



## Vector autoregressive model for monitoring carbondioxide (CO<sub>2</sub>) emission from the consumption of fossil fuels

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### ARTICLE INFO

#### Article history:

Received: 20 November 2014;

Received in revised form:

11 January 2015;

Accepted: 20 January 2015;

#### Keywords

Vector Autoregressive Model, Carbondioxide, Granger-causality, Portmanteau test.

### ABSTRACT

The main source of the Nigerian energy generation is the non-renewable source (which includes the burning of Fossil fuels). Fossil fuels (Coal, Natural gas and Petroleum) are being consumed and certain chemical compounds such as CO<sub>2</sub> are emitted into the atmosphere. The objective of this research is to examine the trend of CO<sub>2</sub> emission from the consumption of Fossil fuels and fitting a model for monitoring the process. Vector Autoregressive Model (VAR) was developed. The Portmanteau test for serial correlation and the Wald test for Granger-causality were carried out. VAR (1) fit the data. The Portmanteau test showed that error term are serially uncorrelated. The Wald test, showed that CO<sub>2</sub> emission from Coal Granger-cause emission from Natural gas and Petroleum and vice-versa. The emission from Natural gas does not Granger-cause emission from Petroleum.

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### Introduction

Energy sources have different impacts on the environment. Some of these include: emission of chemical compounds and release of waste. Some of the chemical compounds emitted from these energy sources make up the Greenhouse gas which later give rise to the Greenhouse effect i.e heating of the Earth due to the presence of Greenhouse gases. When shorter wavelength solar radiation passes through Earth's atmosphere, it is absorbed by the surface of the Earth, causing it to warm. Part of the absorbed energy is then re-radiated back to the atmosphere as long wave infrared radiation. As a result of the presence of the Greenhouse gases in the atmosphere, little of the long wave radiation will be able to escape back to space hence the Greenhouse gases absorb these waves and trap the waves downward, causing the lower atmosphere to warm.

### Objectives of the Study

The main objective of this paper is to examine the trend of CO<sub>2</sub> emission from the consumption of Fossil fuels and fitting a model for monitoring the process

### Source and type of data

The data on carbon dioxide emitted from the consumption of Fossil fuels in Nigeria, measured in million metric tons from 1980 to 2009 were extracted from internet, [www.open data for Nigeria.com](http://www.open data for Nigeria.com).

### Literature Review

The main source of energy generation is the non-renewable source (which includes the burning of Fossil fuels). Between 1959 and 2004, the atmospheric concentration of carbon dioxide at Mauna Loa increased from 316 ppm to 377 ppm. Most of this increase is associated with the combustion of fossil fuels and anthropogenic changes in land-use (Conway *et al.*, 1994; Keeling *et al.*, 1995; Houghton, 2000; Marland *et al.*, 2001; Prentice *et al.*, 2001). Within each year, the atmospheric concentration of carbon dioxide rises and falls through an intrannual cycle, with the amplitude increasing with latitude (Keeling *et al.*, 1996). The intrannual cycle generally is associated with the balance between photosynthesis and respiration in the terrestrial biota (Keeling *et al.*, 1996; Francey *et al.*, 1995). The intrannual cycle changes from year to year, with a general increase in amplitude over time, and changes in monthly values (Dettinger and Gil, 1998). To date, changes in the intrannual cycle of atmospheric carbon dioxide are attributed to changes in atmospheric circulation, the exchange between the atmosphere and ocean, and/or the exchange between the atmosphere and the terrestrial vegetation.

Several analysts focus on the timing of changes in the intrannual cycle of atmospheric CO<sub>2</sub>. Keeling *et al.*, (1996) find that the amplitude of the intrannual cycle has increased most rapidly at high latitudes and that the phasing has advanced by about seven days. Based on correlations between these changes and temperature, Keeling *et al.* (1996) argue that these observations are associated with temperature-driven changes in the terrestrial biota, specifically an earlier spring.

Dettinger and Ghil (1998) use singular-spectrum analysis to analyze monthly patterns at both Mauna Loa and the South Pole. They identify several months in which the intrannual cycle changes. Based on their timing and correlations with other time series, Dettinger and Ghil (1998) argue that these changes are associated with tropical processes, especially sea-surface temperature. Nonetheless, these correlations cannot be used to differentiate among possible causal variables, such as sea surface temperature, oceanic circulation, or changes in biotic activity. Muryama *et al.*, (2004) find that interannual changes in atmospheric circulation disrupt the link between biotic changes and measurements taken at distant stations. Based on these results, Muryama *et al.*, (2004) argue that Northern Hemisphere observational sites are not sensitive to changes in terrestrial carbon fluxes, except as they occur close to the site.

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Yang and Wang (2000) argue for the importance of clouds, which reduce solar radiation and may suppress photosynthetic rates. Rayner et al., (1999) argue for both ocean and terrestrial factors—oceanic changes generate a negative anomaly while terrestrial responses generate a positive anomaly. Vegetation may affect the intrannual cycle via several mechanisms that include a fertilization effect due to the increased concentration of atmospheric carbon dioxide (Kolmaier et al., 1989), seasonal shifts in the phasing of photosynthesis and respiration (Chapin et al., 1996), and/or changes in the length or intensity of the growing season at high latitudes (Myneni et al., 1997). Tanja et al., (2003) find that the start of photosynthesis in the spring is correlated with air temperature. Keyser et al., (2000) find that higher spring temperatures elongate the growing season and increase net primary production by up to 20 percent for sites in Alaska. Hollinger et al., (2004) find that carbon storage in spruce forests in Maine (USA) is related positively to warmer spring and falls. This effect is simulated by Randerson et al., (1999), who find a correlation between spring temperatures and early season net ecosystem uptake. Churkina et al., (2005) suggest a linear relationship between annual net ecosystem exchange and the carbon uptake period. Zhou et al., (2003) find that summer precipitation has a large effect on interannual variations in satellite measures of surface greenness. Consistent with this result, Nemani and White (2003) find that precipitation during the growing season has a greater effect on net ecosystem exchange than the length of the growing season. Correlations between tree rings and NDVI in June and July (as opposed to months in spring or fall) seem to suggest the importance of a greener summer (Kaufmann et al., 2004). Angert et al., (2005) argue that drier summers suppress carbon uptake.

**Methodology**

**Multivariate Time Series**

Consider n time series variables  $\{x_{1t}\}, \dots, \{x_{nt}\}$ . A multivariate time series is an  $(n \times 1)$  vector time series  $\{X_t\}$  where the *ith* row of  $\{X_t\}$  is  $\{x_{it}\}$ . i.e for any time  $t$ ,  $X_t = (x_{1t}, \dots, x_{nt})'$

Multivariate time series analysis is used when one wants to model and explain the interactions and co-movements among a group of time series variables.

For a stationary multivariate time series,

$$E(X_t) = \mu = (\mu_1, \mu_2, \dots, \mu_n)' \dots\dots\dots(1)$$

$$Var(X_t) = \Gamma_0 = E[(X_t - \mu)(X_t - \mu)'] = \begin{pmatrix} \text{var}(x_{1t}) & \text{cov}(x_{1t}, x_{2t}) & \dots & \text{cov}(x_{1t}, x_{nt}) \\ \text{cov}(x_{2t}, x_{1t}) & \text{var}(x_{2t}) & \dots & \text{cov}(x_{2t}, x_{nt}) \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \text{cov}(x_{nt}, x_{1t}) & \text{cov}(x_{nt}, x_{2t}) & \dots & \text{var}(x_{nt}) \end{pmatrix}$$

The parameters  $\mu$ ,  $\Gamma_0$  and  $R_0$  are estimated from data  $(X_1, X_2, \dots, X_n)$  using the sample moments,

$$\bar{X}_t = \frac{1}{N} \sum_{t=1}^N X_t \Leftrightarrow E(X_t) = \mu$$

$$\hat{\Gamma}_0 = \frac{1}{N} \sum_{t=1}^N (X_t - \bar{X}_t)(X_t - \bar{X}_t)' \Leftrightarrow \text{var}(X_T) = \Gamma_0$$

$$\hat{R}_0 = D^{-1} \hat{\Gamma}_0 D^{-1} \Leftrightarrow \text{corr}(X_t) = R_0$$

The correlation matrix of  $X_t$  is the  $(n \times n)$  matrix

$$\text{Corr}(X_t) = R_0 = D^{-1} \Gamma_0 D^{-1}$$

Where  $D$  is an  $(n \times n)$  matrix with *jth* diagonal element  $(\gamma_{jj}^0)^{1/2} = \text{var}(x_{jt})^{1/2}$ .

**Cross Covariance and Correlation Matrices**

With a multivariate time series  $X_t$ , each component has autocovariances and autocorrelations but there are also cross lead-lag covariance and correlations between all possible pairs of components. The autocovariances and autocorrelations of  $y_{jt}$  for  $j=1, \dots, n$  are defined as

$$\gamma_{jj} = \text{cov}(y_{jt}, y_{jt-k})$$

$$\rho_{jj}^k = \text{corr}(x_{jt}, x_{jt-k}) = \frac{\gamma_{jj}^k}{\gamma_{jj}^0}$$

The cross covariance matrix and correlation matrix are symmetric in  $k$ , ie they are time invariant since  $\gamma_{jj}^k = \gamma_{jj}^{-k}$  and  $\rho_{jj}^k = \rho_{jj}^{-k}$

The cross lag covariances and cross lag correlations between  $x_{it}$  and  $x_{jt}$  are defined as

$$\gamma_{ij}^k = \text{COV}(x_{it}, x_{jt-k})$$

$$\rho_{ij}^k = \text{corr}(x_{jt}, x_{jt-k}) = \frac{\gamma_{ij}^k}{\sqrt{\gamma_{ii}^0 \gamma_{jj}^0}}$$

The cross lag covariances and cross lag correlation are not necessarily symmetric, i.e.

$$\gamma_{ij}^k \neq \gamma_{ji}^{-k}$$

All of the lag  $k$  cross covariances and correlations are summarized in the  $(n \times n)$  lag  $k$  cross covariance and lag  $k$  cross correlation matrices

$$\Gamma_k = E(X_t - \mu)(X_{t-k} - \mu)' = \begin{pmatrix} \text{COV}(x_{1t}, x_{1t-k}) & \text{COV}(x_{1t}, x_{2t-k}) & \dots & \text{COV}(x_{1t}, x_{nt-k}) \\ \text{COV}(x_{2t}, x_{1t-k}) & \text{COV}(x_{2t}, x_{2t-k}) & \dots & \text{COV}(x_{2t}, x_{nt-k}) \\ \cdot & \cdot & \dots & \cdot \\ \cdot & \cdot & \dots & \cdot \\ \cdot & \cdot & \dots & \cdot \\ \text{COV}(x_{nt}, x_{1t-k}) & \text{COV}(x_{nt}, x_{2t-k}) & \dots & \text{COV}(x_{nt}, x_{nt-k}) \end{pmatrix}$$

$$R_k = D^{-1} \Gamma_k D^{-1}$$

The matrices  $\Gamma_k$  and  $R_k$  are estimated from data  $(X_1, \dots, X_N)$  using

$$\hat{\Gamma}_k = \frac{1}{N} \sum_{t=k+1}^N (X_t - \bar{X})(X_{t-k} - \bar{X})'$$

$$\hat{R}_k = D^{-1} \hat{\Gamma}_k D^{-1}$$

### Vector Autoregressive Models

The vector autoregressive (VAR) model is one of the most successful, flexible, and easy to use models for the analysis of multivariate time series. It is a natural extension of the univariate autoregressive model to dynamic multivariate time series.

In its basic form, a VAR consists of a set of endogenous variables  $X_t = (x_{1t}, \dots, x_{nt})'$ . The  $\text{VAR}(p)$  – process is defined as

$$X_t = C + \Pi_1 X_{t-1} + \Pi_2 X_{t-2} + \dots + \Pi_p X_{t-p} + \varepsilon_t$$

Where  $\varepsilon_t \sim N(0, \Sigma)$  and  $\Pi_i$  is an  $(n \times n)$  matrix of co-efficient for  $i = 1, \dots, p$

In lag operator notation, the  $\text{VAR}(p)$  is written as

$$\Pi(B)X_t = C + \varepsilon_t$$

$$\Rightarrow \Pi(B) = I_n - \Pi_1 B - \dots - \Pi_p B^p$$

The  $\text{VAR}(p)$  is stable if the roots of

$$\det(I_n - \Pi_1 B - \dots - \Pi_p B^p) = 0$$

lie outside the complex unit circle (have modulus greater than one), or, equivalently, if the eigenvalues of the companion matrix

$$F = \begin{pmatrix} \Pi_1 & \Pi_2 & \dots & \Pi_p \\ I_n & 0 & \dots & 0 \\ 0 & \cdot & 0 & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ 0 & 0 & I_n & 0 \end{pmatrix}$$

have modulus less than one. A stable  $\text{VAR}(p)$  process is stationary with time invariant means, variances and autocovariances.

### Estimation of Vector Autoregressive Model of order $p$ [VAR (p)]

Assume that the  $\text{VAR}(p)$  model is covariance stationary, and there are no restrictions on the parameters of the model. In SUR notation, each equation in the  $\text{VAR}(p)$  may be written as

$$x_i = Z\pi_i + e_i \quad i = 1, 2, \dots, n$$

Where  $x_i$  is a  $(N \times 1)$  vector of observations on the  $i$ th equation.

$Z_i$  is an  $(N \times k)$  matrix with the  $t^{th}$  row given by  $Z'_t = (1 - X'_{t-1}, \dots, X'_{t-p})$

$$k = np + 1$$

$\pi_i = (k \times 1)$  Vector of parameters

$e_i = (N \times 1)$  Error with covariance matrix  $\sigma^2 I_N$

### Lag Length Selection for VAR

The lag length for the  $VAR(p)$  model may be determined using model selection criteria. The general approach is to fit  $VAR(p)$  models with orders  $p = 0, \dots, p_{max}$  and choose the value of  $p$  which minimizes some model selection criteria.

Generally, Model Selection Criteria (MSC) are usually in this form

$$MSC = \ln|\bar{\Sigma}(p)| + c_N \cdot \varphi(n, p)$$

$$\text{Where } \bar{\Sigma}(p) = \frac{1}{N} \sum_{t=1}^N \varepsilon_t \varepsilon_t'$$

$c_N =$  Function of sample size

$\varphi(n, p) =$ penalty function.

The three most common Model selection criteria are

i. Akaike information criterion (AIC)

$$AIC(p) = \ln|\bar{\Sigma}(p)| + \frac{2}{N} pn^2$$

ii. Schwarz-Bayesian(BIC)

$$BIC(p) = \ln|\bar{\Sigma}(p)| + \frac{\ln N}{N} pn^2$$

iii. Hannan-Quinn(HQ)

$$HQ(p) = \ln|\bar{\Sigma}(p)| + \frac{2 \ln(\ln N)}{N} pn^2$$

Under the MSC assumption, the  $VAR(p)$ , ( $p = 0, \dots, p_{max}$ ) model with the smallest information criteria is selected as the best.

### Diagnostic Tests

Once a VAR-model has been estimated, it is of pivotal interest to see whether the residuals obey the model's assumptions. That is, one should check for the absence of serial correlation and heteroscedasticity and see if the error process is normally distributed. As a final check, one can conduct structural stability tests; i.e., CUSUM, CUSUM-of-squares, and/or fluctuation tests. The latter tests can be applied on a per-equation basis.

For this study, the test statistic to be considered for testing serial correlation is the Portmanteau test.

### Portmanteau test

The test statistic for a portmanteau test is

$$\hat{Q}_h = N^2 \sum_{j=1}^h \frac{1}{N-j} \text{tr}(\hat{C}'_j \hat{C}_0^{-1} \hat{C}_j C_0^{-1}) \quad (\text{For small-sample})$$

$$Q_h = N \sum_{j=1}^h \text{tr}(\hat{C}'_j \hat{C}_0^{-1} \hat{C}_j C_0^{-1}) \quad (\text{For large sample})$$

$$\text{Where } \hat{C}_i = \frac{1}{N} \sum_{t=i+1}^N \hat{u}_t \hat{u}'_{t-i}$$

$$Q_h \sim X^2_{(K^2 h - n^*)}$$

### Granger Causality

Consider two variables  $x_1, x_2$

If a variable,  $x_1$  is found to be helpful for predicting another variable  $x_2$ , then  $x_1$  is said to Granger-cause  $x_2$ ; otherwise it is said to fail to Granger-cause

The notion of Granger causality does not imply true causality. It only implies forecasting ability.

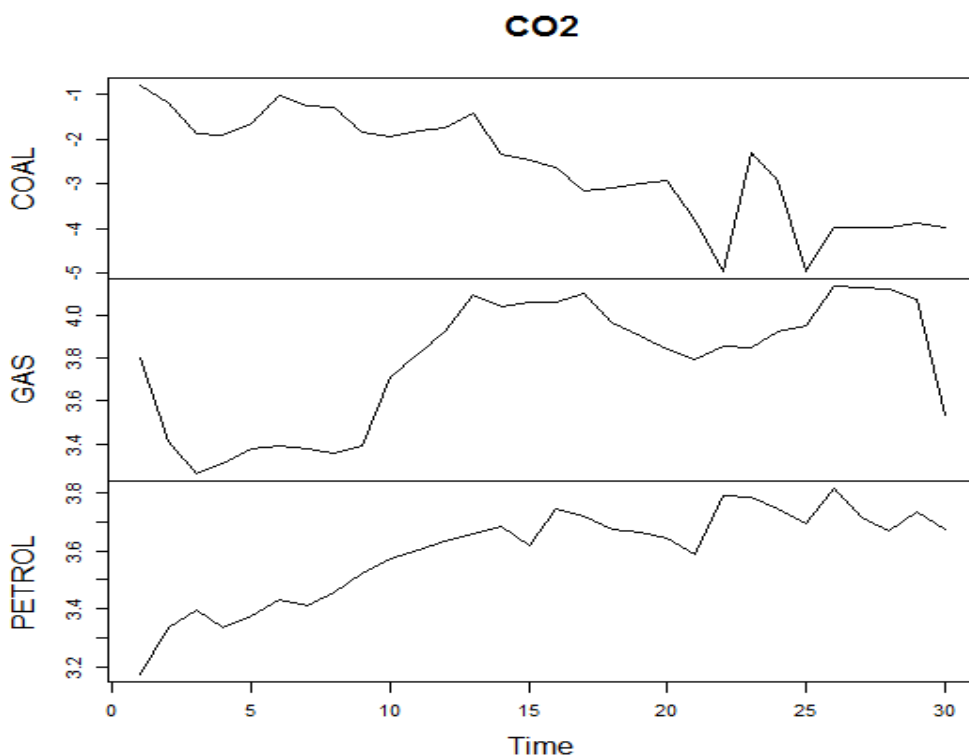
Another good approach of detecting forecasting ability is the use of Wald statistic

$$\text{Wald} = (R \cdot \text{vec}(\hat{\Pi}) - r)' \{R [a \text{var}(\text{vec}(\hat{\Pi}))] R'\}^{-1} \times (R \cdot \text{vec}(\hat{\Pi}) - r)'$$

A Wald-type instantaneous causality test can be used, too. It is characterized by testing for non-zero correlation between the errors processes of the cause and effect variables.

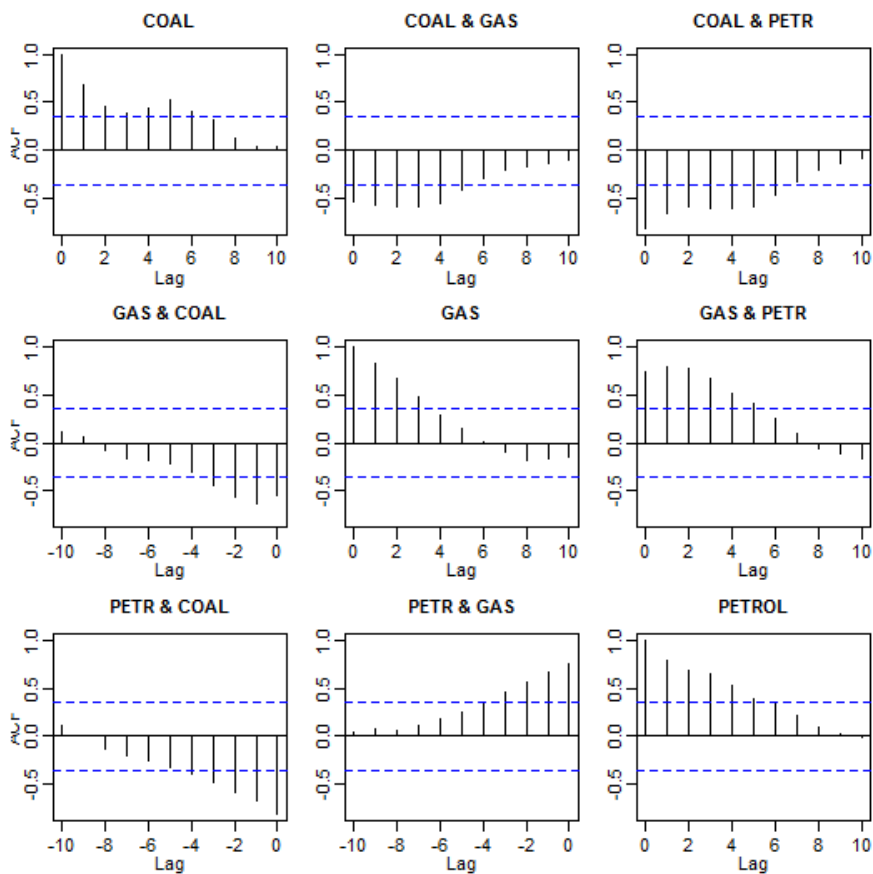
Data Analysis And Discussion

Figure 1: Time Plot



It was observed from the time plot above that even after the natural log of the original data were taken, there is still presence of trend in the process. The CO<sub>2</sub> emission from coal consumption tends to be a decreasing trend while CO<sub>2</sub> emission from natural gas and petroleum consumption tend to increase over time. Since there is still presence of trend in the process, it implies that the whole process is not stationary

Figure 2: Autocorrelation



This slow decay in the value of the autocorrelations is an evidence that the whole process is not stationary.

**Vector autoregressive model for CO<sub>2</sub> emission from fossil fuels**

To fit a VAR model, the process must be stationary. To make the process stationary, the first difference of the log transformed were taken and a unit root test was performed on the result using the Kwiatkowski-Philips-Schmidt-Shin (KPSS) test.

**KPSS Test for Level Stationarity**

H<sub>0</sub>: The series has no unit root (i.e. the process is stationary)

H<sub>1</sub>: The series has unit root (i.e. the process is not stationary)

At α=0.05

**Test statistic**

process	p-value
CO <sub>2</sub> and Coal	0.1
CO <sub>2</sub> and Gas	0.1
CO <sub>2</sub> and Petrol	0.1

Since the p-values are greater than 0.05, H<sub>0</sub> cannot be rejected. Hence we can conclude that the three processes are stationary at the first difference. Since the first order differencing eliminates stationarity, this implies that the maximum order of integration is I(1). The next thing is to select the lag length of the vector autoregressive model.

**Lag Length Selection For VAR(p)**

Model selection criteria of p=2, 3, 4 and 5 were examined and the following results were obtained

Lag length	Model selection criteria	
	AIC	HQ
1	1	1
2	2	1
3	3	3
4	4	3
5	5	5

It is observed that the lag length order with the smallest model selection criteria is 1. Therefore, the vector autoregressive model to be considered is of order 1, VAR(1)

Hence the VAR(1) is given as:

$$X_t = C + \Pi X_{t-1} + \varepsilon_t \dots\dots\dots (2)$$

Where

$$X_t = \begin{pmatrix} X_{1t} \\ X_{2t} \\ X_{3t} \end{pmatrix}, \quad C = \begin{pmatrix} C_1 \\ C_2 \\ C_3 \end{pmatrix}, \quad \Pi = \begin{pmatrix} \pi_{11}^1 & \pi_{12}^1 & \pi_{13}^1 \\ \pi_{21}^1 & \pi_{22}^1 & \pi_{23}^1 \\ \pi_{31}^1 & \pi_{32}^1 & \pi_{33}^1 \end{pmatrix}, \quad X_{t-1} = \begin{pmatrix} X_{1t-1} \\ X_{2t-1} \\ X_{3t-1} \end{pmatrix} \text{ and } \varepsilon_t = \begin{pmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \\ \varepsilon_{3t} \end{pmatrix}$$

Therefore equation (2) becomes

$$\begin{pmatrix} X_{1t} \\ X_{2t} \\ X_{3t} \end{pmatrix} = \begin{pmatrix} C_1 \\ C_2 \\ C_3 \end{pmatrix} + \begin{pmatrix} \pi_{11}^1 & \pi_{12}^1 & \pi_{13}^1 \\ \pi_{21}^1 & \pi_{22}^1 & \pi_{23}^1 \\ \pi_{31}^1 & \pi_{32}^1 & \pi_{33}^1 \end{pmatrix} \begin{pmatrix} X_{1t-1} \\ X_{2t-1} \\ X_{3t-1} \end{pmatrix} + \begin{pmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \\ \varepsilon_{3t} \end{pmatrix} \dots\dots\dots (3)$$

Equation (3) is the selected model which is VAR(1).

**Parameter estimation for VAR(1)**

The lags 1 cross covariance matrix and cross correlation matrix are thus respectively:

$$\Gamma_1 = \begin{pmatrix} 0.524323 & 0.009892 & -0.002964 \\ 0.009892 & 0.019309 & 0.003971 \\ -0.002964 & 0.003971 & 0.003630 \end{pmatrix}$$

$$R_1 = \begin{pmatrix} 1.0000 & 0.09831 & -0.06794 \\ 0.09831 & 1.0000 & 0.47439 \\ -0.06794 & 0.47439 & 1.0000 \end{pmatrix}$$

Estimation results for equation COAL:

	estimates	Standard error	t-value	p-value
Coal X <sub>1t</sub>	0.5475	0.1728	3.169	0.00401
Gas X <sub>2t</sub>	-0.7529	0.7336	-1.026	0.31459
Petroleum X <sub>3t</sub>	-0.8747	1.5432	-0.567	0.57593
constant	4.7316	4.1310	1.145	0.26289

Estimation results for equation GAS:

	estimates	Standard error	t-value	p-value
Coal $X_{1t}$	0.02229	0.03316	0.672	0.507699
Gas $X_{2t}$	0.55869	0.14078	3.969	0.000537
Petroleum $X_{3t}$	0.8472	0.29616	2.861	0.008417
constant	-1.32196	0.79275	-1.668	0.107884

Estimation results for equation PETROLEUM:

	estimates	Standard error	t-value	p-value
Coal $X_{1t}$	-0.02401	0.01438	-1.670	0.107410
Gas $X_{2t}$	0.09416	0.06104	1.543	0.135446
Petroleum $X_{3t}$	0.52652	0.12840	4.101	0.000383
constant	1.29994	0.34371	3.782	0.000865

From the estimates of the 3 equation the following is deduced

$$1. C = \begin{pmatrix} C_1 \\ C_2 \\ C_3 \end{pmatrix} = \begin{pmatrix} 4.7316 \\ -1.32196 \\ 1.29994 \end{pmatrix}$$

$$2. \Pi = \begin{pmatrix} \pi_{11}^1 & \pi_{12}^1 & \pi_{13}^1 \\ \pi_{21}^1 & \pi_{22}^1 & \pi_{23}^1 \\ \pi_{31}^1 & \pi_{32}^1 & \pi_{33}^1 \end{pmatrix} = \begin{pmatrix} 0.5475 & -0.7529 & -0.8747 \\ 0.02229 & 0.55869 & 0.84720 \\ -0.02401 & 0.094167 & 0.52652 \end{pmatrix}$$

Hence equation (3) becomes

$$\begin{pmatrix} X_{1t} \\ X_{2t} \\ X_{3t} \end{pmatrix} = \begin{pmatrix} 4.7316 \\ -1.32196 \\ 1.29994 \end{pmatrix} + \begin{pmatrix} 0.5475 & -0.7529 & -0.8747 \\ 0.02229 & 0.55869 & 0.84720 \\ -0.02401 & 0.094167 & 0.52652 \end{pmatrix} \begin{pmatrix} X_{1t-1} \\ X_{2t-1} \\ X_{3t-1} \end{pmatrix} + \begin{pmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \\ \varepsilon_{3t} \end{pmatrix} \dots\dots\dots(4)$$

Hence from equation (4), we have

$$X_{1t} = 4.7316 + 0.5475X_{1t-1} - 0.7529X_{2t-1} - 0.8747X_{3t-1} + \varepsilon_{1t} \dots\dots\dots(5)$$

$$X_{2t} = 0.02229X_{1t-1} + 0.55869X_{2t-1} + 0.84720X_{3t-1} - 1.32196 + \varepsilon_{2t} \dots\dots\dots(6)$$

$$X_{3t} = 1.29994 - 0.02401X_{1t-1} + 0.094167X_{2t-1} + 0.52652X_{3t-1} + \varepsilon_{3t} \dots\dots\dots(7)$$

Equation (5), (6) and (7) are therefore the selected model for CO<sub>2</sub> emission from the consumption of Coal, Natural gas and petroleum respectively.

**Diagnostic Test**

**Portmanteau test for serial correlation**

H<sub>0</sub>: residual are serially uncorrelated

H<sub>1</sub>: the residual are serially correlated.

At  $\alpha=0.05$

Test statistic

Chi-squared	Degree of freedom	p-value
77.7884	81	0.5805

Since the p-value of the test statistic is greater than 0.05, H<sub>0</sub> cannot be rejected and we conclude the residuals are serially uncorrelated. This implies that the model fit the data.

**Wald test**

It is characterized by testing for non-zero correlation between the errors processes of the cause and effect variables.

**1. For coal emission and Natural gas emission**

H<sub>0</sub>:  $X_{1t}$  does not Granger-cause  $X_{2t}$

H<sub>1</sub>: H<sub>0</sub> is not true

$\alpha = 0.05$

Test statistic:

Chi-squared	Degree of freedom	p-value
14.3	2	0.0008

Since the p-value of the test statistic is less than 0.05, H<sub>0</sub> will be rejected and we conclude that  $X_{1t}$  Granger-cause  $X_{2t}$

**2. For coal emission and petrol emission**

H<sub>0</sub>:  $X_{1t}$  does not Granger-cause  $X_{3t}$

$H_1$ :  $H_0$  is not true

$\alpha = 0.05$

Test statistic:

Chi-squared	Degree of freedom	p-value
12.1	2	0.0023

Since the p-value of the test statistic is less than 0.05,  $H_0$  will be rejected and conclude that  $X_{1t}$  Granger-cause  $X_{3t}$

### 3. For Natural gas and petrol

$H_0$ :  $X_{2t}$  does not Granger-cause  $X_{3t}$

$H_1$ :  $H_0$  is not true

$\alpha = 0.05$

Test statistic:

Chi-squared	Degree of freedom	p-value
5.6	2	0.062

Since the p-value of the test statistic is greater than 0.05,  $H_0$  will be rejected and conclude that  $X_{2t}$  does not Granger-cause  $X_{3t}$

### Interpretation of the Wald test

From the display of the analysis in R-package, it was observed Granger-causality test is vice versa for this study.

1. For CO<sub>2</sub> emission from Coal consumption ( $X_{1t}$ ) and Natural gas consumption ( $X_{2t}$ ), the Wald test shows that  $X_{1t}$  Granger-cause  $X_{2t}$ . This means that  $X_{1t}$  is found to be helpful for predicting  $X_{2t}$  and vice versa.
2. For CO<sub>2</sub> emission from Coal consumption ( $X_{1t}$ ) and petrol consumption ( $X_{3t}$ ), the Wald test shows that  $X_{1t}$  Granger-cause  $X_{3t}$ . This means that  $X_{1t}$  is found to be helpful for predicting  $X_{3t}$  and vice versa.
3. For CO<sub>2</sub> emission from Coal consumption ( $X_{2t}$ ) and petrol consumption ( $X_{3t}$ ), the Wald test shows that  $X_{2t}$  does not Granger-cause  $X_{3t}$ . This means that  $X_{2t}$  cannot be used to predict  $X_{3t}$  and vice versa.

### Conclusion

Carbon dioxide emission from fossil fuels is highly significance; hence its emission can be monitored using the model obtained in this research.

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