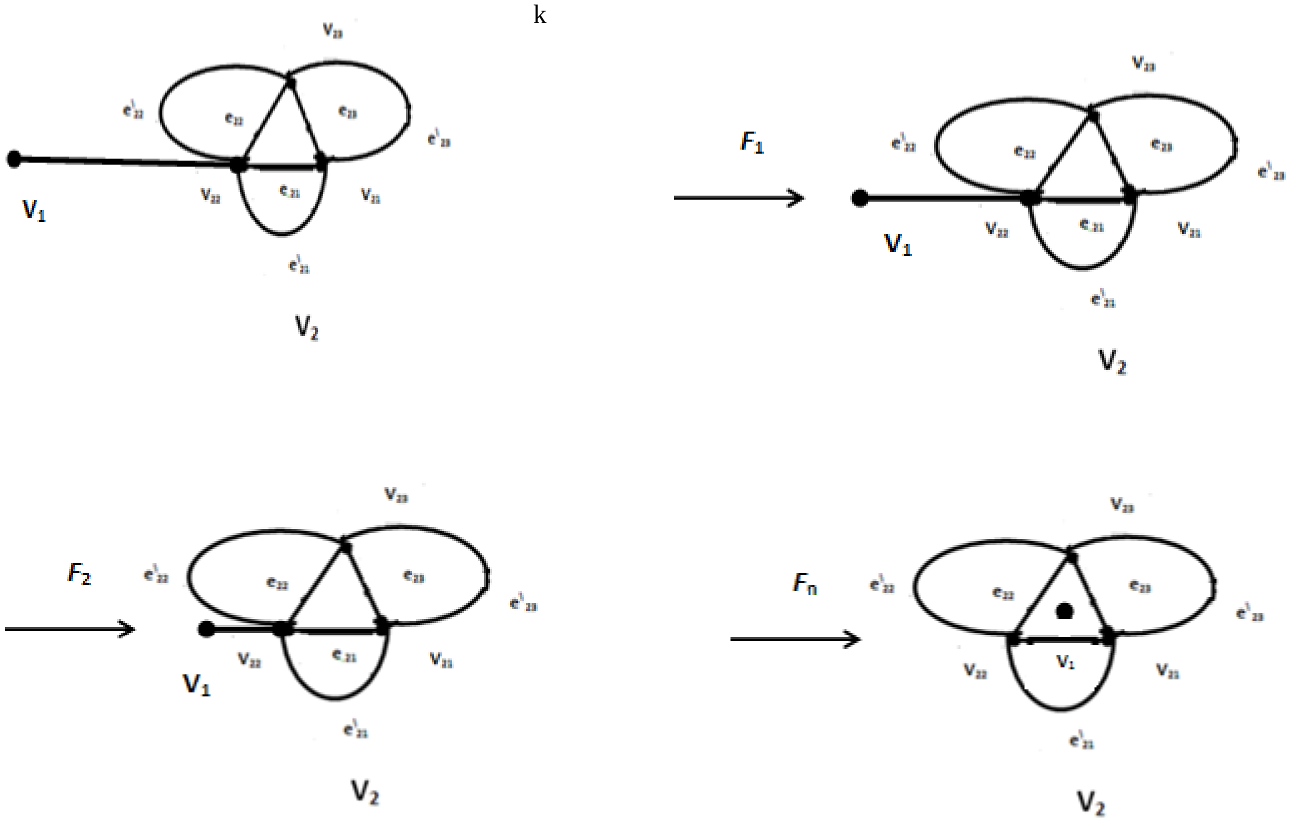

Fig 20

Example 2: For graph consists of simple and graph vertices for which V_1 is simple vertex and V_2 is graph vertex if: $f : G \rightarrow G$ and $f(V_1) < V_1$, $f(V_2) < V_2$ then the result of folding is shown in Fig.(21).

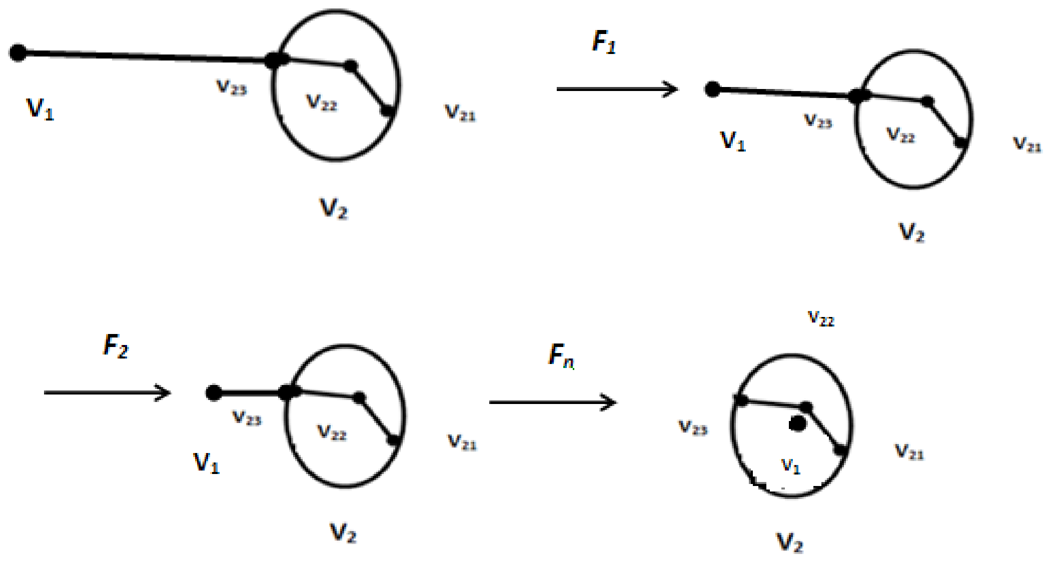

Fig 21

Applications:

1. In cars there is a lot of machine things which depend on these graphs.
2. Electrical transformers are an examples of this type of graph.
3. In human body the Neuron may be an example for this graph.

References:

[1]El-Ghoul M; F. Salama ; A. S. Pelila . :On new type of graph. Tanta University ,Tanta,Egypt,2012.
[2] El-Ghoul,M ; El-Zohny,H; Khalil,M,M.: New types of graphs with graphs vertices. Al-Azhar University ,Cairo,Egypt,2011.
[3] El-Kholy, E. M. and El-Esawy, A.: Graph folding of some special graphs, J. Math.And Statis. I (1), Sci. Pub. ,(2005) , 66-70.
[4]Fournier, Jean-Claude, Graph Theory and applications with Exercises and problems, ISTE Ltd,2009.
[5] Giblin P.J :Graphs, surfaces and homology, an introduction to algebraic topology. Champman and Hall Ltd,London 1977.
[6] Susanna S.Epp, Discrete Mathematics With Application, Third Edition, Thomson Learning, Inc , 2004.
[7] Wilson R.J.:Introduction to graph theory. Oliver and Boyed, Edinburgh 1972.
[8]P.Grimadi Ralph,“ Discrete and combinatorial mathematics:An Applied Introduction”, Fifth Edition, Pearson Education, Inc., 2004.