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Element-free Galerkin method with wavelet basis and its application in electromagnetic field

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ABSTRACT
Part or all of the mesh can be eliminated by using the compact support, the orthogonality and other properties of the wavelet function and the meshless algorithm to enhance the computing capability. A meshless algorithm was proposed based on the wavelet basis function. Combine the advantages of both wavelet function and meshless method. Wavelet transform is introduced into the element-free method to build the form that wavelet basis function meshless method formula. The method is applied in analysis of electromagnetic field. We give the method of discrete model, and use Lagrange multiplier method realizing essential boundary conditions. Numerical examples show the new method is effective for solving electromagnetic field.

Introduction

Recently, meshless method has become a hot spot of computational mechanics. Some meshless method successively put forward and significant progress, for example, smooth particles fluid dynamics (SPH), scattering element method (DEM), no unit method (EFG) and local Petrov-Galerkin method (MLPG) [1] etc. In addition, meshless method with other coupling numerical methods are proposed, such as coupling with EFG and FEM, MLPG and FEM, EFG and BEM [2-3]. Meshless method mentioned above, the critical and essential step build the shape functions, shape function of construction process is complicated, time consuming, and in some cases more difficult to achieve. Additionally, shape function of the complexity of the computation process will lead to a very high computational cost. However, this looks like a complex work. Therefore, it is necessary to turn to other mathematical tools.

Wavelet analysis is quickly developed new analysis method in recent years, it has been widely applied in many practical areas, The biggest advantage is changed to actual needs of any size, High-resolution. Wavelet function with compact support is a good tool in the problem of decrypting the characteristics of high gradient. There are many scholars at home and abroad such as Liu [4-5] and the Yin [6] , who has put meshless method is applied to electromagnetic field of numerical computation [7-10], but meshless method numerical is still at the primary stage of the electromagnetic field. In this paper, we combine the advantages of both wavelet function and meshless method. Wavelet transform is introduced into the element-free method to build the form that wavelet basis function meshless method formula [11-13]. Through the electromagnetic field, the direct use of wavelet analysis of scaling functions construct 2D function in the tensor product of the way to approximate the unknown field function, discrete model of the method are given and numerical examples show the new method is effective for solving electromagnetic field.

Multi-resolution analysis

The key aspect of wavelets in numerical analysis is that wavelets are suitable for multi-resolution analysis. Before discussing the B-spline wavelets, a brief review of the multi-resolution analysis is given.

The multi-resolution analysis can be described by a sequence of closed subspaces $\{V_k\}_{k\in z}$ of the finite energy function space $L^2(R)$ defined on the whole real line R that satisfies the following conditions:

① The monotonicity: $\cdots \subset V_{-1} \subset V_0 \subset V_1 \subset \cdots \in L^2$

② Approximation: $\bigcap_{k\in z} V_k = \{0\}, \bigcup_{k\in z} V_k = L^2(R)$

③ Scalability: $f(t)\in V_j \Leftrightarrow f(2t)\in V_{j+1}$

④ Translation invariant: $f(t)\in V_j \Rightarrow f(t-k)\in V_j, k\in z$

⑤ Exist function: $\phi(t)\in L^2(R)$, make $\{\phi_{0,k}(t), k\in z\}$ constitute V_0 a Riesz base, says $\phi(t)$ for scaling function..

⑥ For 2D multi-resolution analysis $\{V_j^{\,2}\}_{j\in z}$ scale function $\phi(x,y)=\phi(x)\phi(y)$. Among ϕ is one-dimensional $\{V_j\}_{j\in z}$ multi-resolution analysis scale function. For each $j\in z$, function systems $\{\phi_{j,k_1,k_2}=\phi_{j,k_1}(x)\phi_{j,k_2}(y)\,|\,(k_1,k_2)\in z^2\}$ constitute $V_j^{\,2}$ the standard orthogonal basis. The space $L^2(R^2)$ such a multi-resolution analysis $\{V_j^{\,2}\}_{j\in z}$ call a separable.

For multi-resolution analysis. Scale function $\phi(x)\in V$. Expressed as

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$$\phi(x)=\sum_{-\infty}^{+\infty}c_k\phi(2x-k) \quad (1)$$

Definition wavelet space $W_j^2=(V_j^2)$, namely $V_j^2\oplus W_j^2=V_{j+1}{}^2$, where V_j and V_{j+1} denote two different adjacent space. The space constructed function W_j is called wavelet base $\varphi(x)$ and $\varphi_{j,k}(x)=\sqrt{2^j}\varphi(2^jx-k\mid j,k\in z)$. Then wavelet base content relation

$$\varphi(x)=\sum_{-\infty}^{+\infty}g_k\varphi(2x-k) \quad (2)$$

Then one coefficient g_k can have expressed on c_k type.

Moving Least-squares based

Meshless method uses basic formula with polynomial based approximate function to approximate true solution

$$u^h(x)=\sum_{j=1}^{m}p_j(x)a_j(x)=P^T(x)a(x) \quad (3)$$

where $p_i(x)$ as to the space coordinates $x^T=[x,y]$. By using the base, the basic functions, making shape functions meet the completeness. But for high power questions, polynomial matrix has some defects. So we will introduce wavelet meshless method, based on wavelet base to replace conventional polynomial matrix. Because wavelet has the feature of excellent local change, so we do not have to increase the node number in the calculation.

Wavelet base meshless methods

Based on wavelet basis functions of replacing (3) polynomial function, with m -order B-spline function $N_m(x)$ as wavelet base. Then Haar wavelet multi-scale functions $\varphi_{Haar}(x)$ said.

$$\varphi_{Haar}(x)=N_1(t)=\chi_{[0,1]}=\begin{cases}1 & 0\le t\le 1\\ 0 & \text{else}\end{cases} \quad (4)$$

$$N_2(t)=\mathrm{N}_1*\mathrm{N}_1(t)=\begin{cases}t & 0\le t\le 1\\ 2-t & 1\le t\le 2\\ 0 & else\end{cases} \quad (5)$$

$$N_m(t)=\mathrm{N}_{m-1}*\mathrm{N}_1(t)=\int_0^1 N_{m-1}(t-\tau)d\tau,(m\ge 2) \quad (6)$$

The equation is available as follows

$$N_m(x)=\sum_{k=0}^{m}2^{-m+1}\binom{m}{k}N_m(2x-k) \quad (7)$$

Let $\varphi(x)=N_m(x)$, and $\varphi_{j,k}(x)=\sqrt{2^j}N_m(2^jx-k)$. Therefore V_j space can have born $\varphi_{j,k}(x)$ become

$$V_j=\overline{span}=\{\varphi_{j,k}(x)\mid j,k\in z\} \quad (8)$$

where j is scale parameter, k is translation parameters.

For the whole W_j structure of wavelet basic functions $\varphi(x)$ can be expressed as

$$\varphi_m(x)=\sum_{k=0}^{3m-2}[\frac{(-1)^k}{2^{m-1}}\sum_{j=0}^{m}\binom{m}{j}N_{2m}(k-j+1)]N_m(2x-k) \quad (9)$$

With three order B-spline function as multi-scale functions, then $\varphi(x)=N_3(x)$. Constitute V_j a group of standard orthogonal basis. Then the approximate formulas for meshless

$$u^h(x)=\sum_{j=1}^{m}p_j(x)a_j(x)=P^T(x)a(x) \quad (10)$$

By least-squares approximation can be expressed as

$$J=\sum_{i=1}^{n}w(x-x_i)[a^T(x)p(x)-u_i]^2 \quad (11)$$

where $w(x-x_i)$ for i node x in place of weight function, u_i for i point a displacement. Then

$$w(r)=\begin{cases}1-6r^2+8r^3-3r^4 & (0\le r\le 1)\\ 0 & 1<r\end{cases} \quad (r=|x-x_i|) \quad (12)$$

Take extreme

$$\frac{\partial J}{\partial u}=0 \quad (13)$$

then

$$A(x)a(x)=B(x)u \quad (14)$$

Approximate function $u^h(x)$ has node values u_i mean for

$$A(x)a(x)=B(x)u^h(x)=P^T(x)A^{-1}(x)B(x)u=\varphi^T(x)u=\sum_{i=1}^{N}\varphi_i(x)u_i \quad (15)$$

where $$A(x)=\sum_{i=1}^{n}w(x-x_i)\varphi(x_i)\varphi^T(x_i) \quad (16)$$

$$B(x)=[w_1(x)\varphi(x_1),w_2(x)\varphi(x_2)\mathrm{L}\ \ w_n(x)\varphi(x_n)] \quad (17)$$

In dealing with two-dimensional problems. Using scale tensor integral form gets $L^2(R^2)$ scale function for

$$\varphi_{j,k,l}(x,y)=\varphi_{j,k}(x)\varphi_{j,l}(y)=2^{-\frac{j}{2}}\varphi(2^{-j}x-k)2^{-\frac{j}{2}}\varphi(2^{-j}y-l),k,l\in z \quad (18)$$

With three-orders B-spline function for scaling function, then

$$\varphi(x)=N_3(x),\varphi(y)=N_3(y) \quad (19)$$

Let Eq.(19) as MLS base function and by 2D wavelet base to unfold

$$u^h(x)=\sum\sum a_{j;k,l}\varphi_{j,k}(x)\varphi_{j,l}(y)=a^T\varphi(x,y) \quad (20)$$

In 2D Poisson equation

$$\begin{cases}\Omega: & \nabla^2=-f\\ \Gamma_D: & \Phi=\bar{\Phi}\\ \Gamma_N: & \dfrac{\partial\Phi}{\partial n}=q\end{cases} \quad (21)$$

where Ω for solving the domain. Let Γ_D be Dirichlet boundary, Γ_N be the Raman boundary for north Φ be the Potential.

The corresponding weak conditions

$$F(A)=\int_\Omega\{\frac{1}{2}[(\frac{\partial\Phi}{\partial x})^2+(\frac{\partial\Phi}{\partial y})^2]-f\Phi\}d\Omega-(\int_{\Gamma_N}\bar{q}\Phi ds+\int_{\Gamma_D}\frac{\partial\Phi}{\partial n}\Phi ds) \quad (22)$$

In the above discrete model, the boundary conditions using Lagrange multiplier method introducing Dirichlet boundary conditions, then $\lambda(x)=\sum_{i=1}^{n}N_i(s)\lambda_i$, where $N_i(s)$ be Lagrange interpolation function, λ be undetermined coefficients, s be boundary arc length. The generation of the boundary value problems to the discrete equations for

$$\begin{bmatrix} K & G \\ G^T & 0 \end{bmatrix} \begin{Bmatrix} u \\ \lambda \end{Bmatrix} = \begin{Bmatrix} F \\ q \end{Bmatrix} \qquad (23)$$

where

$K_{i,j} = \int_\Omega B_i^T D B_k d\Omega$, $G_{i,j} = -\int_{\Gamma_D} \phi_i N_k d\Gamma$, $F_i = \int_\Omega \phi_i f d\Omega + \int_\Omega \phi_i \bar{q} d\Omega$.

$q = -\int_{\Gamma_D} N_k \bar{u} d\Gamma$. and $B_i = \begin{bmatrix} \frac{\partial \phi_i}{\partial x} & 0 \\ 0 & \frac{\partial \phi_i}{\partial y} \\ \frac{\partial \phi_i}{\partial y} & \frac{\partial \phi_i}{\partial x} \end{bmatrix}$ be the heft Strain matrix.

One dimension parameters j value directly related to the discretion of the resolution.

Numerical examples

Consider Poisson equation

$$\nabla^2 u(x,y) = -2(x+y-x^2-y^2), \quad \text{in}\Omega: x \in [0,1], y \in [0,1] \qquad (24)$$

$$u(x,y) = 0 \qquad on: \partial\Omega \qquad (25)$$

The precise solution for

$$u(x,y) = (x-x^2)(y-y^2) \qquad (26)$$

In domain Ω decorate 11×11 rules node (Fig.1). Using 3×3 order Gaussian integral. Accumulate molecular domain sizes 0.5×0.25. Weight functions choose three times spline function, with two-dimensional wavelet base for the MLS of the basement. Scaling function j take 4. Boundary conditions use Lagrange multiplier method, compared with accurate solution, EFGM and wavelet numerical solutions (Fig. 2), and the wavelet accurate solution (Fig. 3).

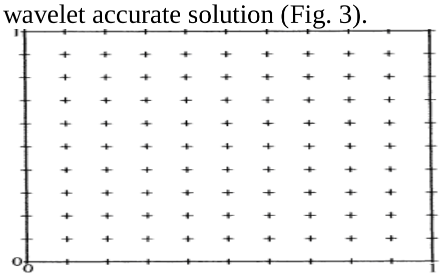

Fig.1 Nodal arrangement

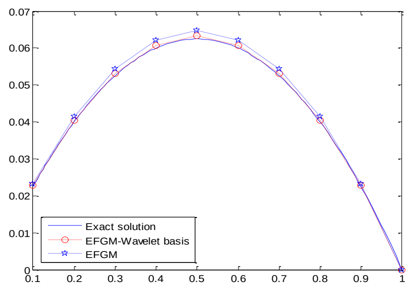

Fig. 2 When x = 0.5 when the accurate solution and numerical solutions

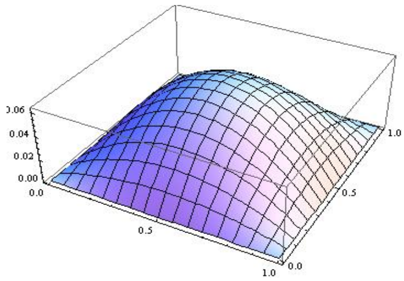

Fig. 3 Wavelet exact solutions

Set of electromagnetic field meet Poisson equation

$$\nabla^2\Phi = \sin(\pi x)\sin(\pi y), x \in (0,2), y \in (0,2) \qquad (27)$$

Solving the domain for square, in the border satisfy Dirichlet conditions $\Phi|_\Gamma = 0$.

In the area of cloth 9×9 node, 64 a deposition of molecular domain. take wavelet base, scaling function j take 4, second order Gaussian integral. That each product molecules domain existence 4 Gaussian points. Ask for its numerical solution and exact solutions $\Phi = -\frac{1}{2\pi^2}\sin(\pi x)\sin(\pi y)$. In each node place function values as shown in figure 4 below.

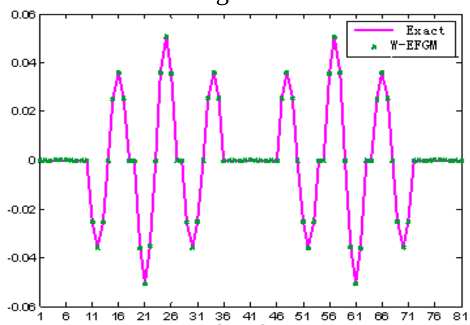

Fig. 4 The exact solution and the numerical solution of the nodes

Conclusions

In this paper, based on wavelet meshless method, we analyzed the electromagnetic field problem. Wavelet meshless method is a new meshless method. By using the wavelet multi-scale functions of approximate unknown field function, avoiding the traditional meshless method using node information structure shape function of complex process. It makes that the formula in this method is concise and easy to be used in application. In this paper, the method is applied in analysis of electromagnetic field. We gave the method of discrete model, and use Lagrange multiplier method realizing essential boundary conditions. Numerical examples show the new method for solving electromagnetic field is effective.

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