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Radiation and chemical reaction effects on MHD mixed convective flow from a vertical surface with ohmic heating and viscous dissipation

V. Seetamaha lakshmi[1], G.V.Ramana Reddy[2] and B.D.C.N.Prasad[3]
[1,3]P.V.P.Siddartha Institute of Technology, Vijayawada, A.P (India)-521007.
[2]Department of Mathematics, KL University, Vaddeswarsam, A.P-522502.

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ABSTRACT

The paper investigated the effect of radiation and chemical reaction on unsteady MHD mixed convection flow past an infinite vertical plate with Ohmic heating and viscous dissipation has been is discussed. Approximate solutions have been derived for the velocity, temperature field, concentration profiles, skin friction and Nusselt number using multi-parameter perturbation technique. The obtained results are discussed with the help of the graphs to observe the effect of various parameters like Schmidt number (Sc), Prandtl number (Pr), Magnetic parameter (M), radiation parameter (F) and porosity parameter (K).

Introduction

The hydromagnetic convection with heat and mass transfer in porous medium has been studied due to its importance in the design of MHD generators and accelerators in geophysics, in design of under ground water energy storage system, soil-sciences, astrophysics, nuclear power reactors and so on. Magnetohydrodynamics is currently undergoing a period of great enlargement and differentiation of subject matter. The interest in these new problems generates from their importance in liquid metals, electrolytes and ionized gases. Because of their varied importance, these flow have been studied by several authors-notable amongst them are Shercliff [1], Ferraro and Plumpton [2] and Crammer and Pai [3]. Elbashbeshy [4] studied heat and mass transfer along a vertical plate in the presence of magnetic field. Hossian and Rees [5] examined the effects of combined buoyancy forces from thermal and mass diffusion by natural convection flow from a vertical wavy surface. Combined heat and mass transfer in MHD free convection from a vertical surface has been studied by Chein [6]. Further, the effect of Hall current on the fluid flow with variable concentration has many applications in MHD power generation, in several astrophysical and meteorological studies as well as in plasma flow through MHD power generators. From the point of application, model studies on the Hall Effect on free and forced convection flows have been made by several investigators. Aboeldahab [7], Datta et al. [8], Acharya et al.[9] and Biswal et al.[10] have studied the Hall effect on the MHD free and forced convection heat and mass transfer over a vertical surface. Hossain and Alim [11] studied the radiation effect on free and forced convection flows past a vertical plate, including various physical aspects. Aboeldhab [12] studied the radiation effect in heat transfer in an electrically conducting fluid at stretching surface. A. Y. Ghaly and E. M. E. Elbarbary, [13], were examined by radiation effect on MHD free convection flow of a gas at a stretching surface with uniform free stream. Heat and mass transfer effects on moving plate in the presence of thermal radiation have been studied by Muthukumaraswamy [14] using Laplace technique. For the problem of coupled heat and mass transfer in MHD free convection, the effect of both viscous dissipation and Ohmic heating are not studied in the above investigations. However, it is more realistic to include these two effects to explore the impact of the magnetic field on the thermal transport in the boundary layer. With this awareness, the effect of Ohmic heating on the MHD free convection heat transfer has been examined for a Newtonian fluid by Hossain [15]. Chen [16] studied the problem of combined heat and mass transfer of an electrically conducting fluid in MHD natural convection, adjacent to a vertical surface with Ohmic heating. Ganesan and Palani [17] obtained numerical solution of Unsteady MHD flow past a semi- infinite isothermal vertical plate. Ganesan and Palani [18] studied numerical solution of transient free convection MHD flow of an incompressible viscous fluid flow past a

Tele:
E-mail addresses: gvrr2020@ymail.com

semi- infinite inclined plate with variable surface heat and mass flux. The set of governing equations are solved by using an implicit finite difference scheme. Orhan Aydin and Ahmet Kaya [19] investigates mixed convection heat transfer about a permeable vertical plate in the presence of magneto and thermal radiation effects, The set of governing equations of the problem are solved using similarity variables. The problem of steady laminar magneto hydrodynamic(MHD) mixed convection heat transfer about a vertical plate is solved numerically by Orhan Aydin and Ahmet Kaya [20] taking into account the effect of ohmic heating and viscous dissipation.

The propagation of thermal energy through mercury and electrolytic solution in the presence of magnetic field and radiation has wide range of applications. Hence, our object in the present paper is to study the effect of radiation on heat and mass transfer in mercury (Pr = 0.025) and electrolytic solution (Pr = 1.0) past an infinite porous hot vertical plate in the presence of Ohmic heating and transverse magnetic field.

Formation of the problem

We consider the mixed convection flow of an incompressible and electrically conducting viscous fluid such that x^* - axis is taken along the plate in upwards direction and y^* -axis is normal to it. A transverse constant magnetic field is applied that is in the direction of y^* -axis. Since the motion is two dimensional and length of the plate is large therefore all the physical variables are independent of x^*. Let u^* and v^* be the components of velocity in x^* and y^* directions respectively, taken along and perpendicular to the plate. The governing equations of continuity, momentum, energy and mass diffusion for a flow of an electrically conducting fluid along a hot, non-conducting porous vertical plate in the presence of concentration and radiation is given by

$$\frac{\partial v^*}{\partial y^*} = 0 \Rightarrow v^* = -v_0 \text{ (constant)} \tag{1}$$

$$\frac{\partial p^*}{\partial y^*} = 0 \Rightarrow p^* \text{ is independent of } \mathrm{y}^* \tag{2}$$

$$\rho v^* \frac{\partial u^*}{\partial y^*} = \mu \frac{\partial^2 u^*}{\partial y^{*2}} + \rho g \beta (T^* - T_\infty) + \rho g \beta^* (C^* - C_\infty) - \sigma B_0^2 u^* - \frac{\nu}{K^*} u^* \tag{3}$$

$$\rho C_p v^* \frac{\partial T^*}{\partial y^*} = \kappa \frac{\partial^2 T^*}{\partial y^{*2}} + \mu \left(\frac{\partial u^*}{\partial y^*} \right)^2 - \frac{\partial q_r^*}{\partial y^*} + \sigma B_0^2 u^{2*} \tag{4}$$

$$v^* \frac{\partial C^*}{\partial y^*} = D \frac{\partial^2 C^*}{\partial y^{*2}} - K_r^* C^* \tag{5}$$

Here, g is the due to gravity, T^* the temperature of the fluid near the plate, T_∞ the free stream temperature, C^* - the concentration, β - the coefficient of thermal expansion, κ the thermal conductivity, p^* the pressure, c_p the specific heat of constant pressure, B_0 the magnetic field coefficient, μ the viscosity of the fluid, q_r^* the radiative heat flux, ρ the density, σ the magnetic permeability of the fluid, v_0 - the constant suction velocity, ν the kinematic viscosity , D molecular diffusivity and K_r^* – chemical reaction parameter.

The radiative heat flux is given by [21]

$$\frac{\partial q_r^*}{\partial y^*} = 4(T^* - T_\infty) I' \tag{6}$$

where $I^{'} = \int_0^{\infty} K \frac{\partial e_{b\lambda}}{\partial T^*} d\lambda,\ K\lambda w$ is the absorption coefficient at wall and $e_{b\lambda}$ is Planck's function.

The boundary conditions are

$$u^* = 0,\ T^* = T_w,\quad C^* = C_w \qquad y^* = 0$$
$$u^* \to 0,\ T^* \to T_\infty,\quad C^* \to C_\infty \qquad y^* \to \infty \tag{7}$$

Introducing the following non-dimensional quantities are

$$y = \frac{v_0 y^*}{\nu},\ u = \frac{u^*}{v_0},\ T = \frac{T^* - T^*_\infty}{T_w - T_\infty},\ C = \frac{C^* - C^*_\infty}{C_w - C_\infty},\ \Pr = \frac{\mu C_p}{\kappa}$$

$$Gr = \frac{\rho g \beta \nu (T_w - T_\infty)}{v_0^3},\ Gm = \frac{\rho g \beta^* \nu (C_w - C_\infty)}{v_0^3},\ Sc = \frac{\nu}{D},\ K_r = \frac{K_r^* \nu}{v_0^2}$$

$$M = \frac{\sigma B_0^2 \nu}{\rho v_0^2},\ K = \frac{K^* v_0^2}{\nu},\ E = \frac{v_0^2}{C_p\left(T_w - T_\infty\right)},\ F = \frac{4\nu I^{'}}{\rho C_p v_0^2} \tag{8}$$

In the equations (3), (4), (5) and (7), we get

$$\frac{\partial^2 u}{\partial y^2} + \frac{\partial u}{\partial y} - (M + 1/K)u = -\left[GrT + GmC\right] \tag{9}$$

$$\frac{\partial^2 T}{\partial y^2} + \Pr \frac{\partial T}{\partial y} - F \Pr T + \Pr E \left(\frac{\partial u}{\partial y}\right)^2 + \Pr EMu^2 = 0 \tag{10}$$

$$\frac{\partial^2 C}{\partial y^2} + Sc \frac{\partial C}{\partial y} + K_r ScC = 0 \tag{11}$$

where Gr is the Grashof number, Gm -the modified Grashof number, Pr- the Prandtl number, F - the radiation parameter, Sc - the Schmidt number, E - the Eckert number, M – the magnetic parameter.

The corresponding boundary conditions in dimensionless form are reduced to

$$u = 0,\ T = 1,\quad C = 1 \qquad y = 0$$
$$u \to 0,\ T \to 0,\quad C \to 0 \qquad y \to \infty \tag{12}$$

The physical variable *u*, *T* and *C* can expand in the power of Eckert number (*E*). This can be possible physically as E for the flow of an incompressible fluid is always less than unity. It can be interpreted physically as the due to the Joules dissipation is super imposed on the main flow.

Solution of the problem

To reduce the above system of partial differential equations to a system of ordinary differential equations in a dimensionless form, we may represent the translational velocity, temperature and concentration as

$$u(y) = u_0(y) + Eu_1(y) + o(E^2)$$
$$T(y) = T_0(y) + ET_1(y) + o(E^2) \tag{13}$$
$$C(y) = C_0(y) + EC_1(y) + o(E^2)$$

Using equation (13) in equation (9)-(11) and equating the coefficient of like power of E, we have

$$u_0^{''} + u_0^{'} - \left(M + 1/K\right)u_0 = -GrT_0 - GmC_0 \tag{14}$$

$$T_0^{''} + \Pr T_0^{'} - F \Pr T_0 = 0 \tag{15}$$

$$C_0'' + ScC_0' + K_r ScC_0 = 0 \tag{16}$$

$$u_1'' + u_1' - \left(M + 1/K\right)u_1 = -GrT_1 - GmC_1 \tag{17}$$

$$T_1'' + \Pr T_1' - F\Pr T_1 + pru_0'^2 + Mu_0^2 = 0 \tag{18}$$

$$C_1'' + ScC_1' + K_r ScC_1 = 0 \tag{19}$$

The corresponding boundary conditions are

$$\begin{aligned} &u_0 = 0,\ \ u_1 = 0,\ \ T_0 = 1,\ T_1 = 0,\ \ \ C_0 = 1,\ C_1 = 0 \qquad \text{at} \quad y = 0 \\ &u_0 \to 0,\ \ u_1 \to 0,\ \ T_0 \to 0,\ T_1 \to 0,\ \ \ C_0 \to 0,\ C_1 \to 0 \qquad \text{as} \quad y \to \infty \end{aligned} \tag{20}$$

Solving equations (14)-(19) with the help of (20), we get

$$u_0(y) = m_6(e^{-m_4 y} - e^{-Scy}) + m_5(e^{-m_4 y} - e^{-m_1 y})$$

$$T_0(y) = e^{-m_1 y}$$

$$C_0(y) = e^{-Scy}$$

$$u_1(y) = D_{17}e^{-m_9 y} - D_{10}e^{-m_1 y} + D_{11}e^{-2m_1 y} + D_{12}e^{-2m_4 y} - D_{13}e^{-m_4 y} + D_{14}e^{-2Scy} - D_{15}e^{-D_1 y} + D_{16}e^{-D_2 y}$$

$$T_1(y) = D_9 e^{-m_1 y} - D_3 e^{-2m_1 y} - D_4 e^{-2m_4 y} + D_5 e^{-m_{10} y} - D_6 e^{-2Scy} + D_7 e^{-D_1 y} - D_8 e^{-D_2 y}$$

The skin-friction, Nusselt number and Sherwood number are important physical parameters for this type of boundary layer flow. Knowing the velocity field, the ski-friction at the plate can be obtained, which in non-dimensional form is given by

$$\tau = \left(\frac{\partial u}{\partial y}\right)_{y=0} = m_6(Sc - m_4) + m_5(m_1 - m_4) - E\begin{pmatrix} m_4 D_7 - m_1 D_{10} + 2m_1 D_{11} + 2m_4 D_{12} \\ -m_4 D_{13} + 2ScD_{14} - D_1 D_{15} + D_2 D_{16} \end{pmatrix}$$

Knowing the temperature field, the rate of heat transfer coefficient can be obtained, which in non-dimensional form, in terms of Nusselt number, is given by

$$Nu = -\left(\frac{\partial T}{\partial y}\right)_{y=0} = m_1 + E\left(m_1 D_9 - 2m_1 D_3 - 2m_4 D_4 + m_{10} D_5 - 2ScD_6 + D_1 D_7 - D_2 D_8\right)$$

Here constants are not given due to shake of brevity.

Results and Discussion

In order to get a physical insight in to the problem the effects of various governing parameters on the physical quantities are computed and represented in Figures 1-16 and discussed in detail.

Fig.1 illustrates the effect of Schmidt number on the velocity field. It is noticed that as the Schmidt number increases the velocity field decreases. The effect of Prandtl number on the velocity field has been illustrated in Fig. 2. It is observed that as the Prandtl number increases the velocity field increases. Fig.3. illustrates the effect of Grashof number on the velocity field. It is noticed that as the Grashof number increases the velocity field increases. Fig.4. illustrates the effect of magnetic field on the velocity profiles. When the applied magnetic field intensity increases, there seems to be a decrease in the velocity field. The effect of modified Grashof number on the velocity field has been illustrated in Fig.5. It is observed that as the modified Grashof number increases the velocity field increases. The effect of thermal radiation parameter on the velocity field has been illustrated in Fig.6. It is seen that as the thermal radiation parameter increases the velocity field increases. The effect of porosity parameter on the velocity field is shown in Fig.7. It is observed that as the porosity parameter increases the velocity field increases.

Fig.8. illustrates the effect of Schmidt number on the temperature field. It is noticed that as the Schmidt number increases the temperature field decreases. The effect of Prandtl number on the temperature field is shown in Fig.9. It is observed that, an increase in the Prandtl number contributes to an increase in the temperature. The effect of magnetic field intensity on the temperature field is

illustrated in Fig.10. It is observed that as the magnetic field increases, the temperature increases. Fig.11 illustrates the effect of Grashof number on the temperature field. It is noticed that the Grashof number increases, the temperature decreases. The effect of modified Grashof number on the temperature field is illustrated in Fig.12. It is observed that as the modified Grashof number increases, the temperature increases. Fig.13 illustrates the effect of thermal radiation parameter on the temperature field. It is noticed that as the thermal radiation parameter increases, the temperature of the fluid medium increases. Fig.14 illustrates the effect of Schmidt number on the concentration field. It is noticed that as the Schmidt number increases, the concentration of the fluid medium decreases.

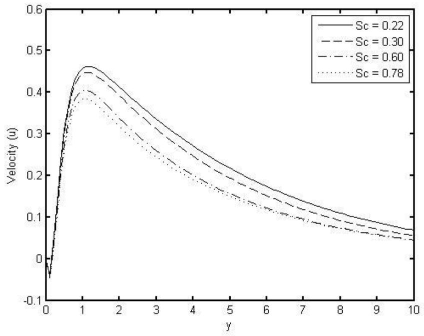

Fig.1. Effects of Schmidt number on the velocity profiles. (ε=0.001, Pr=0.025, M=2, Gr=5, Gm=2, K=1, F=3)

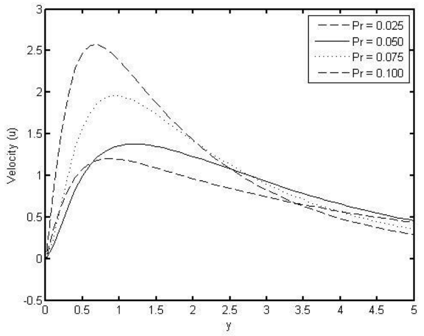

Fig.2. Effects of Prandtl number on velocity profiles. (ε=0.001, Sc=0.22, M=2, Gr=5, Gm=2, K=1, F=3)

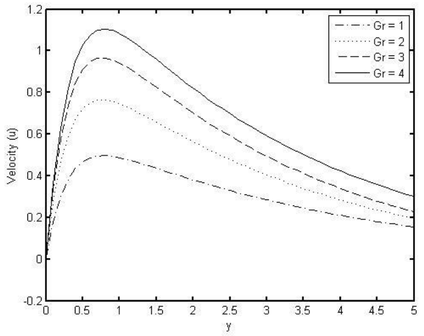

Fig.3. Effects of Grashof number on velocity profiles. (ε=0.001, Pr=0.025, M=2, F=3, Gm=2, Sc=0.22, K=1)

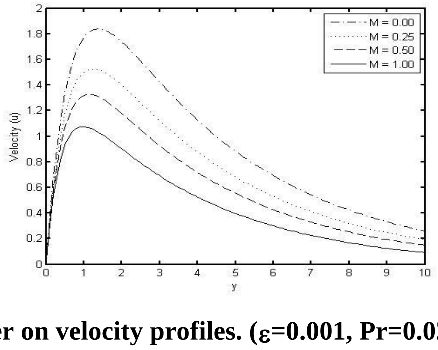

Fig.4. Effects of magnetic parameter on velocity profiles. (ε=0.001, Pr=0.025, Gm=2, Gr=2, F=3, Sc=0.22, K=1)

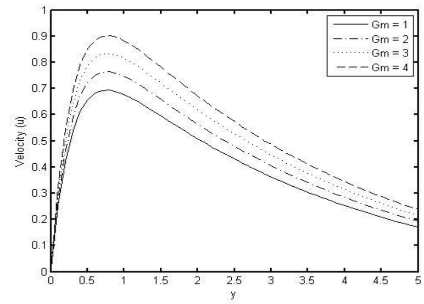

Fig.5. Effects of modified Grashof number on velocity profiles. (ε=0.001, Pr=0.025, M=2, Gr=2, F=3, Sc=0.22, K=1)

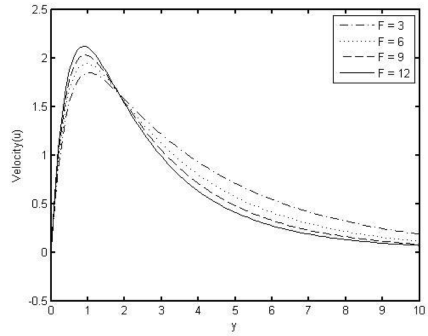

Fig.6. Effects of radiation parameter on velocity profiles. (ε=0.001, Pr=0.025, M=2, Gr=5, Gm=2, Sc=0.22, K=1)

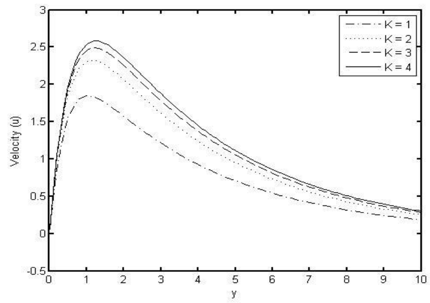

Fig.7. Effects of porosity parameter on velocity profiles. (ε=0.001, Pr=0.025, M=2, Gr=5, Gm=2, Sc=0.22,F=3)

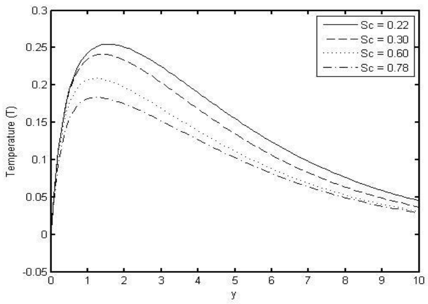

Fig.8. Effects of Schmidt number on temperature profiles. (ε=0.001, Pr=0.025, M=0.5, Gr=2, Gm= - 2, K=1 ,F=3)

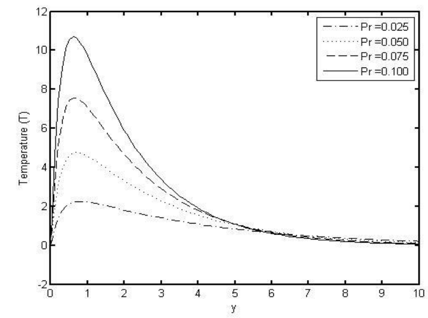

Fig.9. Effects of Prandtl number on temperature profiles. (ε=0.001, Sc=0.22, M=0.5, Gr=2, Gm= 2, K=1 ,F=3)

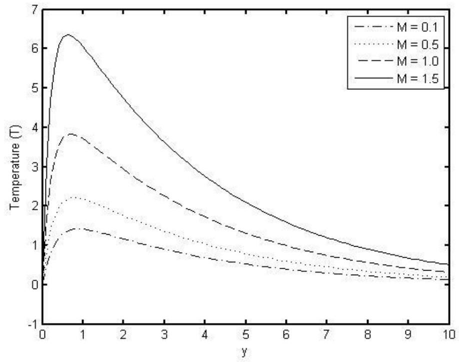

Fig.10. Effects of magnetic parameter on temperature profiles. (ε=0.001, Sc=0.60, Pr=0.025, Gr=2, Gm= 2, K=1,F=3)

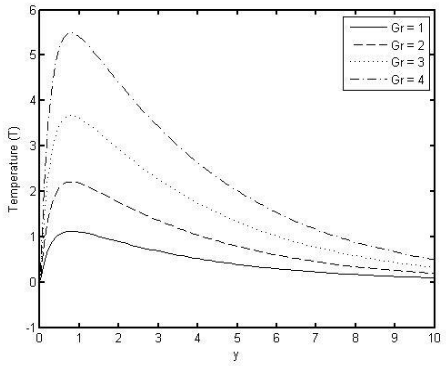

Fig.11. Effects of Grashof number on temperature profiles. (ε=0.001, Sc=0.60, Pr=0.025, M=0.5, Gm= 2, K=1,F=3)

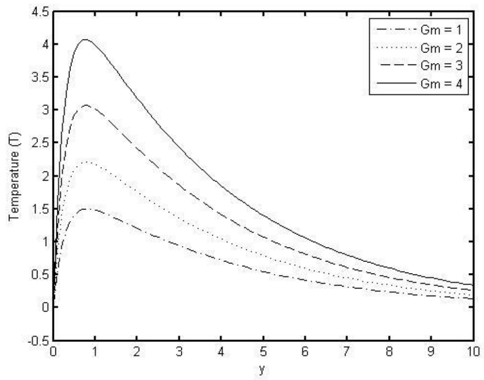

Fig.12. Effects of modified Grashof number on temperature profiles. (ε=0.001, Sc=0.60, Pr=0.025, M=0.5, Gr= 2, K=1,F=3)

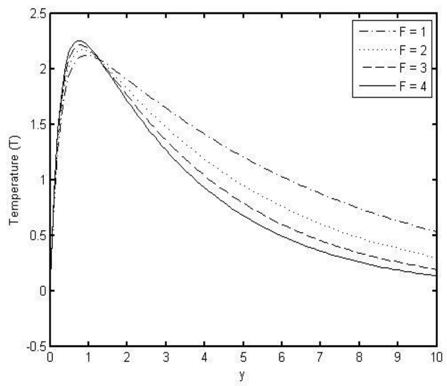

Fig.13. Effects of radiation parameter on temperature profiles. (ε=0.001, Sc=0.60, Pr=0.025, M=0.5, Gr= 2, Gm=2,F=3)

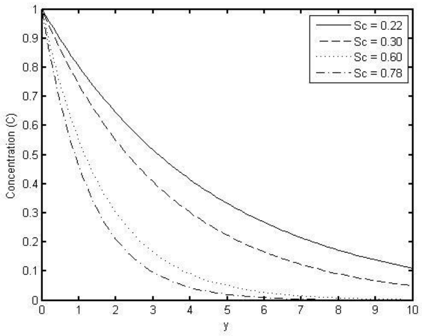

Fig.14. Effects of Schmidt number on concentration profiles.

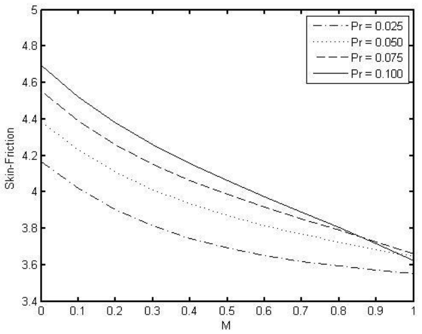

Fig.15. Effects of Prandtl number on skin-friction. (ε=0.001, Sc=0.22, K=1, Gr= 2, Gm=2,F=3)

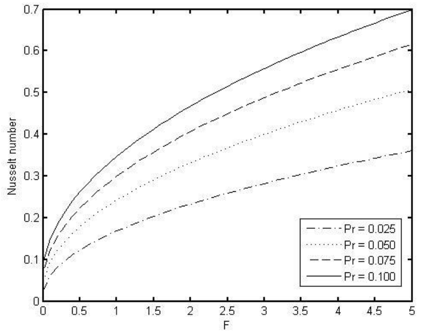

Fig.16. Effects of Prandtl number on the Nusselt number. (ε=0.001, M=2, K=1, Gr= 2, Gm=2, Sc=0.22)

Fig.15 illustrates the effect of Prandtl number on the skin-friction of the fluid under consideration. As the Prandtl number increases the ski-friction is found to be increasing. Fig.16 illustrates the effect of the Prandtl number on the Nusselt number of the fluid under consideration. As the Prandtl number increases, the Nusselt number increases.

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