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Multi objective fuzzy linear programming technique for weighted additive model for supplier selection in supply chain management

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ABSTRACT

Supplier selection and allocating orders to supplier is a complex multi objective problem which includes both quantitative and qualitative factors. In order to achieve an efficient solution in the quantitative factors, a Multi Objective Fuzzy Linear Programming [MOFLP] model with fuzziness in objective, resources, technological coefficient and resources for the proposed weighted additive model and it is compared with zimmermann method to help the management to allocate the optimum order quantities, in which the three objectives are to optimize purchasing cost, quality, and service and satisfy constraints like supplier's capacity, supply chain demand etc are considered. The model has been applied to supplier selection of a high technology company named Multi-Flex Lami-Print Ltd which manufactures Flexible Packaging materials. In fuzzy supplier selection problem four different cases are considered to incorporate the uncertainty by zimmermann and weighted additive method. The result shows that the model is effective and applicable to industries.

Introduction

Supplier selection plays a key role in an organization because order allocation to the suppliers and purchasing cost of a product plays a vital role in supply chain. In addition to cost, the emphasis on quality and timely delivery in today's competitive market place also adds complexities to supplier selection decisions. To have an efficient and effective organization purchasing department plays a key role because it has direct effect on cost reduction, profitability and flexibility of a company. Supplier selection decision is often made in an uncertain environment filled with multiple objectives and incomplete information. Deterministic model cannot take this vagueness into account. In this case fuzzy set is one of the best tools for dealing with uncertainty. Fuzzy set theories are used because of the presence of vagueness and imprecision of information in the supplier selection problem. Theory of fuzzy sets has been employed to solve fuzzy linear programming problems by zimmermann (1978). While maximizing the objective functions and constraints maximum membership values for the fuzzy parameters are given by zarafat et.al (2006). In real cases, if the Decision-Makers (DMs) face uncertain data and situations, an additive weighted model is presented for fuzzy multi objective supplier selection problem with fuzzy weights which help DMs to find out the appropriate ordering from each supplier, and allows purchasing manager(s) to manage supply chain performance on cost, quality, on time delivery, etc by Amin Amid & .S. H. Ghodsypour (2008).

A multi-objective linear programming model (MOLP) for the special issues of purchasing the raw materials, selecting vendors and deciding ordering quantity as the key issue in optimizing purchasing policies, the point estimate weighted-sums, is used to solve this model Zhen Gao & Lixin Tang (2003). A linear programming model which proposed a compensatory fuzzy approach for finding the optimum Strategy to select suppliers by Soroush Avakh Darestani and Samane Ghavami (2013).

In this paper a fuzzy multi objective with fuzzy constraint and deterministic constraint model has been developed to allocate order to the suppliers in which different weights can be considered for various objectives such as cost, quality, service and constraint such as demand. To include the uncertainty, imprecise, vagueness in supply chain four cases are considered by both zimmermann and weighted additive method. In case-I objectives are fuzzy and constraints are crisp. In case-II availability of resources are fuzzy .In case-III technological coefficients are fuzzy and in case-IV both technological and resources are fuzzy.

Multi objective linear programming:

A general multi objective model for the supplier selection problem can be stated as follows:

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$$\min z_1.z_2,.............z_k,$$
$$\max z_{k+1}, z_{k+2},.......z_p$$
$$such\ that\ x \in X_d, X_d = \{x / g(x) \le b_r, r = 1,2,...........m\} \tag{1}$$

Where $z_1,z_2,\ldots\ldots\ldots z_k$ are the negative objectives and $z_{k+1},z_{k+2},\ldots\ldots z_p$ are the positive objectives and X_d is the set of feasible solutions which satisfy the constraints.

Fuzzy Linear Programming with Fuzzy Multi objective functions:

Step 1: FMOLP model is represented as

$$\max \sum_{j=1}^{n} \tilde{c}_j x_j \qquad \min \sum_{j=1}^{n} \tilde{c}_j x_j$$

$$s.t \quad \sum_{j=1}^{n} a_{ij} x_j \le b_i, i = 1,2......m$$

$$x_j \ge 0, j = 1,2..........n \tag{2}$$

Step 2: Multi objective problem is solved as a single objective by separating every objective function into its maximum and minimum linear programming problem.

$$\max \sum_{j=1}^{n} \tilde{c}_j x_j \ / \ \min \sum_{j=1}^{n} \tilde{c}_j x_j$$

$$s.t \quad \sum_{j=1}^{n} a_{ij} x_j \le b_i, i = 1,2......m, \quad x_j \ge 0, j = 1,2..........n \tag{3}$$

Step 3: Corresponding lower bound and upper bound for each objective is obtained as Max value = Upper bound, Min value = Lower bound

Step 4: Establish the linear membership function for each objective, linear membership for minimization goals (Z_k) where Z_k^+ represents the upper bound and Z_k^- represents the lower bound.

$$\mu_{z_k}(x) = \begin{cases} 1 & \text{for } Z_k \le Z_k^-, \\ \dfrac{(Z_k^+ - Z_k(x))}{(Z_k^+ - Z_k^-)} & \text{for } Z_k^- \le Z_k(x) \le Z_k^+, \\ & k = 1,2\ldots p, \\ 0 & \text{for } Z_k \ge Z_k^+ \end{cases} \tag{4}$$

Linear membership for maximization goals (Z_l) Z_l^+ represents the upper bound and Z_l^- represents the lower bound.

$$\mu_{z_l}(x) = \begin{cases} 1 & \text{for } Z_l \ge Z_l^+, \\ \dfrac{(Z_l(x) - Z_l^-)}{(Z_l^+ - Z_l^-)} & \text{for } Z_l^- \le Z_l(x) \le Z_l^+, \\ & l = p+1, p+2\ldots q, \\ 0 & \text{for } Z_l \le Z_l^- \end{cases} \tag{5}$$

Step 5: Introduce the new variable λ, which transforms the fuzzy linear program to crisp linear program.

Maximize λ

Such that

$\lambda \le \mu_{z_k}(x)$, k = 1, 2…..p (objective functions),

$\lambda \le \mu_{z_l}(x)$, l = p+1, p+2…q (objective functions)

$\sum_{j=1}^{n} a_{ij}x_j \le b_i, i = 1,2......m$, $x_j \ge 0,\ j = 1,2......n$ (deterministic constraints) and λ ε [0,1]. (6)

Step 6: Optimal soln of the crisp LP is (x^* , λ^*) where x^* is the optimal soln of the problem and λ^* is the degree to which the decision makers want to achieve.

Weighted additive model for Fuzzy Multi objective function:

Step 1: Same steps 1 to 4 as FMOLP with objective function.

Step 2: Maximize $\sum_{k=1}^{p} w_k \lambda_k + \sum_{l=p+1}^{q} w_l \lambda_l$

such that

$\lambda_k \le \mu_{z_k}(x)$, k = 1, 2…..p (objective functions),

$\lambda_l \le \mu_{z_l}(x)$, l = p+1, p+2……q(objective functions)

$\sum_{j=1}^{n} a_{ij}x_j \le b_i, i = 1,2......m$, $x_j \ge 0,\ j = 1,2.........n$

(deterministic constraints)

and λ ε [0,1]. $\sum_{k=1}^{p} w_k + \sum_{l=p+1}^{q} w_l = 1, w_k, w_l \ge 0$. (7)

Where w_k *and* w_l are the weight given by the preference of the decision maker.

Fuzzy Linear Programming with Multi objective functions and Fuzzy resources $(\tilde{b}_i)$:

Step 1: MOFLP model is represented as

$$\max \sum_{j=1}^{n} c_j x_j$$

$$\min \sum_{j=1}^{n} c_j x_j$$

$$s.t \ \sum_{j=1}^{n} a_{ij}x_j \le \tilde{b}_i, i = 1,2......m, \ \sum_{j=1}^{n} a_{ij}x_j \le b_i, i = 1,2......m$$

$$x_j \ge 0,\ j = 1,2.........n \tag{8}$$

Where $\tilde{b}_i$ is a fuzzy number with linear membership function as

$$\mu_{b_i}(x) = \begin{cases} 1 & if \quad x \le b_i \\ \dfrac{b_i + p_i - x}{p_i} & if \ b_i < x < b_i + p_i \\ 0 & if \ \ b_i + p_i \le x \end{cases} \tag{9}$$

where $x \in R$.

For each vector x = ($x_1, x_{2.....} x_n$) first calculate the degree $D_i(x)$ to which x satisfies the ith constraint as $D_i(x) = B_i\left(\sum_{j=1}^{n} a_{ij} x_j\right)$.

These degrees are fuzzy set on R^n and their intersection $\bigcap_{i=1}^{m} D_i$ is a fuzzy feasible set.

Step 2: Calculate the lower bound of optimal values by solving the linear programming problem

$$\max \sum_{j=1}^{n} c_j x_j \; / \; \min \sum_{j=1}^{n} c_j x_j$$

$$s.t \sum_{j=1}^{n} a_{ij} x_j \le b_i, i = 1,2......m, \; x_j \ge 0, j = 1,2..........n \tag{10}$$

Step 3: Calculate the Upper bound of optimal values by solving the linear programming problem

$$\max \sum_{j=1}^{n} c_j x_j \; / \; \min \sum_{j=1}^{n} c_j x_j$$

$$s.t \sum_{j=1}^{n} a_{ij} x_j \le b_i + p_i, i = 1,2......m, \; \sum_{j=1}^{n} a_{ij} x_j \le b_i, i = 1,2......m, \; x_j \ge 0, j = 1,2....n \tag{11}$$

Step 4: Establish the linear membership function for each objective, linear membership for minimization goals (Z_k) where Z_k^+ represents the upper bound and Z_k^- represents the lower bound.

$$\mu_{z_k}(x) = \begin{cases} 1 & \text{for } Z_k \le Z_k^-, \\ \dfrac{(Z_k^+ - Z_k(x))}{(Z_k^+ - Z_k^-)} & \text{for } Z_k^- \le Z_k(x) \le Z_k^+, \\ & k = 1,2..p, \\ 0 & \text{for } Z_k \ge Z_k^+ \end{cases} \tag{12}$$

Linear membership for maximization goals (Z_l) where Z_l^+ represents the upper bound and Z_l^- represents the lower bound.

$$\mu_{z_l}(x) = \begin{cases} 1 & \text{for } Z_l \ge Z_l^+, \\ \dfrac{(Z_l(x) - Z_l^-)}{(Z_l^+ - Z_l^-)} & \text{for } Z_l^- \le Z_l(x) \le Z_l^+, \\ & l = p+1, p+2......q, \\ 0 & \text{for } Z_l \le Z_l^- \end{cases} \tag{13}$$

Step 5: Linear membership function for ith constraint as

$$\mu_{C_i}(x) = \begin{cases} 1 & if \;\; \sum_{j=1}^{n} a_{ij} x_j < b_i \\ \dfrac{b_i + p_i - \sum_{j=1}^{n} a_{ij} x_j}{p_i} & if \; b_i < \sum_{j=1}^{n} a_{ij} x_j \le b_i + p_i \\ 0 & if \;\; b_i + p_i \le \sum_{j=1}^{n} a_{ij} x_j \end{cases} \tag{14}$$

Step 6: Introduce the new variable λ, which transforms the fuzzy linear program to crisp linear program.

Maximize λ

Such that

$\lambda \le \mu_{z_k}(x)$, k = 1, 2…….p (objective functions),

$\lambda \le \mu_{z_l}(x)$, l = p+1, p+2…q (objective functions)

$\lambda \le \mu_{C_i}(x)$, (fuzzy constraints)

$\sum_{j=1}^{n} a_{ij}x_j \le b_i, i = 1,2......m$, $x_j \ge 0, j = 1,2.....n$ (deterministic constraints) and λ ε [0,1]. (15)

Step 7: Optimal soln of the crisp LP is (x^* , λ^*) where x^* is the optimal soln of the problem and λ^* is the degree to which the decision makers want to achieve.

Weighted additive model for FLP with Multi objective functions and Fuzzy resources $(\tilde{b}_i)$:

Step 1: Same steps 1 to 5 as FLP with multi objective function and fuzzy resources.

Step 2: Maximize $\sum_{k=1}^{p} w_k\lambda_k + \sum_{l=p+1}^{q} w_l\lambda_l + \sum_{i=1}^{m} w_i\lambda_i$ such that

$\lambda_k \le \mu_{z_k}(x)$, k = 1, 2..p (objective functions),

$\lambda_l \le \mu_{z_l}(x)$, l = p+1, p+2…q (objective functions)

$\lambda_i \le \mu_{C_i}(x), i = 1,2......m$ (Fuzzy constraints)

$\sum_{j=1}^{n} a_{ij}x_j \le b_i, i = 1,2......m$, $x_j \ge 0, j = 1,2.........n$

(deterministic constraints) and λ ε [0,1].

$$\sum_{k=1}^{p} w_k + \sum_{l=p+1}^{q} w_l + \sum_{i=1}^{m} w_i = 1, w_k, w_l, w_i \ge 0 \quad (16)$$

Where w_k , w_l *and* w_i are the weight given by the preference of the decision maker.

Fuzzy Linear Programming with Multi objective functions and Fuzzy Technological coefficients $(\tilde{a}_{ij})$:

Step 1: MOFLP model is represented as

$$\max \sum_{j=1}^{n} c_j x_j \qquad \min \sum_{j=1}^{n} c_j x_j$$

$$s.t \ \sum_{j=1}^{n} \tilde{a}_{ij}x_j \le b_i, i = 1,2......m \ , \sum_{j=1}^{n} a_{ij}x_j \le b_i, i = 1,2......m \quad x_j \ge 0, j = 1,2.........n \quad (17)$$

Where $\tilde{a}_{ij}$ is a fuzzy number with linear membership function as

$$\mu_{a_{ij}}(x) = \begin{cases} 1 & if \quad x < a_{ij} \\ \dfrac{a_{ij} + d_{ij} - x}{d_{ij}} & if \ a_{ij} \le x < a_{ij} + d_{ij} \\ 0 & if \quad x \ge a_{ij} + d_{ij} \end{cases} \quad (18)$$

where $x \in R$ and $d_{ij}>0$ for all i & j

Step 2: Calculate the lower bound and upper bound of optimal values by solving the linear programming problem

$$Z_1 = \max \sum_{j=1}^{n} c_j x_j \;/\; \min \sum_{j=1}^{n} c_j x_j$$

$$s.t \;\sum_{j=1}^{n} a_{ij} x_j \le b_i, i = 1,2......m, \; x_j \ge 0, j = 1,2..........n \tag{19}$$

$$Z_2 = \max \sum_{j=1}^{n} c_j x_j \;/\; \min \sum_{j=1}^{n} c_j x_j$$

$$s.t \;\sum_{j=1}^{n} (a_{ij} + d_{ij}) x_j \le b_i, i = 1,2......m, \; \sum_{j=1}^{n} a_{ij} x_j \le b_i, i = 1,2......m, \; x_j \ge 0, j = 1,2......n \tag{20}$$

Step 3: Let $Z_k = \min(Z_1, Z_2)$ and $Z_l = \max(Z_1, Z_2)$. Then Z_k and Z_l are called the lower and upper bounds of the optimal values.

Step 4: Establish the linear membership function for each objective, linear membership for minimization goals (Z_k) where Z_k^+ represents the upper bound and Z_k^- represents the lower bound.

$$\mu_{z_k}(x) = \begin{cases} 1 & \text{for } Z_k \le Z_k^-, \\ \dfrac{(Z_k^+ - Z_k(x))}{(Z_k^+ - Z_k^-)} & \text{for } Z_k^- \le Z_k(x) \le Z_k^+, \\ & k = 1,2..................p \\ 0 & \text{for } Z_k \ge Z_k^+ \end{cases} \tag{21}$$

Linear membership for maximization goals (Z_l) where Z_l^+ represents the upper bound and Z_l^- represents the lower bound.

$$\mu_{z_l}(x) = \begin{cases} 1 & \text{for } Z_l \ge Z_l^+, \\ \dfrac{(Z_l(x) - Z_l^-)}{(Z_l^+ - Z_l^-)} & \text{for } Z_l^- \le Z_l(x) \le Z_l^+, \\ & l = p+1, p+2......q, \\ 0 & \text{for } Z_l \le Z_l^- \end{cases} \tag{22}$$

Step 5: Linear membership function for ith constraint as

$$\mu_{C_i}(x) = \begin{cases} 1 & \text{if } b_i \ge \sum_{j=1}^{n} (a_{ij} + d_{ij}) x_j \\ \dfrac{b_i - \sum_{j=1}^{n} a_{ij} x_j}{\sum_{j=1}^{n} d_{ij} x_j} & \text{if } \sum_{j=1}^{n} a_{ij} x_j \le b_i < \sum_{j=1}^{n} (a_{ij} + d_{ij}) x_j \\ 0 & \text{if } b_i < \sum_{j=1}^{n} a_{ij} x_j \end{cases} \tag{23}$$

Step 6: Introduce the new variable λ, which transforms the fuzzy linear program to crisp linear program.

Maximize λ

Such that

$\lambda \leq \mu_{z_k}(x)$, k = 1, 2…….p (objective functions),

$\lambda \leq \mu_{z_l}(x)$, l = p+1, p+2…q (objective functions) $\lambda \leq \mu_{C_i}(x)$, (fuzzy constraints)

$\sum_{j=1}^{n} a_{ij}x_j \leq b_i, i = 1,2......m$, $x_j \geq 0, j = 1,2......n$ (deterministic constraints) and λ ε [0,1]. (24)

Step 7: Optimal soln of the crisp LP is (x^* , λ^*) where x^* is

the optimal soln of the problem and λ^* is the degree to which the decision makers want to achieve.

Weighted additive model for FLP with Multi objective functions and Fuzzy Technological coefficients $(\tilde{a}_{ij})$:

Step 1: Same steps 1 to 5 as FLP with multi objective function and fuzzy technological coefficients

Step 2: Maximize $\sum_{k=1}^{p} w_k\lambda_k + \sum_{l=p+1}^{q} w_l\lambda_l + \sum_{i=1}^{m} w_i\lambda_i$ such that

$\lambda_k \leq \mu_{z_k}(x)$, k = 1, 2…..p (objective functions),

$\lambda_l \leq \mu_{z_l}(x)$, l = p+1, p+2……q (objective functions) $\lambda_i \leq \mu_{C_i}(x), i = 1,2.......m$ (Fuzzy constraints)

$\sum_{j=1}^{n} a_{ij}x_j \leq b_i, i = 1,2......m$, $x_j \geq 0, j = 1,2.........n$ (deterministic constraints) and λ ε [0,1].

$$\sum_{k=1}^{p} w_k + \sum_{l=p+1}^{q} w_l + \sum_{i=1}^{m} w_i = 1, w_k, w_l, w_i \geq 0 \tag{25}$$

Where w_k , w_l *and* w_i are the weight given by the preference of the decision maker

Fuzzy Linear Programming with Multi objective functions and Fuzzy Technological coefficients $(\tilde{a}_{ij})$ and Fuzzy resources $(\tilde{b}_i)$:

Step 1: MOFLP model is represented as

$$\max \sum_{j=1}^{n} c_j x_j$$

$$\min \sum_{j=1}^{n} c_j x_j$$

$$s.t \ \sum_{j=1}^{n} \tilde{a}_{ij}x_j \leq \tilde{b}_i, i = 1,2......m, \ \sum_{j=1}^{n} a_{ij}x_j \leq b_i, i = 1,2......m \quad x_j \geq 0, j = 1,2.........n \tag{26}$$

Where $\tilde{a}_{ij}$ is a fuzzy number with linear membership function as

$$\mu_{a_{ij}}(x) = \begin{cases} 1 & if \quad x < a_{ij} \\ \dfrac{a_{ij} + d_{ij} - x}{d_{ij}} & if \ a_{ij} \leq x < a_{ij} + d_{ij} \\ 0 & if \quad x \geq a_{ij} + d_{ij} \end{cases} \tag{27}$$

where $x \in R$ and $d_{ij}>0$ for all i & j

Where $\tilde{b}_i$ is a fuzzy number with linear membership function as

$$\mu_{b_i}(x)=\begin{cases}1 & if \quad x\le b_i\\ \dfrac{b_i+p_i-x}{p_i} & if \; b_i<x<b_i+p_i\\ 0 & if \;\; b_i+p_i\le x\end{cases} \tag{28}$$

where $x\in R$.

Step 2: Calculate the lower bound and upper bound of optimal values by solving the linear programming problem

$$Z_1=\max\sum_{j=1}^{n}c_jx_j \,/\, \min\sum_{j=1}^{n}c_jx_j$$

$$s.t \;\; \sum_{j=1}^{n}(a_{ij}+d_{ij})x_j\le b_i, i=1,2......m, \;\; \sum_{j=1}^{n}a_{ij}x_j\le b_i, i=1,2......m, \;\; x_j\ge 0, j=1,2.....n \tag{29}$$

$$Z_2=\max\sum_{j=1}^{n}c_jx_j \,/\, \min\sum_{j=1}^{n}c_jx_j$$

$$s.t \;\; \sum_{j=1}^{n}a_{ij}x_j\le b_i+p_i, i=1,2......m, \;\; \sum_{j=1}^{n}a_{ij}x_j\le b_i, i=1,2.....m,$$

$$x_j\ge 0, j=1,2..........n \tag{30}$$

$$Z_3=\max\sum_{j=1}^{n}c_jx_j \,/\, \min\sum_{j=1}^{n}c_jx_j$$

$$s.t\sum_{j=1}^{n}(a_{ij}+d_{ij})x_j\le b_i+p_i, i=1,2..m, \;\; \sum_{j=1}^{n}a_{ij}x_j\le b_i, i=1,2....m, \;\; x_j\ge 0, j=1,2.....n \tag{31}$$

$$Z_4=\max\sum_{j=1}^{n}c_jx_j \,/\, \min\sum_{j=1}^{n}c_jx_j$$

$$s.t \;\; \sum_{j=1}^{n}a_{ij}x_j\le b_i, i=1,2......m, \;\; x_j\ge 0, j=1,2..........n \tag{32}$$

Step 3: Let Z_k = min (Z_1 , Z_2 , Z_3 ,Z_4) and

Z_l = max (Z_1 , Z_2, Z_3, Z_4). Then Z_k and Z_l are called the lower and upper bounds of the optimal values.

Step 4: Establish the linear membership function for each objective, linear membership for minimization goals (Z_k) where Z_k^+ represents the upper bound and Z_k^- represents the lower bound.

$$\mu_{z_k}(x)=\begin{cases}1 & \text{for } Z_k\le Z_k^-,\\ \dfrac{(Z_k^+-Z_k(x))}{(Z_k^+-Z_k^-)} & \text{for } Z_k^-\le Z_k(x)\le Z_k^+,\\ & k=1,2.........p,\\ 0 & \text{for } Z_k\ge Z_k^+\end{cases} \tag{33}$$

Linear membership for maximization goals (Z_l) where Z_l^+ represents the upper bound and Z_l^- represents the lower bound.

$$\mu_{z_l}(x)=\begin{cases}1 & \text{for } Z_l \geq Z_l^+, \\ \dfrac{(Z_l(x)-Z_l^-)}{(Z_l^+ - Z_l^-)} & \text{for } Z_l^- \leq Z_l(x) \leq Z_l^+, \\ & l = p+1, p+2\ldots\ldots q, \\ 0 & \text{for } Z_l \leq Z_l^- \end{cases} \quad (34)$$

Step 5: Linear membership function for ith constraint as

$$\mu_{C_i}(x)=\begin{cases}1 & \text{if } b_i \geq \sum_{j=1}^{n}(a_{ij}+d_{ij})x_j + p_i \\ \dfrac{b_i - \sum_{j=1}^{n} a_{ij}x_j}{\sum_{j=1}^{n} d_{ij}x_j + p_i} & \text{if } \sum_{j=1}^{n} a_{ij}x_j \leq b_i < \sum_{j=1}^{n}(a_{ij}+d_{ij})x_j + p_i \\ 0 & \text{if } b_i < \sum_{j=1}^{n} a_{ij}x_j \end{cases} \quad (35)$$

Step 6: Introduce the new variable λ, which transforms the fuzzy linear program to crisp linear program.

Maximize λ

Such that

$\lambda \leq \mu_{z_k}(x)$, k = 1, 2……..p (objective functions),

$\lambda \leq \mu_{z_l}(x)$, l = p+1, p+2…q (objective functions) $\lambda \leq \mu_{C_i}(x)$, (fuzzy constraints)

$\sum_{j=1}^{n} a_{ij}x_j \leq b_i, i=1,2\ldots\ldots m$, $x_j \geq 0,\ j=1,2\ldots\ldots\ldots\ldots n$ (deterministic constraints) and λ ε [0,1]. (36)

Step 7: Optimal soln of the crisp LP is (x^* , λ^*) where x^* is

the optimal soln of the problem and λ^* is the degree to which the decision makers want to achieve.

Weighted additive model for FLP with Multi objective functions and Fuzzy Technological coefficients $(\tilde{a}_{ij})$ and Fuzzy resources $(\tilde{b}_i)$:

Step 1: Same steps 1 to 5 as FLP with multi objective function and fuzzy technological coefficients

Step 2: Maximize $\sum_{k=1}^{p} w_k\lambda_k + \sum_{l=p+1}^{q} w_l\lambda_l + \sum_{i=1}^{m} w_i\lambda_i$

such that

$\lambda_k \leq \mu_{z_k}(x)$, k = 1, 2…..p (objective functions),

$\lambda_l \leq \mu_{z_l}(x)$, l = p+1, p+2……q (objective functions) $\lambda_i \leq \mu_{C_i}(x), i=1,2\ldots\ldots\ldots\ldots m$ (Fuzzy constraints)

$\sum_{j=1}^{n} a_{ij}x_j \leq b_i, i=1,2\ldots\ldots m$, $x_j \geq 0,\ j=1,2\ldots\ldots\ldots\ldots n$ (deterministic constraints) and λ ε [0,1].

$$\sum_{k=1}^{p} w_k + \sum_{l=p+1}^{q} w_l + \sum_{i=1}^{m} w_i = 1, w_k, w_l, w_i \geq 0 \quad . \quad (37)$$

Where w_k , w_l *and* w_i are the weight given by the preference of the decision maker.

In weighted additive fuzzy linear program, there is no difference between the fuzzy goals and fuzzy constraints. The weighted additive model is widely used in vector-objective optimization problems; the basic concept is to use a single utility function to express the overall preference of DM to draw out the relative importance of criteria (Lai and Hawang, 1994).

Mathematical Model for Supplier Selection:

Notations:

i- index for suppliers, i= 1,2……m

j- index for periods, j= 1,2………n

k- index for products, k= 1,2……..p

x_{ijk} - Order quantity of kth product from ith supplier in jth period.

q_{ijk} - quality level of kth product purchased from ith supplier in jth period.

t_{ijk} - on-time delivery rate of kth product purchased from ith supplier in jth period.

$\tilde{D}_k$ - fuzzy demand quantity of kth product.

P_{ijk} – Unit price of the kth product from the ith supplier in jth period.

B_{kj} – purchasing budget of the kth product in jth period.

MC_{ijk} – Maximum supply capacity of the kth product from the ith supplier in jth period.

Q_{kmax} – buyer's maximum acceptable defective rate of kth product.

T_{kmin} – buyer's minimum acceptable on time delivery rate on kth product.

Model formulation:

To allocate the optimum order quantities to the suppliers, we use a MOLP model with three objectives to optimize total purchasing costs, quality and and service, with constraint as purchasing budget, production demand, suppliers' capacity, and quality control and delivery reliability control constraints. The objective functions are as follows:

Purchasing cost: To minimize the total purchasing cost. Purchasing cost includes the price, transportation cost and ordering cost.

$$Min\ Z_1 = \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{p} c_{ijk} x_{ijk}$$

Quality: To maximize the number of non defective items for improving product quality.

$$Max\ Z_2 = \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{p} q_{ijk} x_{ijk}$$

Service: To maximize the number of items delivered on time.

$$Max\ Z_3 = \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{p} t_{ijk} x_{ijk}$$

Constraints are as follows:

Purchasing Budget Constraints: Total purchasing payment for each product cannot exceed the budget of each product in that period.

$$\sum_{i=1}^{m}\sum_{k=1}^{p} p_{ijk} x_{ijk} \le B_{kj},\ j = 1,2...n$$

Demand Constraints: The assigned order quantity of each product from all suppliers must meet the demand quantity of each product in the total period.

$$\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{p} x_{ijk} \ge \tilde{D}_k$$

Capacity Constraints: The order quantity of the kth product from the ith supplier cannot exceed each supplier's capacity.

$$x_{ijk} \le \mathrm{MC}_{ijk},\ \mathrm{i} = 1,2........\mathrm{m},\ \mathrm{j} = 1,2,..........\mathrm{n},\ \mathrm{k} = 1,2...........\mathrm{p}$$

Quality Control Constraints: The total defect quantity of each product cannot exceed maximum acceptable defective quantity of each product

$$\sum_{i=1}^{m}\sum_{j=1}^{n} q_{ijk}\ \mathrm{x}_{\mathrm{ijk}}\ \leq\ \mathrm{Q}_{\mathrm{kmax}}\tilde{D}_k,\ \mathrm{k}=1,2.............\mathrm{p}$$

Delivery Constraint: The total late delivery on each product cannot exceed minimum acceptable late delivery.

$$\sum_{i=1}^{m}\sum_{j=1}^{n} t_{ijk}\ \mathrm{x}_{\mathrm{ijk}} \leq T_{k\min}\tilde{D}_k,\ \mathrm{k}=1,2.......\mathrm{p}$$

Variable non-negativity Constraints: Non-negativity restrictions on the decision variables is

$$x_{ijk} \geq 0\,,\ \mathrm{i}=1,2...........\mathrm{m},\ \mathrm{j}=1,2..........\mathrm{n},\ \mathrm{k}=1,2............\mathrm{p}$$

The above multi objective supplier selection problem is solved as a single objective supplier selection problem using each time only one objective. This value is the best value for this objective as other objectives are absent. After obtaining the results determine the corresponding values for every objective at each solution derived. Then for each objective function find a lower bound and an upper bound corresponding to the set of solutions for each objective. Membership function values are calculated for the objective functions and fuzzy constraints. Using membership function and DM's preferences, based on fuzzy convex decision-making formulate the equivalent crisp model of the fuzzy optimization problem .Find the optimal solution vector x^*, where x^* is the efficient solution of the original multi objective supplier selection problem with the DM's preferences based on weighted additive model.

Numerical calculations and graphical representations:

Fuzzy multi objective linear programming model has applied to a professionally managed company namely Multi-Flex Lami-Print Ltd., who manufactures Quality Flexible Packaging Materials against specific orders from their customers like Hindustan Unilever Ltd., ITC Ltd., Tata Tea Ltd., Cavinkare Pvt Ltd….. will produce important raw materials like Polyester film, Bi-axially oriented Poly Propylene film, Polyethylene film and also Printing inks, Lamination adhesives, Diluting Solvents……….from the best suitable supplier's for various production processes such as Printing-Lamination-Slitting-Finishing.

Before selecting / finalizing a best suitable, ideal, reliable supplier from many of them in that category, Multi-Flex will carefully analyse important factors/ parameters in many of their suppliers suiting to their requirements. Finally suppliers meeting / fulfilling all their requirements in terms of most important basic criteria like Cost-Quality-Service will be selected from many of the suppliers in that category.

Basic process of manufacturing Polyester film is Polyester chips will be coextruded to bi-axially oriented thin film in the range of 10m to 200m for the various applications. From these range only 10micron and 12micron thickness of polyester film only are used in packaging industry.

Table 1: Supplier's quantitative information

Supplier	cost		Defects (rate)		On time Delivery (rate)		Capacity		Price	
	j=1	j=2	j=1	j=2	j=1	j=2	j=1	j=2	j=1	j =2
1	65	64	0.03	0.05	0.15	0.18	90000	40000	2	1
2	45	48	0.04	0.01	0.09	0.06	85000	95000	2	1
3	50	56	0.03	0.02	0.17	0.07	65000	45000	4	2
4	54	60	0.04	0.02	0.15	0.1	50000	65000	4	2

Suppliers of Polyester film are Garware Polyesters Ltd (Supplier 1), U Flex Films Ltd (Supplier 2), Jindal Films Ltd (Supplier 3), Polyplex Ltd (Supplier 4), MTZ Polyester Ltd, Venlon Polyester Ltd etc.

Cost, Quality, on time delivery, Capacity, Demand, Budget are said to be the criteria's and the de-fuzzified data's corresponding to each criteria given in Table 1 for four suppliers, two products in one period. The demand is a fuzzy number and is predicted to be about 500000. Maximum total damaging rate which can be accepted is 0.05. Minimum acceptable late delivery is 0.15.

The multi objective linear formulation is presented as min Z_1, max Z_2, Z_3.Then the linear membership function is used for fuzzifing the objective functions and demand constraint. If the data's are uncertain, imprecise and vague fuzzy mathematical program is applied by fuzzy membership function. In MOFLP model, fuzziness is considered in four types. Fuzziness is calculated by membership function. Linear membership function is developed for the maximization of decision maker's level of satisfaction. Fuzziness in objective function is calculated by the membership function, considering only one objective at a time as lower bound and upper bound. Similarly for fuzziness in resources, fuzziness in technological coefficient and fuzziness in both technological coefficient, resources and these values are represented in Table 2.

FMOLP with Fuzzy objective function			
	μ=0	**μ=1**	**μ=0**
Purchasing cost	-	26890000	27590000
Quality	13450	14850	-
Service	55950	60150	-

FMOLP with Fuzzy resources $(\tilde{b}_i)$		
μ=0	**μ=1**	**μ=0**
-	26890000	28190000
14850	15050	-
60150	61350	-

FMOLP with Fuzzy Technological coefficients $(\tilde{a}_{ij})$			
	μ=0	**μ=1**	**μ=0**
Purchasing cost	-	23935455	27590000
Quality	11581.82	14850	-
Service	48172.73	60150	-

FMOLP with $(\tilde{a}_{ij})$ **and** $(\tilde{b}_i)$		
μ=0	**μ=1**	**μ=0**
-	23935455	28490000
11581.82	15050	-
48172.73	61350	-

Table 2: Data set for membership functions

In MOFLP with fuzziness in the objective functions, the degree of truth for compromised solution for three conflicting objective cost is 27098102, quality is 14527.59, service is 58860.34 under fuzzy environment is $\lambda = 0.6929$ and in weighted additive FLP, objective cost is 27485000, quality is 14850, service is 60150 by giving weights for cost as 0.26, quality as 0.37, service as 0.37 to obtain degree of truth as $\lambda = 0.779$ and the order quantity to supplier is represented in figure 1.

In MOFLP with fuzziness in the resources, the degree of truth for compromised solution for three conflicting objective cost is 27784469, quality is 14912.39, service is 60524.34,demand under fuzzy environment is $\lambda = 0.311948$ and in weighted additive FLP, objective cost is 27784469, quality is 14912.39, service is 60524.34 by giving weights for minimizing the cost as 0.16, maximizing the quality as 0.23, maximizing the service as 0.16 for each resource of demand as 0.15 degree of truth is $\lambda = 0.574779$ and the order quantity to supplier is represented in figure 2.

In MOFLP with fuzziness in the technological coefficient, the degree of truth for compromised solution for three conflicting objective cost is 25152836, quality is 13951.7, service is 56541.25, demand, quality, delivery under fuzzy environment is $\lambda = 0.666886$ and in weighted additive FLP ,objective cost is 27466495, quality is 14846.14, service is 60126.87 by giving weights for minimizing the cost as 0.16, maximizing the quality as 0.23, maximizing the service as 0.16 and demand, quality, delivery for each technological coefficient as 0.15 degree of truth is $\lambda = 0.844827$ and the order quantity to supplier is represented in figure 3.

In MOFLP with fuzziness in the technological coefficient and resources, the degree of truth for compromised solution for three conflicting objective cost is 25152836, quality is 13951.7, service is 56541.25,demand, quality, delivery under fuzzy environment is λ

= 0.635072 and in weighted additive FLP objective cost is 27466495, quality is 14846.14, service is 60126.87 by giving weights for minimizing the cost as 0.16, maximizing the quality as 0.23, maximizing the service as 0.16 and demand, quality, delivery for each technological coefficient and resources as 0.15 degree of truth is λ = 0.700476 and the order quantity to supplier is represented in figure 4.

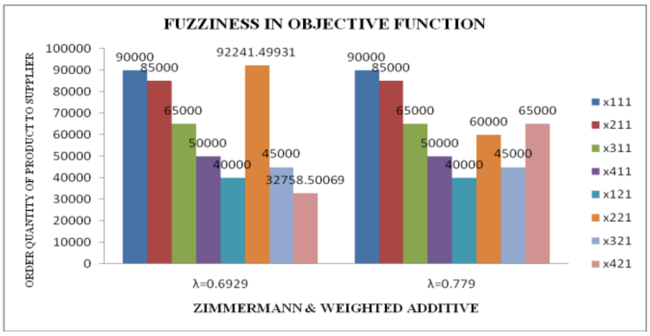

Figure 1: Order Quantity to Supplier Using Fuzziness In Objective Function

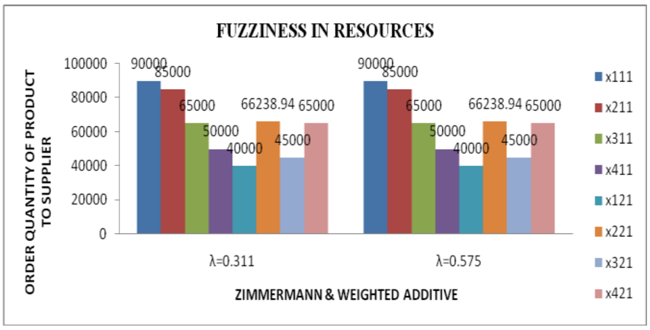

Figure 2: Order Quantity to Supplier Using Fuzziness In Resources

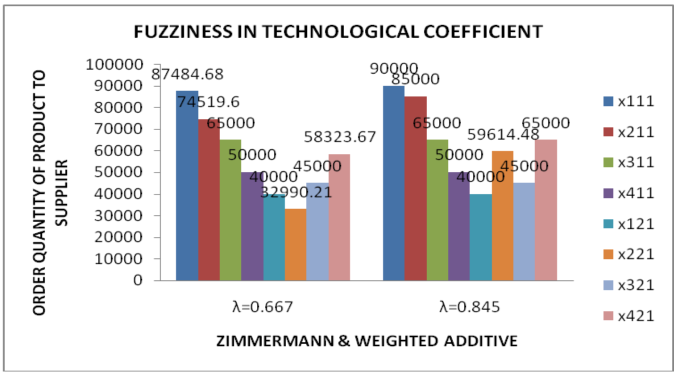

Figure 3: Order Quantity To Supplier Using Fuzziness In Technological Coefficients

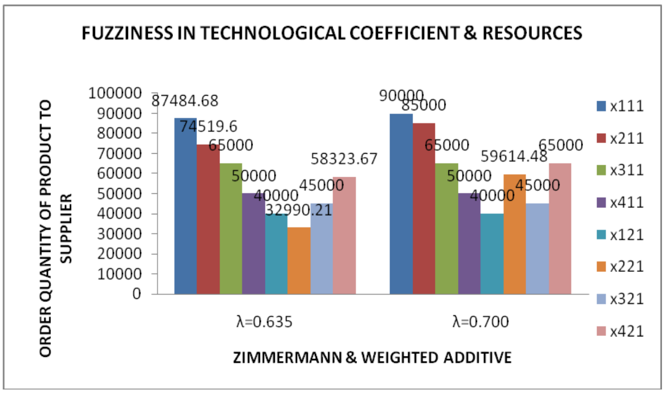

Figure 4: order quantity to supplier using fuzziness in technological coefficients & resources

The membership values of weighted additive for fuzziness in objective as follows:

$\mu_{z_1}(x) = \lambda_1 = 0.15$, $\mu_{z_2}(x) = \lambda_2 = 1$, $\mu_{z_3}(x) = \lambda_3 = 1$. It means that the achievement level of the objective functions is consistent with quality and service with the DM's preferences.

The membership values of weighted additive for fuzziness in resources as follows: $\mu_{z_1}(x)=0.31=\mu_{z_2}(x)=\mu_{z_3}(x)=\lambda_1=\lambda_2=\lambda_3$, $\mu_{C_1}(x)=\lambda_4=0.68$, $\mu_{C_2}(x)=\lambda_5=1$, $\mu_{C_3}(x)=\lambda_6=1$. It means that the achievement level of the resources is consistent with the DM's preferences.

The membership values of weighted additive for fuzziness in technological coefficients as follows: $\mu_{z_1}(x)=0.03=\lambda_1, \mu_{z_2}(x)=0.99=\lambda_2, \mu_{z_3}(x)=0.99=\lambda_3$, $\mu_{C_1}(x)=\lambda_4=1$, $\mu_{C_2}(x)=\lambda_5=1$, $\mu_{C_3}(x)=\lambda_6=1$. It means that the achievement level of the technological coefficient is consistent with the DM's preferences.

The membership values of weighted additive for fuzziness in technological coefficients and resources as follows: $\mu_{z_1}(x)=0.22=\lambda_1, \mu_{z_2}(x)=0.94=\lambda_2, \mu_{z_3}(x)=0.91=\lambda_3$, $\mu_{C_1}(x)=\lambda_4=0.02$, $\mu_{C_2}(x)=\lambda_5=1$ $\mu_{C_3}(x)=\lambda_6=1$.It means that the achievement level of the technological coefficient is consistent with the DM's preferences.

Conclusion:

To allocate order quantity among the selected suppliers is one of the major areas in supply chain. Supply chain includes tangible and intangible factors which should be determined based on organization requirements. In real case, many input data are not known precisely for decision making. In this paper the proposed MOFLP model with three objectives and constraints are formulated to handle the uncertainty, vagueness and imprecision in the objective functions, resources, technological coefficients, technological coefficients and resources for weighted additive method and it is compared with zimmermann's method in supply chain model. The model has applied to professionally well managed company and the results were found out consistent and reliable. By putting the weights of objectives it help the DM to find out the appropriate order for the suppliers and allows purchasing manager to manage the performance on the objectives in supply chain model. Complexity due to vagueness in allocating the order quantity to the supplier is easily handled by multi objective fuzzy linear program. Fuzzy multi objective supplier selection problem transforms into a convex (weighted additive) fuzzy programming model and its equivalent to crisp single objective linear programming. This transformation reduces the dimension of the system giving less computational complexity and makes the application of fuzzy methodology more understandable.

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