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Decomposition of a weaker form of fuzzy continuity

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ABSTRACT

The aim of this paper is to give decomposition of a weaker form of continuity, namely fuzzy g'''-continuity, by providing the concepts of fuzzy g'''_p-closed set, fuzzy g'''_t- set, fuzzy g'''_p-continuity and fuzzy g'''_t-continuity.

Introduction

Various types of generalizations of fuzzy continuous functions were introduced and studied by various authors in the recent development of fuzzy topology. The decomposition of fuzzy continuity is one of many problems in fuzzy topology. Tong [16] obtained a decomposition of fuzzy continuity by introducing two weak notions of fuzzy continuity namely, fuzzy strong semi-continuity and fuzzy precontinuity. Rajamani [9] obtained a decomposition of fuzzy continuity.

In this paper, we obtain decompositions of fuzzy g'''-continuity in topological spaces using fuzzy g'''_p-continuity and fuzzy g'''_t-continuity.

Preliminaries

Definition 2.1 [14, 18]:

If X is a set, then any function A: X→ [0, 1] (from X to the closed unit interval [0, 1]) is called a fuzzy set in X.

Definition 2.2 [9]:

If X is a set, then A, B : X→ [0,1] are fuzzy sets in X.

(i) The complement of a fuzzy set A, denoted by A^c, is defined by $A^c(x) = 1 - A(x)$, $\forall x \in X$.

(ii) Union of two fuzzy sets A and B, denoted by A∨B, is defined by (A∨B) (x)= max{A(x),B(x)}, $\forall x \in X$.

(iii) Intersection of two fuzzy sets A and B, denoted by A∧B, is defined by (A∧B) (x)= min {A(x), B(x)}, $\forall x \in X$.

Definition 2.3 [14, 18]:

Let f: X→Y be a function from a set X into a set Y. Let A be a fuzzy subset in X and B be a fuzzy subset in Y. Then the Zadeh's function f(A) and $f^{-1}(B)$ are defined by

(i) f(A) is a fuzzy subset of Y where

$$f(A)=\begin{cases} \sup_{z \in f^{-1}(y)} A(z), & \text{if } f^{-1}(y) \neq \emptyset \\ 0, & \text{otherwise} \end{cases}$$

for each y ∈ Y.

(ii) $f^{-1}(B)$ is a fuzzy subset of X where $f^{-1}(B)(x) = B(f(x))$, for each x ∈ X.

Definition 2.4 [5, 14]:

Let X be a set and τ be a family of fuzzy sets in X. Then τ is called a fuzzy topology if τ satisfies the following conditions:

(i) 0, 1 $\in \tau$.

(ii) If $A_i \in \tau$, $i \in I$ then $\bigcup_{i\in I} A_i \in \tau$ or $\bigvee_{i \in I} A_i \in \tau$.

(iii) If A, B $\in \tau$ then A∩B $\in \tau$ or A∧B $\in \tau$.

The pair (X, τ) is called a fuzzy topological space (briefly fts). The elements of τ are called fuzzy open sets. Complements of fuzzy open sets are called fuzzy closed sets.

Definition 2.5 [14]:

Let A be a fuzzy set in a fts (X, τ). Then,

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(i) the closure of A, denoted by cl(A), is defined by

cl(A)= $\wedge$ { F:A $\leq$ F and F is a fuzzy closed };

(ii) the interior of A, denoted by int(A), is defined by

int(A)= $\vee${G:G $\leq$ A and G is a fuzzy open}.

Definition 2.6:

A subset A of a fts (X, τ) is called:

(i) fuzzy semi-open set [1] if A ≤ cl(int(A));

(ii) fuzzy preopen set [4] if A ≤ int(cl(A));

(iii) fuzzy α-open set [4] if A ≤ int(cl(int(A)));

The complements of the above mentioned fuzzy open sets are called their respective fuzzy closed sets.

For a subset A of a fuzzy topological space X, the fuzzy α-closure (resp. fuzzy semi-closure, fuzzy pre-closure) of A, denoted by αcl(A) (resp. scl(A), pcl(A)), is the intersection of all fuzzy α-closed(resp. fuzzy semi-closed, fuzzy preclosed) subsets of X containing A. Dually, the fuzzy α-interior (resp. fuzzy semi-interior, fuzzy pre-interior) of A, denoted by αint(A) (resp. sint(A), pint(A)), is the union of all fuzzy α-open (resp. fuzzy semi-open, fuzzy preopen) subsets of X contained in A.

Definition 2.7 [5, 9]:

A function f: X→Y is said to be fuzzy continuous if $f^{-1}(\lambda)$ is fuzzy open in X for each fuzzy open set λ in Y.

Definition 2.8 :

Let (X, τ) be a fuzzy topological space. A fuzzy set λ in X is called:

(i) a fuzzy generalized-semi closed (briefly fuzzy gs-closed)set [3] if scl(λ)$\leq \mu$ whenever $\lambda \leq \mu$ and μ is fuzzy open in (X, τ). The complement of fuzzy gs-closed set is called fuzzy gs-open set;

(ii) a fuzzy g'''-closed set [7] if cl(λ)$\leq \mu$ whenever $\lambda \leq \mu$ and μ is a fuzzy gs-open in (X, τ). The complement of fuzzy g''' -closed set is called fuzzy g''' -open.

(iii) a fuzzy g'''_α-closed set [7] if αcl(λ)$\leq \mu$ whenever $\lambda \leq \mu$ and μ is a fuzzy gs-open in (X, τ). The complement of fuzzy g'''_α -closed set is called fuzzy g'''_α -open.

Definition 2.9:

A fuzzy subset A of a space (X, τ) is called fuzzy t-set [17] if int(A) = int(cl(A)).

Definition 2.10:

A function f: (X, τ) → (Y, σ) is said to be

(i) fuzzy α -continuous[4] if for each fuzzy open set λ of Y, $f^{-1}(\lambda)$ is fuzzy α -open in X.

(ii) fuzzy g''' -continuous [8] if for each fuzzy open set λ of Y, $f^{-1}(\lambda)$ is fuzzy g''' -open in X.

(iii) fuzzy g'''_α -continuous [8] if for each fuzzy open set λ of Y, $f^{-1}(\lambda)$ is fuzzy g'''_α -open in X.

Proposition 2.11[7]:

(i) Every fuzzy closed set is fuzzy g''' -closed as well as fuzzy α-closed but not conversely.

(ii) Every fuzzy g''' -closed set is fuzzy g'''_α -closed but not conversely.

(iii) Every fuzzy α-closed is fuzzy g'''_α -closed but not conversely.

Example 2.12:

(i) Let X = {a, b} with τ = {0_x, A, 1_x} where A is fuzzy set in X defined by A(a)=1,A(b)=0 .Then (X, τ) is a fuzzy topological space. Clearly B defined by B(a)=0.5,B(b)=1 is fuzzy g''' -closed set but not fuzzy closed.

(ii) Let X = {a, b} with τ = {0_x, λ, 1_x} where λ is fuzzy set in X defined by λ(a)=0.6, λ(b)=0.5 .Then (X, τ) is a fuzzy topological space. Clearly μ defined by μ(a)=0.4, μ(b)=0.4 is fuzzy g'''_α -closed as well as fuzzy α-closed set but neither a fuzzy closed set nor a fuzzy g''' -closed set in (X, τ) .

(iii) Let X = {a, b} with τ = {0_x, α, 1_x} where α is fuzzy set in X defined by α(a)=0.4, α(b)=0.5 .Then (X, τ) is a fuzzy topological space. Clearly α is fuzzy g'''_α -closed but not fuzzy α -closed set in (X, τ).

Proposition 2.13:

(i) Every fuzzy continuous function is fuzzy g''' - continuous as well as fuzzy α-continuous function but not conversely.

(ii) Every fuzzy g''' -continuous function is fuzzy g'''_α - continuous but not conversely.

(iii) Every fuzzy α-continuous function is fuzzy g'''_α - continuous function but not conversely.

Example 2.14:

(i) Let X=Y= {a, b} with τ = {0_x, A, 1_x} where A is fuzzy set in X defined by A(a)=1, A(b)=0 and σ ={0_x, B, 1_x} where B is fuzzy set in Y defined by B(a)=0.5, B(b)=0. Then (X, τ) and (Y, σ) are fuzzy topological spaces. Let f: (X, τ) → (Y, σ) be the identity fuzzy function. Clearly f is fuzzy g''' - continuous but not fuzzy continuous.

(ii) Let X=Y= {a, b} with τ = {0_x, λ, 1_x} where λ is fuzzy set in X defined by λ(a)=0.6, λ(b)=0.5 and σ ={0_x, β, 1_x} where β is fuzzy set in Y defined by β(a)=0.6, β(b)=0.6. Then (X, τ) and (Y, σ) are fuzzy topological spaces. Let f: (X, τ) → (Y, σ) be the identity fuzzy function. Clearly f is fuzzy g'''_α -continuous as well as fuzzy α -continuous but it is neither fuzzy continuous nor fuzzy g''' -continuous.

(iii) Let X=Y= {a, b} with τ = {0_x, α, 1_x} where α is fuzzy set in X defined by α(a)=0.4, α(b)=0.5 and σ ={0_x, β, 1_x} where β is fuzzy set in Y defined by β(a)=0.6, β(b)=0.5. Then (X, τ) and (Y, σ) are fuzzy topological spaces. Let f: (X, τ) →(Y, σ) be the identity fuzzy function. Clearly f is fuzzy g'''_α -continuous but not fuzzy α -continuous.

Remark 2.15:

The following example shows that

(i) fuzzy g''' -closed sets and fuzzy α-closed sets are independent.

(ii) fuzzy g'''_α -closed sets and fuzzy ω-closed sets are independent.

Example 2.16:

(i) Let X = {a, b} with $\tau = \{0_x, A, 1_x\}$ where A is fuzzy set in X defined by A(a)=1,A(b)=0 .Then (X, τ) is a fuzzy topological space. Clearly B defined by B(a)=0.5, B(b)=1 is fuzzy g'''-closed but not fuzzy α -closed in (X, τ).

(ii) Let X = {a, b} with $\tau = \{0_x, A, 1_x\}$ where A is fuzzy set in X defined by A(a)=1,A(b)=0 .Then (X, τ) is a fuzzy topological space. Clearly C defined by C(a)=0, C(b)=0.5 is fuzzy α -closed in (X, τ) but it is not fuzzy g'''-closed in (X, τ).

(iii) Let X = {a, b} with $\tau = \{0_x, A, 1_x\}$ where A is fuzzy set in X defined by A(a)=1,A(b)=0 .Then (X, τ) is a fuzzy topological space. Clearly C defined by C(a)=0, C(b)=0.5 is fuzzy g'''_α-closed in (X, τ) but it is not fuzzy ω-closed in (X, τ).

(iv) Let X = {a, b} with $\tau = \{0_x, \alpha, 1_x\}$ where A is fuzzy set in X defined by α(a)=0.4, α(b)=0.5 .Then (X, τ) is a fuzzy topological space. Clearly C defined by β(a)=0.6, β(b)=0.6 is fuzzy ω-closed in (X, τ) but it is not fuzzy g'''_α-closed in (X, τ).

Lemma 2.17[8]:

A fuzzy subset A of (X, τ) is fuzzy g'''-open if and only if F ≤ int(A) whenever F is fuzzy gs-closed and F ≤ A.

Proof:

Suppose that F≤ int(A) such that F is fgs-closed set and F ≤ A. Let $A^c \leq U$ where U is fsg-open . Then $U^c \leq A$ and U^c is fsg-closed . Therefore $U^c \leq$ int(A) by hypothesis. Since $U^c \leq$ int(A), we have $(\text{int}(A))^c \leq$ U. i.e.,$\text{cl}(A^c) \leq$ U, since $\text{cl}(A^c) = (\text{int}(A))^c$. Thus A^c is f g'''-closed set. i.e., A is f g'''-open.

Conversely, suppose that A is f g'''-open such that F ≤ A and F is fgs-closed. Then F^c is fsg-open and $A^c \leq F^c$. Therefore, $\text{cl}(A^c) \leq F^c$ by definition of f g'''-closedness and so F ≤ int(A) , $\text{cl}(A^c) = (\text{int}(A))^c$.

On Fuzzy g'''_p Closed Set And Fuzzy g'''_t Set

We introduce the following definition.

Definition 3.1:

A subset λ in a fuzzy topological space X is called a fuzzy g'''_p-closed set if pcl(λ)≤ μ whenever $\lambda \leq \mu$ and μ is a fuzzy gs-open in (X, τ). The complement of fuzzy g'''_p-closed set is called fuzzy g'''_p-open.

Definition 3.2:

A subset λ in a fuzzy topological space X is called fuzzy g'''_t-set if $\lambda = \alpha \wedge \beta$ where α is a fuzzy g'''-open in X and β is a fuzzy t-set in X.

The family of all g'''_t-sets in a space (X, τ) is denoted by g'''_t (X, τ).

Example 3.3:

Let X = {a, b} with $\tau = \{0_x, \lambda, 1_x\}$ where λ is fuzzy set in X defined by λ(a)=0.6, λ(b)=0.5 .Then (X, τ) is a fuzzy topological space. Clearly λ_1 defined by λ_1(a)=0.4, λ_1(b)=0.5 is fuzzy g'''_t-set.

Proposition 3.4:

Every fuzzy g'''_α-closed set is fuzzy g'''_p-closed.

Proof:

If A is a fuzzy g'''_α-closed subset of (X, τ) and G is any fgs-open set such that A ≤ G, then pcl(A) ≤ αcl(A) ≤ G. Hence A is fuzzy g'''_p-closed in (X, τ).

The converse of Proposition 3.4 need not be true as seen from the following example.

Example 3.5:

Let X = {a, b} with $\tau = \{0_x, \lambda, 1_x\}$ where λ is fuzzy set in X defined by λ(a)=0.6, λ(b)=0.5 .Then (X, τ) is a fuzzy topological space. Clearly β defined by β(a)=0.5, β(b)=0.5 is fuzzy g'''_p-closed set but not fuzzy g'''_α-closed set in (X, τ) .

Proposition 3.6:

Every fuzzy g'''-closed set is fuzzy g'''_t-set but not conversely.

Example 3.7:

Let X = {a, b} with $\tau = \{0_x, \lambda, 1_x\}$ where λ is fuzzy set in X defined by λ(a)=0.6, λ(b)=0.5 .Then (X, τ) is a fuzzy topological space. Clearly μ defined by μ(a)=0.4, μ(b)=0.4 is fuzzy g'''_t-set but not fuzzy g'''-closed set in (X, τ) .

Remark 3.8:

Fuzzy g'''_p-open sets and fuzzy g'''_t-sets are independent of each other.

Example 3.9:

Let X= {a, b} with $\tau = \{0_x, A, 1_x\}$ where A is fuzzy set in X defined by A(a)=1, A(b)=0. Then (X, τ) is a fuzzy topological space. Clearly B defined by B(a)=0.5, B(b)=1 is fuzzy g'''_p-open set but it not fuzzy g'''_t-set in (X, τ) .

Example 3.10:

Let X = {a, b} with $\tau = \{0_x, \beta_1, 1_x\}$ and β_1 and β_2 are fuzzy sets in X defined by β_1(a)= 0.6, β_1(b)= 0.5; and β_2(a)= 0.4, β_2(b)= 0.5. Then (X, τ) is a fuzzy topological space. Clearly β_2 is fuzzy g'''_t-set but not fuzzy g'''_p-open set in (X, τ).

Remark 3.11:

The union of two fuzzy g'''_t-sets need not be a fuzzy g'''_t-set.

Example 3.12:

Let X= {a, b} with $\tau = \{0_x, A, 1_x\}$ and A, B, C and D are fuzzy sets in X defined by A(a)=1, A(b)=0 and B(a)=0.5, B(b)=0 and C(a)=0, C(b)=1 and D(a)= 0, D(b)= 0.5. Then (X, τ) is a fuzzy topological space. Clearly B and C are fuzzy g'''_t-sets but B $\vee$ C = E defined by E(a)= 0.5, E(b)= 1 is not fuzzy g'''_t-sets .

Lemma 3.13:

A fuzzy subset A of (X, τ) is fuzzy g'''_p-open if and only if F ≤ pint(A) whenever F is fuzzy gs-closed and F ≤ A.

Proof:

The proof follows immediately from Lemma 2.17.

Theorem 3.14:

A fuzzy subset S is fuzzy g'''-open in (X, τ) if and only if it is both fuzzy g'''_p-open and an fuzzy g'''_t-set in (X, τ).

Proof:

Necessity. The proof is obvious.

Sufficiency. Let S be a fuzzy g'''_p-open set and a fuzzy g'''_t-set. Since S is a fuzzy g'''_t-set, S = A $\wedge$ B, where A is fuzzy g'''-open and B is a fuzzy t-set. Assume that F ≤ S, where F is fuzzy gs-closed in X. Since A is fuzzy g'''-open, by Lemma 2.17, F ≤ int(A). Since S is fuzzy g'''_p-open in X, by Lemma 3.13,

F ≤ pint(S) = S $\wedge$ int(cl(S)) = (A $\wedge$ B) $\wedge$ int(cl(A $\wedge$ B)) ≤ A $\wedge$ B $\wedge$ int(cl(A)) $\wedge$ int(cl(B)) = A $\wedge$ B$\wedge$ int(cl(A)) $\wedge$ int(B) ≤ int(B). Therefore, we obtain F ≤ int(B) and hence F ≤ int(A) $\wedge$ int(B) = int(S). Hence S is fuzzy g'''-open, by Lemma 2.17.

Decomposition of fuzzy g''' continuity

Definition 4.1:

A function f : (X, τ) → (Y,σ) is said to be

(i) fuzzy g'''_p - continuous if for each fuzzy open set λ of Y, $f^{-1}(\lambda)$ is fuzzy g'''_p-open in X.

(ii) fuzzy g'''_t -continuous if for each fuzzy open set λ of Y, $f^{-1}(\lambda)$ is fuzzy g'''_t-set in X.

Example 4.2:

Let X=Y={a, b} with τ ={0_x, α_1, 1_x} where α_1 is fuzzy set in X defined by α_1 (a)=0.6, α_1 (b)=0.5 and σ={0_x, α_2, 1_x} where α_2 is fuzzy set in Y defined by α_2 (a)=0.4, α_2 (b)=0.4. Then (X, τ) and (Y, σ) are fuzzy topological spaces. Let f: (X, τ) → (Y, σ) be the fuzzy identity function. Clearly f is fuzzy g'''_{α^*} -continuous as well as fuzzy g'''_t - continuous .

Remark 4.3:

(i) Every fuzzy g'''-continuous function is fuzzy g'''_t-continuous but not conversely.

(ii) Every fuzzy g'''_α-continuous function is fuzzy g'''_p-continuous function but not conversely.

Example 4.4:

(i) Let X=Y={a, b} with τ ={0_x, α_1, 1_x} where α_1 is fuzzy set in X defined by α_1 (a)=0.6, α_1 (b)=0.5 and σ={0_x, α_2, 1_x} where α_2 is fuzzy set in Y defined by α_2 (a)=0.4, α_2 (b)=0.4. Then (X, τ) and (Y,σ) are fuzzy topological spaces. Let f: (X, τ) → (Y, σ) be the fuzzy identity function. Clearly f is fuzzy g'''_t -continuous but not fuzzy g''' -continuous

(ii) Let X=Y= {a, b} with τ = {0_x, λ, 1_x} where λ is fuzzy set in X defined by λ(a)=0.6, λ(b)=0.5 and σ ={0_x, β, 1_x} where β is fuzzy set in Y defined by β(a)=0.5, β(b)=0.5. Then (X, τ) and (Y, σ) are fuzzy topological spaces. Let f: (X, τ) → (Y, σ) be the identity fuzzy function. Clearly f is fuzzy g'''_p -continuous but not fuzzy g'''_α -continuous.

Remark 4.5:

Fuzzy g'''_p-continuity and fuzzy g'''_t - continuity are independent of each other.

Example 4.6:

Let X=Y={a, b} with τ ={0_x, A, 1_x} where A is fuzzy set in X defined by A(a)=1, A(b)=0 and σ={0_x, B, 1_x} where B is fuzzy set in Y defined by B(a)=0.5, B(b)=1. Then (X, τ) and (Y, σ) are fuzzy topological spaces.

Let f: (X, τ) → (Y, σ) be the fuzzy identity function. Clearly f is fuzzy g'''_p -continuous but not fuzzy g'''_t - continuous.

Example 4.7:

Let X=Y={a, b} with τ ={0_x, α_1, 1_x} where α_1 is fuzzy set in X defined by α_1 (a)=0.6, α_1 (b)=0.5 and σ={0_x, α_2, 1_x} where α_2 is fuzzy set in Y defined by α_2 (a)=0.4, α_2 (b)=0.5. Then (X, τ) and (Y,σ) are fuzzy topological spaces.

Let f: (X, τ) → (Y, σ) be the fuzzy identity function. Clearly f is fuzzy g'''_t -continuous but not fuzzy g'''_p - continuous.

Theorem 4.8:

A function f : (X, τ) → (Y, σ) is fuzzy g'''-continuous if and only if it is both fuzzy g'''_p-continuous and fuzzy g'''_t-continuous.

Proof:

The proof follows immediately from Theorem 3.14.

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