



Second minimum weight spanning tree in a network

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ABSTRACT

Minimum weight spanning tree is a well known graph optimization problem, which has a wide range of applications in telecommunications and routing problems. The problem considered in this paper is to find the second minimum weight spanning tree for a given network. A new algorithm is proposed and its computational complexity is discussed with numerical illustrations.

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Keywords

Optimization,
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Introduction

Some of the applications of Minimum Weight Spanning Tree (MWST) include laying of cables in an office connecting all the departments with minimum cost, construction of roads connecting all cities in the network with minimum cost etc. These problems can be formulated as a network G . The vertices and edges represent the cities and feasible connections. The weight of the edge may represent construction cost or time etc. For optimization problems and spanning tree of a network, one can refer [1], [2], [4], [5], [6].

In this paper, a new method is proposed to find the second minimum weight spanning tree. In the proposed method all weights are assumed to be non negative. If there does not exist a link between nodes, weights can be thought of as being infinity.

The Proposed Method

Minimum weight spanning tree for a given network is first determined by using any of the available (Kruskal's or Prim's) methods [3]. It is assumed that, for the given network there is only one MWST. The weight of each edge of the MWST is set in turn to infinity in the given network and the MWST is to be found for each such cases. As a spanning tree with n vertices has $n-1$ edges, we will get $(n-1)$ minimum weight spanning trees for the resulting $(n-1)$ networks. Among, select a spanning tree with minimum weight, which is the second minimum weight spanning tree for the given network.

The proposed Algorithm

Step 1. Find the MWST of a network with n vertices.

Step 2. For $i = 1$ to $n-1$

Set in turn, the weight of i th edge of the MWST to be infinity in the original network.

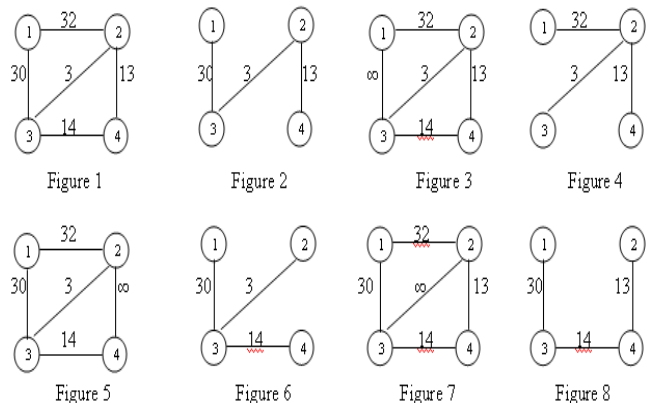
Find the MWST for the network and store its weight in ST_i .

Step 3. Find the minimum of ST_i and its corresponding spanning tree is the second MWST

Example

The minimum weight spanning tree of the network in figure 1 is given in figure 2, with weight 46. The edges of the MWST are 1-3, 2-4 and 2-3. Initially the edge 1-3 is set to infinity (fig-3) and its corresponding MWST is given in fig-4 with weight

48. In the same way the edges 2-4 and 2-3 are set to infinity (fig-5 and fig-7) and its corresponding MWST is given in fig-6 and fig-8 with weights 47 and 57 respectively. The minimum of the weight of these spanning trees is 47, which is the desired second (next) MWST of the original network.



Conclusion

In this paper a new algorithm is proposed to find the second minimum weight spanning tree for a given network. For finding out the minimum weight spanning tree using Kruskal's algorithm, step 1 of this algorithm requires $O(n^2)$ elementary operations. In Step 2 and step 3 of this algorithm, the MWST is found for $(n-1)$ number of times. Hence the computational complexity required by the proposed method is $O(n^3)$.

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