Available online at www.elixirpublishers.com (Elixir International Journal)

Statistics

Elixir Statistics 78 (2015) 29648-29653

ISSN: 2229-712X

On the Comparative Study of Estimators in Seemingly Unrelated Regression Equations

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ARTICLE INFO

Article history:

Received: 7 May 2014;

Received in revised form: 21 December 2014;

Accepted: 5January 2015;

Keywords

Bootstrapping,

Feasible Generalized Least Squares,

Generalized Least Squares,

Ordinary LeastSquares,

Seemingly Unrelated Regressions.

ABSTRACT

This work examined the efficiencies of Ordinary Least Squares (OLS) and Seemingly Unrelated Regression (SUR) estimators in lagged and unlagged models. Literature has shown gain in efficiency of SUR estimator over OLS estimator when the errors are correlated across equations. This paper studied the efficiencies of these estimators in a lagged and unlagged models and also sought a comparative study of these estimators in both models.Data was simulated for sample sizes 50, 100 and 1000 with 5000 bootstrapped replicates in each case with the predictors having Gaussian distribution. Results from the study showed that both estimators were efficient in each model with the SUR estimator being consistently more efficient than the OLS estimator as the sample size increased. On the assessment of the models, the unlagged model was found to be more efficient than the lagged model in small sample but converged as sample size increased.

Introduction

The Seemingly Unrelated Regressions (SUR) model explains the variation of a set of *m* dependent variables, e.g. the monthly consumption expenditures of m consumers or the annual voting behaviour of *m* voters, in terms of the variation of general and specific input or independent variables and error terms specific to each individual problems that are frequently encountered in many sciences. Indeed, Geweke (2003, p. 162) has written, "The Seemingly Unrelated Regressions (SUR) model developed in Zellner (1962) is perhaps the most widely used econometric model after linear regressions. The reason is that it provides a simple and useful representation of systems of demand equations that arise in neoclassical static theories of producer and consumer behaviour."

It is the case that a SUR model is a collection of two or more regression relations that can be analyzed with data on the dependent and independent variables. For many years, the individual regression relations were fitted one by one, usually using least squares techniques and justified by an appeal to single equation estimation optimality properties, e.g. the least squares estimators are best linear unbiased estimators according to the well known Gauss Markov theorem and maximum likelihood estimators when single equation normal likelihood functions are employed. What was overlooked in the pre-1962 literature is the fact that when the error terms in the different regression equations are correlated the regression equations are related and that the sample information in other regressions can be employed to improve the precision of estimation of parameters in any given regression equation under a wide range of conditions. That is, new, operational SUR best linear unbiased estimators for the parameters of a set of say m regression equations were put forward (Zellner 1962) that uniformly dominate the single equation least squares estimators under a broad range of conditions. It was shown that these SUR or Generalized Least Squares (GLS) estimators are best linear unbiased, maximum likelihood and Bayesian estimators under frequently encountered conditions. And in addition, by joint analysis of the set of regression equations rather than equation by equation analysis, more precise estimates and predictions are obtained that lead to better solutions to many applied problems, e.g. portfolio formation procedures in study by Quintana, et al (2003) in which dynamic regression equations with time varying parameters and various input variables were employed to explain the variation of monthly stock prices. By taking account of the fact that the regression equations were related and not unrelated, SUR estimation, prediction and portfolio formation procedures were utilized to yield improved analyses of the variation of stock prices and to form optimal portfolios with very good rates of return. For textbook and other analyses of the SUR model and applications of it, see. e.g., Geweke (2003), Greene (2003), Judge et al. (1985), Meng and Rubin (1996), Percy (1992, 1996), Rossi, Allenby and McCulloch (2005), Quintana, Putnam and Wilford (1998), Srivastava and Giles (1987), Theil (1971), Zellner (1962,1963), and Zellner and Huang (1962). Also, in Zellner and Theil (1962) similar techniques were applied to simultaneous equations models to yield a new joint estimator, the three stage least squares estimator that dominates single equation estimators by taking account of the correlation of error terms in equations of the system by use of joint estimation of coefficients in equations of structural models.

Adebayo (2003) also investigated how large the contemporaneous correlations among disturbances should be in order for SUR to be more efficient than OLS using the Bayesian approach. He asserted that definite gains are obtained when $\rho > 0.333$ which compares well with Dielman (1989) who used a frequentist approach. He further asserted that there is no clear-cut distinction between SUR and univariate models (GLS and OLS are identical) when the same set or subset(and values) of covariates, which may not likely lead to more efficient estimates in SUR than running the model separately, that is, the covariates $X_i = X_j$ are used. On the other side, if the equations are actually uncorrelated or if the design matrices are identical in all equations, SUR (GLS) and Ordinary Least Squares (OLS) regressions give the same results. (See Revankar, 1974).

In all the estimation techniques developed for different SUR situations as reported above, Zellner's basic recommendation for high contemporaneous correlation between the error vectors with uncorrelated explanatory variables within each response equations was also maintained. More recently, Olamide et al. (2013) reaffirmed the gain in efficiency of the SUR estimator over the Ordinary Least Squares (OLS) and the Iterative Ordinary Least Squares (IOLS) estimators when the errors are both contemporaneously and serially correlated. In this paper, we aim at examining the performances of the GLS and OLS estimators for both normal and lagged models using a true covariance matrix for correlated errors.

In Section 2, materials and methods were presented and the structural parametric framework of SUR system is discussed while the simulation studies carried out in the work is discussed in Section 3. Detail discussion of our results is presented in Section 4 and finally Section 5 provides some concluding remarks.

Materials And Methods

Parametric SUR Framework

Consider a complete system of regression equations with *m* response variables each containing *n* observations denoted by the vector $\boldsymbol{Y}' = (\boldsymbol{y_1}, \boldsymbol{y_2}, \cdots, \boldsymbol{y_m})$ with associated distinct vector of explanatory variables $\boldsymbol{X_1}, \boldsymbol{X_2}, \cdots, \boldsymbol{X_m}$respectively. Each of the equations in this system of regression equations is assumed to satisfy the Gauss-Markov properties of homoscedasticity and no serial correlations of the error terms. That is, for each of the response equations the popular distributional assumptions on the error term of

$$\boldsymbol{\varepsilon_i} \sim \text{N}(0, \boldsymbol{\sigma_i^2}) \qquad (2.1)$$

for$i$ = 1,2, $\cdots$,m and

$$\text{Cov}(\boldsymbol{\varepsilon_i}, \boldsymbol{\varepsilon_i'}) = 0 \qquad (2.2)$$

are maintained for $\mathbf{n}_i, \mathbf{n}_i'$ = 1, 2, $\cdots$,n.

The system can therefore be represented by

$$\left.\begin{array}{l} \boldsymbol{y_1} = \boldsymbol{X_1\beta_1} + \boldsymbol{\varepsilon_1} \\ \boldsymbol{y_2} = \boldsymbol{X_2\beta_2} + \boldsymbol{\varepsilon_2} \\ \quad\vdots \\ \boldsymbol{y_m} = \boldsymbol{X_m\beta_m} + \boldsymbol{\varepsilon_m} \end{array}\right\} \qquad (2.3)$$

where ; i = 1, 2, $\cdots$, m, $\boldsymbol{y_i}$is an n $\times$ 1 vector of observations on the $\boldsymbol{i^{th}}$ response variable $\boldsymbol{X_i}$is an n $\times$ $\boldsymbol{p_i}$ matrix of explanatory variables, $\boldsymbol{\beta_i}$ is a $\boldsymbol{p_i}$ $\times$1 vector of regression parameters and $\boldsymbol{\varepsilon_i}$is the corresponding n $\times$ 1 vector of disturbances.

Thus, each set of the $\boldsymbol{y_i}$ regression equations has $\boldsymbol{p_i}$parameters. This system of equations in (2.3) can further be presented in a more compact form as

$$\overbrace{\begin{bmatrix} \boldsymbol{y_1} \\ \boldsymbol{y_2} \\ \vdots \\ \boldsymbol{y_T} \end{bmatrix}}^{Y} = \overbrace{\begin{bmatrix} \mathbf{X_1} & \mathbf{0} & \dots & \mathbf{0} \\ \mathbf{0} & \mathbf{X_2} & \dots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \dots & \mathbf{X_T} \end{bmatrix}}^{X} \overbrace{\begin{bmatrix} \boldsymbol{\beta_1} \\ \boldsymbol{\beta_2} \\ \vdots \\ \boldsymbol{\beta_T} \end{bmatrix}}^{\beta} + \overbrace{\begin{bmatrix} \boldsymbol{\varepsilon_1} \\ \boldsymbol{\varepsilon_2} \\ \vdots \\ \boldsymbol{\varepsilon_T} \end{bmatrix}}^{\varepsilon} \qquad (2.4)$$

$mn\times 1 \; mn\times \sum \boldsymbol{p_i} \; \sum \boldsymbol{p_i} \times 1 \qquad mn\times 1$

and when stacked together the whole system becomes

$$\boldsymbol{y} = \boldsymbol{X\beta} + \boldsymbol{\varepsilon} \qquad (2.5)$$

The regression equations in (2.3) or (2.4) above appear independent (seemingly unrelated) with each other because they do not have common variables or parameters. However, Zellner (1962) was of the opinion that each pair of the system of regression equations above are actually (contemporaneously) correlated through their error terms. Hence, the name *Seemingly Unrelated Regression* (SUR) given to such a system of regression equations as depicted by (2.3) or (2.4).

Estimating each of the equation separately by Ordinary Least Squares (OLS) may yield consistent but inefficient estimates of the parameters. Therefore, in SUR estimation techniques the correlations among the errors in different equations are put into consideration and are used to improve the regression estimates.

The OLS estimation method is given by

$$\boldsymbol{\hat{\beta}_{OLS}} = (\mathbf{X'X})^{-1}\,\text{X}'\text{Y} \qquad (2.6)$$

and this can only provide a set of consistent but less efficient estimates of the regression equations.

For the SUR estimation procedure, we make use of the assumption placed on the variance-covariance matrix of the disturbance in equation (2.5) that

$$\text{E}(\boldsymbol{\varepsilon}, \boldsymbol{\varepsilon}') = \Omega = \boldsymbol{\Sigma} \otimes \boldsymbol{I_n} \qquad (2.7)$$

Where $\boldsymbol{\Sigma}$ is an $m\times m$ matrix of the form

$$\boldsymbol{\Sigma} = \begin{bmatrix} \boldsymbol{\sigma_{11}} & \boldsymbol{\sigma_{12}} & \cdots & \boldsymbol{\sigma_{1M}} \\ \boldsymbol{\sigma_{21}} & \boldsymbol{\sigma_{22}} & \cdots & \boldsymbol{\sigma_{2M}} \\ \vdots & \vdots & \ddots & \vdots \\ \boldsymbol{\sigma_{M1}} & \boldsymbol{\sigma_{M2}} & \cdots & \boldsymbol{\sigma_{MM}} \end{bmatrix} \qquad (2.8)$$

$\boldsymbol{I_n}$is an $n\times n$ identity matrix and $\otimes$ is the Kronecker product which multiplies each element in $\boldsymbol{\Sigma}$ by $\boldsymbol{I_n}$.

From (2.7), if all values of the elements of Ω are known, then the SUR estimation of the regression models yields more efficient regression parameter estimates via the Generalized Least Squares estimation technique given by

$$\boldsymbol{\hat{\beta}_{GLS}} = (\boldsymbol{X'\Omega^{-1}X})^{-1}\boldsymbol{X'\Omega^{-1}}\text{y} \qquad (2.9)$$

i.e.,

$$\hat{\boldsymbol{\beta}}_{GLS} = [\boldsymbol{X}'(\boldsymbol{\Sigma}^{-1} \otimes \mathrm{I})\, \mathbf{X}]^{-1} \boldsymbol{X}'(\boldsymbol{\Sigma}^{-1} \otimes \mathrm{I})\, \mathbf{y}.$$

However, if the values of the elements of Ω are unknown, they are estimated and the resulting estimation procedure known as Feasible Generalized Least Squares (FGLS) or Estimated Generalized Least Squares (EGLS) is given by

$$\hat{\boldsymbol{\beta}}_{FGLS} = (\mathbf{X}'\,(\hat{\boldsymbol{\Sigma}}^{-1} \otimes \mathbf{I})\,\mathbf{X}\,)^{-1} \boldsymbol{X}'(\hat{\boldsymbol{\Sigma}}^{-1} \otimes \mathbf{I})^{-1}\mathrm{Y}.$$

Simulation Studies

The simulation work considers a system of SUR equations containing two distinct linear regression equations each with normal and lagged independent variables.

Thus, with m = 2, we have

$$\left.\begin{aligned} \boldsymbol{y_1} &= \boldsymbol{X_1\beta_1} + \boldsymbol{\varepsilon_1} \\ \boldsymbol{y_2} &= \boldsymbol{X_2\beta_2} + \boldsymbol{\varepsilon_2} \end{aligned}\right\} \quad (3.1)$$

The structural form of the SUR equations for our simulation is

$$\left.\begin{aligned} \boldsymbol{y_1} &= \mathbf{0.8} + \mathbf{0.2}\boldsymbol{x_{11}} + \mathbf{0.6}\boldsymbol{x_{12}} + \boldsymbol{\varepsilon_1} \\ \boldsymbol{y_2} &= \mathbf{0.5} - \mathbf{0.1}\boldsymbol{x_{21}} + \mathbf{0.7}\boldsymbol{x_{22}} + \boldsymbol{\varepsilon_2} \end{aligned}\right\} \quad (3.2)$$

and for the lagged version, we have

$$\left.\begin{aligned} \boldsymbol{y_1} &= \mathbf{0.8} + \mathbf{0.2}\boldsymbol{x}_{\mathbf{11,t-1}} + \mathbf{0.6}\boldsymbol{x}_{\mathbf{12,t-1}} + \boldsymbol{\varepsilon_1} \\ \boldsymbol{y_2} &= \mathbf{0.5} - \mathbf{0.1}\boldsymbol{x}_{\mathbf{21,t-1}} + \mathbf{0.7}\boldsymbol{x}_{\mathbf{22,t-1}} + \boldsymbol{\varepsilon_2} \end{aligned}\right\} \quad (3.3)$$

The true variance-covariance $\boldsymbol{\Sigma}$ used in $\boldsymbol{\varepsilon}$for this simulation is

$$\hat{\boldsymbol{\Sigma}} = \begin{bmatrix} \mathbf{1} & \mathbf{0.8} \\ \mathbf{0.8} & \mathbf{1} \end{bmatrix} \quad (3.4)$$

The variance-covariance matrix (3.4) is a positive definite non-singular symmetric matrix whose Cholesky decomposition is computed as

$$\mathrm{K} = \begin{bmatrix} \mathbf{1} & \mathbf{0} \\ \mathbf{0.8} & \mathbf{0.6} \end{bmatrix} \quad (3.5)$$

Therefore, in establishing a contemporaneous relationship among the error terms in the set of regression equations we now use $\boldsymbol{\varepsilon}^*$ = $(\boldsymbol{\varepsilon}_1^*,\ \boldsymbol{\varepsilon}_2^*)'$ in place of $\boldsymbol{\varepsilon}$ = $(\boldsymbol{\varepsilon_1}, \boldsymbol{\varepsilon_2})\,'$ whose values are determined by the product

$$\boldsymbol{\varepsilon}^* = \mathrm{K} \times \boldsymbol{\varepsilon} = \begin{bmatrix} \boldsymbol{\varepsilon_1} \\ \mathbf{0.8}\boldsymbol{\varepsilon_1} + \mathbf{0.6}\boldsymbol{\varepsilon_2} \end{bmatrix} \quad (3.6)$$

Simulations were performed with n = 50, 100, 1000. The regression equations were case bootstrapped using 5000 replicates in each case.

Discussion of Results

Relevant discussions of the simulation results obtained in this work are presented in this section. The results of the simulation studies (some of which are presented in *Appendix A & B*) generally showed that both SUR and OLS estimators were increasingly more efficient as the sample size increased. This is evident from the decreasing trend of the standard errors of the parameter estimates using the lagged and unlagged (normal) variables as the sample size increased in the two regression equations considered. Table 1 presents results for model with unlagged variables and Table 2 gives results for model with lagged variables. Despite the gain in efficiency by using OLS estimators, SUR estimators were still consistently more efficient than OLS in all cases considered.

It was observed that the estimates of the intercepts tend to approach the true parameter value most at sample size of 100. The slopes in all cases approached true parameter values as sample size increased. The OLS estimators for the models with lagged variables tend to be more efficient at sample size of 100 than that of the unlagged.

The efficiencies of the estimates for both models (lagged and unlagged) converged at large sample size of 1000.

All estimates were consistently efficient in both models as sample size increased.

Appendix B showed the bootstrapped results for the unlagged model and it revealed that the results were consistently efficient as sample size increased.

Conclusion

In this work, we reaffirm that SUR estimator is consistently better than the OLS equation-by-equation method of estimation in a regression equations that are related by their disturbance terms. Though, both OLS and SUR estimators increased in their efficiencies as sample size increased in the two models (*lagged and unlagged*) considered, SUR supremacy over OLS is maintained in both cases.

Finally, there was no loss of information in the lagged model especially for large sample size (1000) as the estimators converged in efficiencies to that of the unlagged model.

Appendix A: Tables of Results

Table 1: Results for Models with Unlagged Variables

	n = 50				**n = 100**				**n = 1000**			
	OLS Estimations		SUR Estimations		OLS Estimations		SUR Estimations		OLS Estimations		SUR Estimations	
True Coefficients	Estimate	Std Error	Estimate	Std Error	Estimate	Std Error	Estimate	Std Error	Estimate	Std Error	Estimate	Std Error
Regression 1												
$\boldsymbol{\beta_{10}}$ = 0.8	0.8440	0.1617	0.8502	0.1605	0.7936	0.1154	0.7801	0.1148	0.8626	0.0321	0.8622	0.0321
$\boldsymbol{\beta_{11}}$ = 0.2	0.3078	0.1945	0.2825	0.1029	-0.0030	0.1282	0.1182	0.0653	0.2456	0.0311	0.2188	0.0190
$\boldsymbol{\beta_{12}}$ = 0.6	0.3759	0.1817	0.4333	0.0956	0.4028	0.1345	0.4292	0.0680	0.5879	0.0322	0.5975	0.0197

Regression 2												
β_{20} = 0.5	0.5606	0.1787	0.5902	0.1746	0.4752	0.1164	0.4906	0.1150	0.5597	0.0315	0.5595	0.0315
β_{21} = -0.1	0.0617	0.1951	-0.0693	0.1032	0.1013	0.1288	-0.0603	0.0655	-0.1057	0.0325	-0.1254	0.0199
β_{22} = 0.7	0.6499	0.1706	0.6641	0.0897	0.7225	0.1139	0.7014	0.0576	0.6948	0.0312	0.7114	0.0191

Table 2: Results for Models with Lagged Variables

	n = 50				n = 100				n = 1000			
	OLS Estimations		*SUR Estimations*		OLS Estimations		*SUR Estimations*		OLS Estimations		*SUR Estimations*	
True Coefficients	Estimate	Std Error	*Estimate*	*Std Error*	Estimate	Std Error	*Estimate*	*Std Error*	Estimate	Std Error	*Estimate*	*Std Error*
Regression 1												
β_{10} = 0.8	0.8937	0.1661	*0.8956*	*0.1653*	0.7709	0.1181	*0.7845*	*0.1174*	0.8628	0.0321	*0.8625*	*0.0321*
β_{11} = 0.2	0.1871	0.2003	*0.0936*	*0.1053*	0.2424	0.1307	*0.1403*	*0.0700*	0.1834	0.0311	*0.1722*	*0.0190*
β_{12} = 0.6	0.6810	0.1871	*0.5528*	*0.0981*	0.7715	0.1368	*0.6271*	*0.0728*	0.5901	0.0322	*0.6236*	*0.0197*
Regression 2												
β_{20} = 0.5	0.6129	0.1843	*0.5889*	*0.1781*	0.5234	0.1158	*0.5118*	*0.1152*	0.5610	0.0315	*0.5617*	*0.0315*
β_{21} = -0.1	-0.0634	0.2078	*0.0758*	*0.1092*	-0.2658	0.1275	*-0.1506*	*0.0683*	-0.1056	0.0325	*-0.0954*	*0.0199*
β_{22} = 0.7	0.6775	0.1722	*0.5070*	*0.0903*	0.5316	0.1132	*0.6416*	*0.0603*	0.6937	0.0312	*0.7088*	*0.0191*

Appendix B

Bootstrapped Results (**Normal Variables**)

For n = 50

	R	original	bootBiasbootSE		bootMed
(Intercept)	5000	0.84399	-0.0080361	0.15817	0.83508
x11	5000	0.30777	0.0188731	0.20147	0.32691
x12	5000	0.37588	0.0197646	0.18714	0.38162

	R	original	bootBiasbootSE		bootMed
(Intercept)	5000	0.560635	-0.01367154	0.19253	0.546653
x21	5000	0.061739	-0.00193235	0.21320	0.066393
x22	5000	0.649905	-0.00082948	0.15287	0.655620

For n = 100

	R	original	bootBiasbootSE		bootMed
(Intercept)	5000	0.7935848	3.3808e-06	0.11813	0.79407348
x11	5000	-0.0030215	4.4851e-03	0.13777	-0.00081494
x12	5000	0.4027738	7.8722e-03	0.13380	0.40739315

	R	original	bootBiasbootSE		bootMed
(Intercept)	5000	0.47522	-0.0053925	0.11782	0.46905
x21	5000	0.10128	0.0023311	0.13429	0.10662
x22	5000	0.72254	-0.0024399	0.10093	0.72257

For n =1000

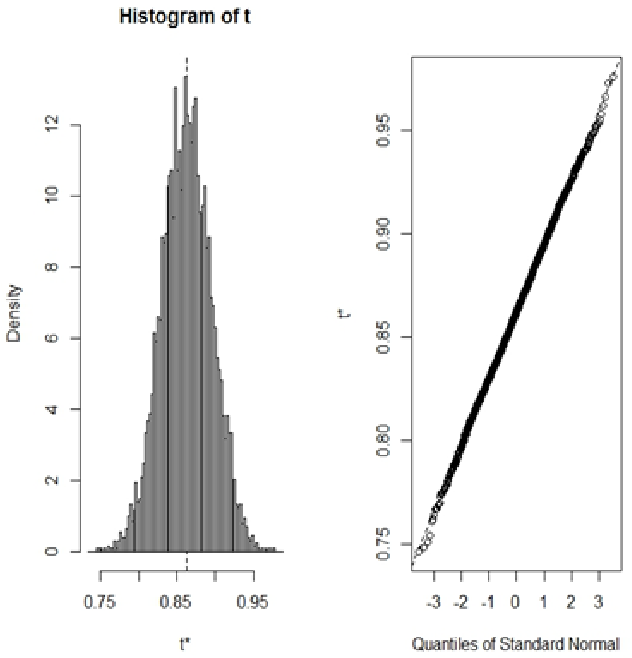
	R	original	bootBiasbootSE		bootMed
(Intercept)	5000	0.86261	-0.00017081	0.032253	0.86227
x11	5000	0.24560	0.00020955	0.031222	0.24643
x12	5000	0.58785	0.00032211	0.031723	0.58729

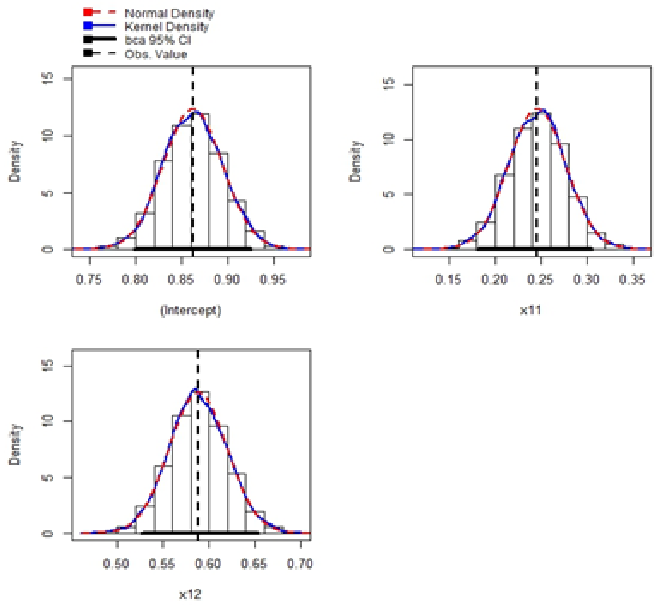
	R	original	bootBiasbootSE		bootMed
(Intercept)	5000	0.55971	-0.00037671	0.031376	0.55907
x21	5000	-0.10568	-0.00093528	0.033601	-0.10662
x22	5000	0.69477	-0.00083633	0.029478	0.69399

Appendix C

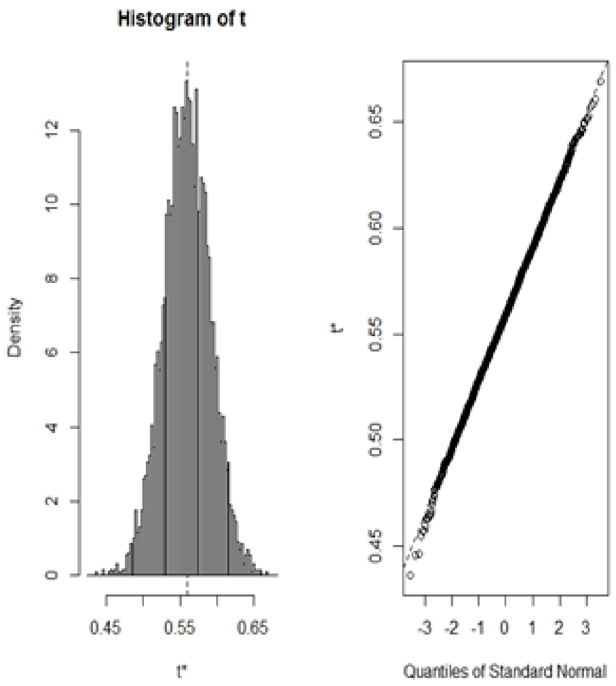
Some List of Figures

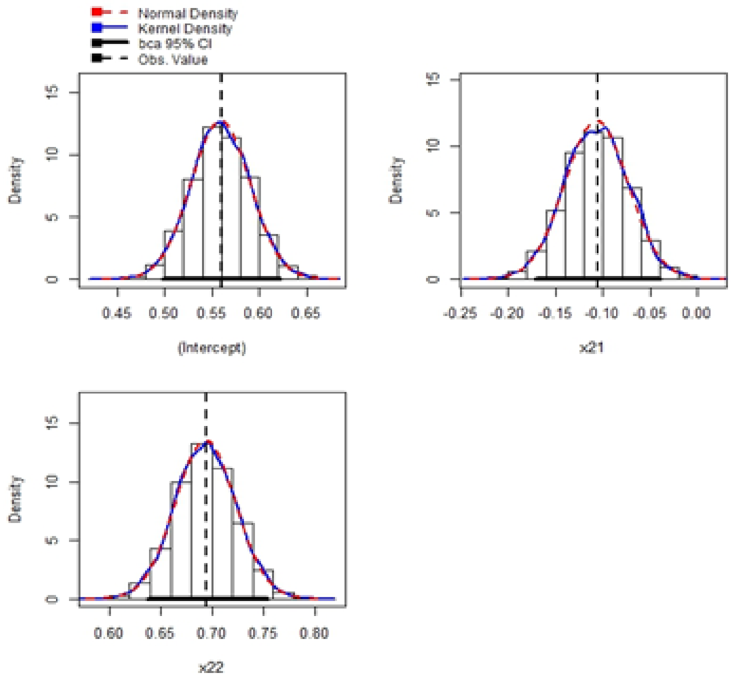
For n =1000





a. Plot of histogram and Standard Normal Quantiles in reg.1 **b. Plot of Standard Normal Density of the coefficients in reg. 1**





c. Histogram and Standard Normal Quantiles for reg.2. **d. Standard Normal Density of the Coefficients in reg.2**

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