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Second-order slip flow on diffusion of chemically reactive species in a non-Newtonian Casson fluid past a vertical stretching sheet

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ABSTRACT

The second order slip boundary condition influence on diffusion of chemically reactive species of non-Newtonian fluid over a vertical permeable linearly stretching or shrinking sheet with surface mass transfer is considered using a second order slip flow model. Casson fluid model is used to characterize the non-Newtonian fluid behavior. The first order chemical reaction is considered. Choosing appropriate similarity variables, the partial differential equations are transformed into a set of non-linear self-similar ordinary differential equations, which are then solved numerically using the function bvp4c from Matlab for different values of the governing parameters. The results clearly show that the second order slip flow model is necessary to predict the flow characteristics accurately. A comprehensive numerical computation is carried out for various values of the parameters that describe the flow characteristics.

Introduction

The mass transfer problems have great importance in extending the theory of separation processes and chemical kinetics. Processes involving the effects of mass transfer have attracted the attention of researchers due to its applications in many engineering applications, such as chemical processing equipments. The driving force for mass transfer is the difference in concentration. The effects of chemically reactive species on fluid flow due to a stretching sheet bear equal importance in engineering research. The diffusion of a chemically reactive species in a laminar boundary layer flow over a flat plate was studied by Chambra and Young [1]. The effect of transfer of chemically reactive species in the laminar flow over a stretching sheet is explained by Andersson et al. [2]. Afify [3] explicated the MHD free convective flow of viscous incompressible fluid and mass transfer over a stretching sheet with chemical reaction. There are some fluids which react chemically with some ingredients present in them. On the other hand, the process of suction and injection (blowing) is also important in many engineering applications such as in the design of thrust bearing and radial diffusers, and thermal oil recovery. Suction is applied to chemical processes to remove reactants. Blowing is used to add reactants, to cool the surfaces, to prevent corrosion or scaling and to reduce the drag. Injection or withdrawal of fluid through a porous heated or cooled bounding wall is of practical interest in problems involving film cooling, control of boundary-layers, and so forth. For example, Gurminder and Chamkha [4] studied the problem of viscous fluid flow and heat transfer with second-order slip at linearly shrinking isothermal sheet in a quiescent medium. Nanadeppanavar et al. [5] analyzed second-order slip flow over a horizontal shrinking sheet with a non-linear Navier boundary condition.

The boundary-layer flows of non-Newtonian fluids have been given considerable attention due to ever increasing engineering applications. In order to obtain a thorough cognition of non-Newtonian fluids and their various applications, it is necessary to study their flow behaviors. It is well known that the mechanics of non-Newtonian fluids present a special challenge to engineers, physicists and mathematicians. The non-linearity can manifest itself in a variety of ways in many fields, such as food processing, drilling operations and bio-engineering. Furthermore, in order to obtain a thorough cognition of non-Newtonian fluids and their various applications, it is necessary to study their flow behaviors. Due to their application in industry and technology, few problems in fluid mechanics have enjoyed the attention that has been accorded to the flow which involves non-Newtonian fluids. It is well known that mechanics of non-Newtonian fluids present a special challenge to engineers, physicists and mathematicians. The non-linearity can manifest itself in a variety of ways in many fields, such as food, drilling operations and bio-engineering. The Navier–Stokes theory is inadequate for such fluids, and no single constitutive equation is available in the literature which exhibits the properties of all fluids. Because of the complexity of these fluids, there is not a single constitutive equation which exhibits all properties of such non-Newtonian fluids. Thus, a number of non-Newtonian fluid models have been proposed. In the literature, the vast majority of non-Newtonian fluid models are concerned with simple models like the power law and grade two or three [6–10]. These simple fluid models have shortcomings that render to results not having accordance with fluid flows in the reality. Casson fluid is another fluid model for non-Newtonian fluid. In the literature, the Casson fluid model is sometimes stated to fit rheological data better than general viscoplastic models for many materials [11,12]. Examples of Casson fluid include jelly, tomato sauce, honey, soup and concentrated fruit juices, etc. Human blood can also be treated as Casson fluid. Due to the presence of several substances like, protein, fibrinogen and globulin in aqueous base plasma, human red blood cells can form a chainlike structure, known as aggregates or rouleaux. If the rouleaux behave like a plastic solid, then there exists a yield stress that can be identified with the constant yield stress in Casson's fluid [13–15]. The non-linear Casson's constitutive equation has been found to describe

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accurately the flow curves of suspensions of pigments in lithographic varnishes used for preparation of printing inks [16] and silicon suspensions [17]. The shear stress–shear rate relation given by Casson satisfactorily describes the properties of many polymers [18] over a wide range of shear rates. Casson fluid can be defined as a shear thinning liquid which is assumed to have an infinite viscosity at zero rate of shear, a yield stress below which no flow occurs, and a zero viscosity at an infinite rate of shear [19]. Eldabe and Salwa [20] have studied the Casson fluid for the flow between two rotating cylinders, and Boyd et al. [21] investigated the Casson fluid flow for the steady and oscillatory blood flow. In this contribution the boundary layer flow due to stretching plane with mass transfer is studied. We venture further in the regime of two-dimensional second order slip flows of a non-Newtonian fluid with first order chemical reaction effect. Casson fluid model is used to characterize the non-Newtonian fluid behavior.

Mathematical analysis

Let us consider a steady two-dimensional, boundary layer, non-Newtonian Casson fluid flowing over a continuous permeable vertical stretching or shrinking sheet in the x – direction with a linear velocity ax, and a concentration distribution undergoing a first-order chemical reaction. The mass transfer velocity at the surface equal to v_w .

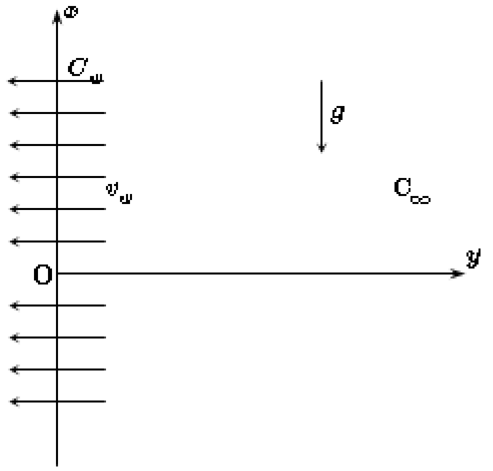

Fig. 1 Problem schematic and coordinate system

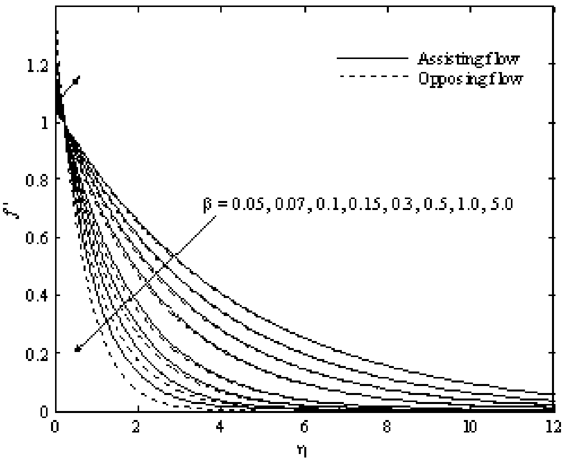

Fig. 2 Velocity distribution versus   for various values of Casson parameter

The Casson fluid model is used to characterize the non-Newtonian fluid behavior, Fig. 1 illustrates the physical model and coordinate system, the x -axis is taken along the plate and the y -axis is taken perpendicular to the plate. The fluid experiences a second-order slip at the sheet surface.

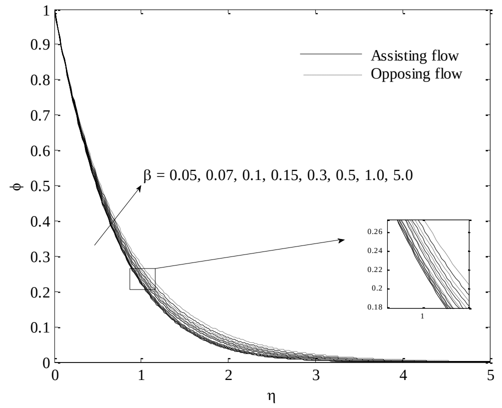

Fig. 3 Concentration distribution versus h for various values of Casson parameter

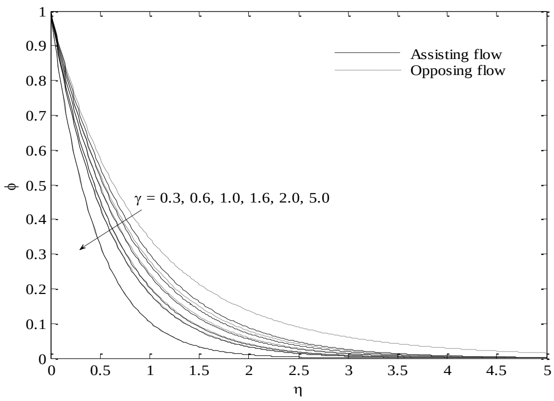

Fig. 4 Concentration distribution versus h for various values of reaction rate parameter

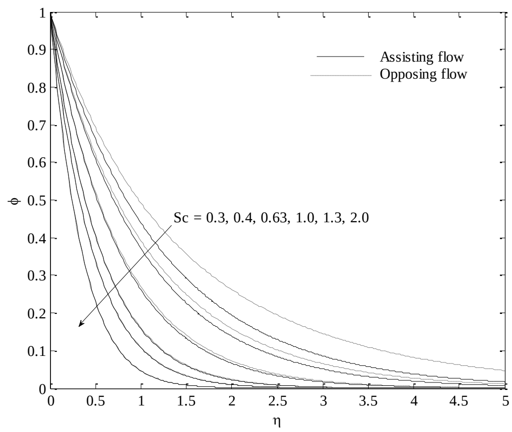

Fig. 5 Concentration distribution versus h for various values of Schmidt number

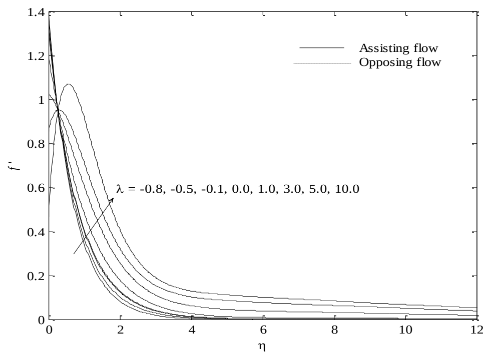

Fig. 6 Velocity distribution versus h for various values of diffusion parameter

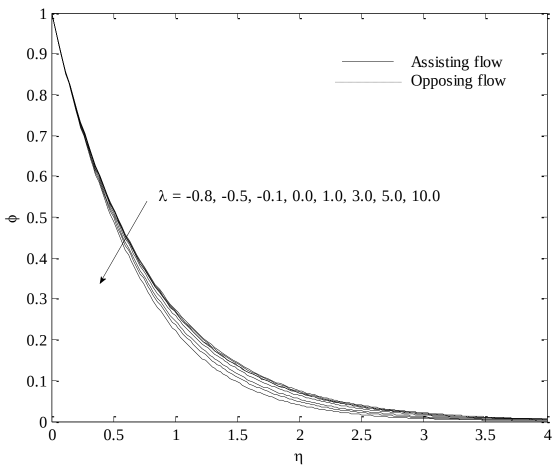

Fig 7. Concentration distribution versus h for various values of diffusion parameter

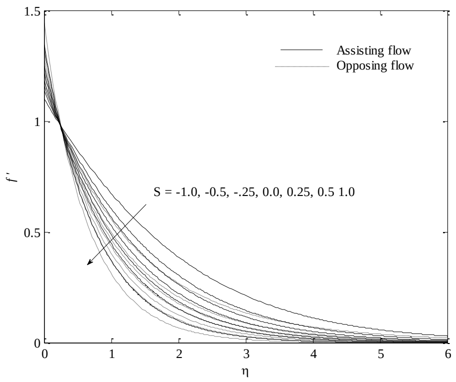

Fig. 8 Velocity distribution versus h for various values of mass transfer parameter

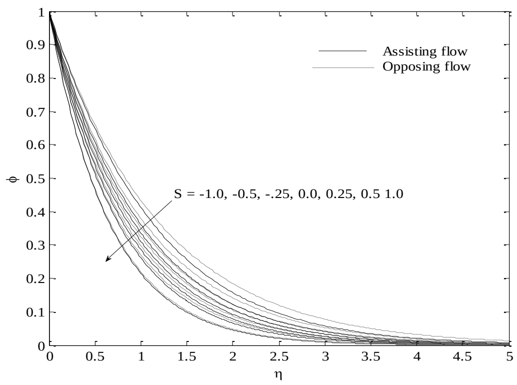

Fig. 9 Concentration distribution versus h for various values of mass transfer parameter

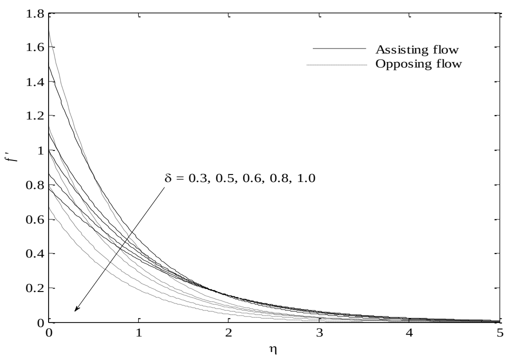

Fig. 10 Velocity distribution versus h for various values of first order slip parameter

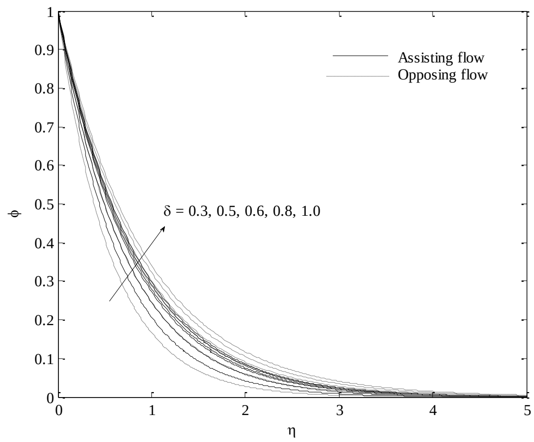

Fig. 11 Concentration distribution versus h for various values of first order slip parameter

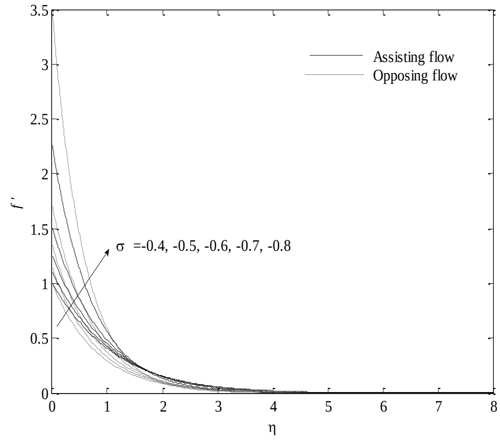

Fig. 12 Velocity distribution versus h for various values of second order slip parameter

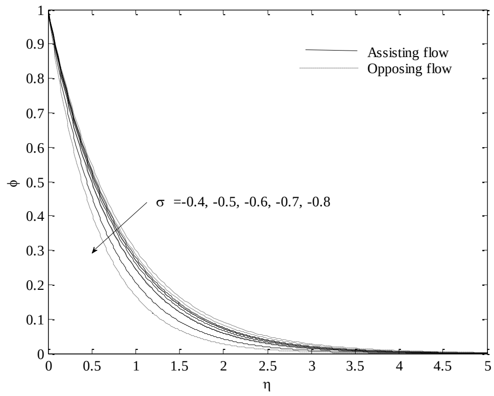

Fig. 13 Concentration distribution versus h for various values of second order slip parameter

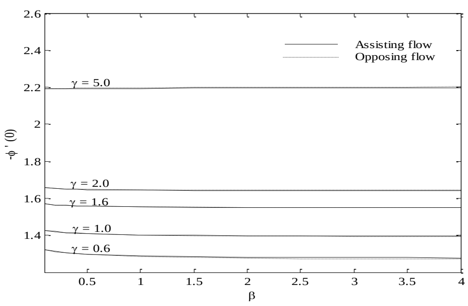

Fig. 14 Variation of f ¢(0) versus b for various values of reaction rate parameter

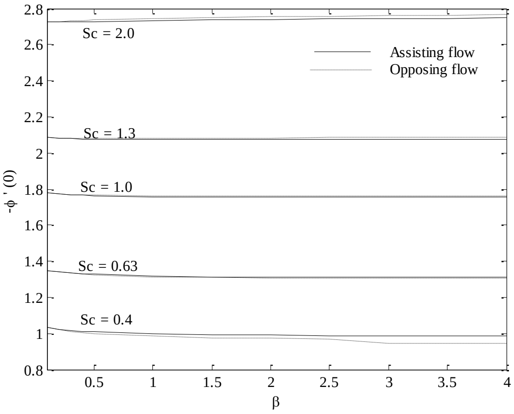

Fig. 15 Variation of $\phi'(0)$ versus β for various values of Schmidt number

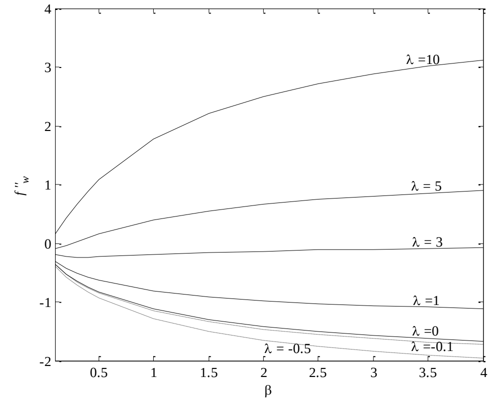

Fig. 16 Variation of $f''(0)$ versus β for various values of diffusion parameter

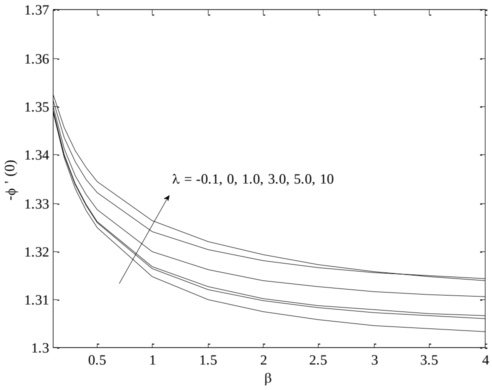

Fig. 17 Variation of $\phi'(0)$ versus β for various values of diffusion parameter

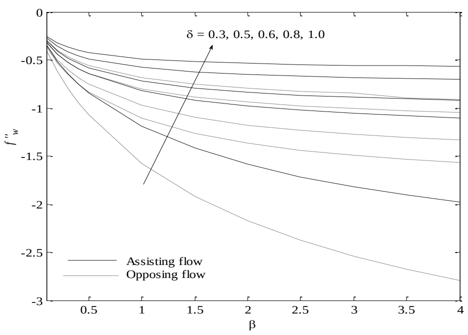

Fig. 18 Variation of $f''(0)$ versus β for various values of first order slip parameter

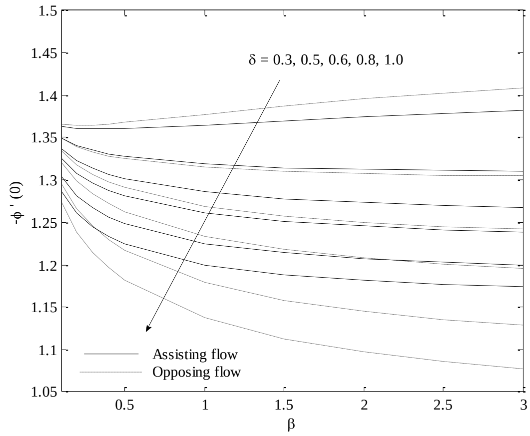

Fig. 19 Variation of $\phi'(0)$ versus β for various values of first order slip parameter

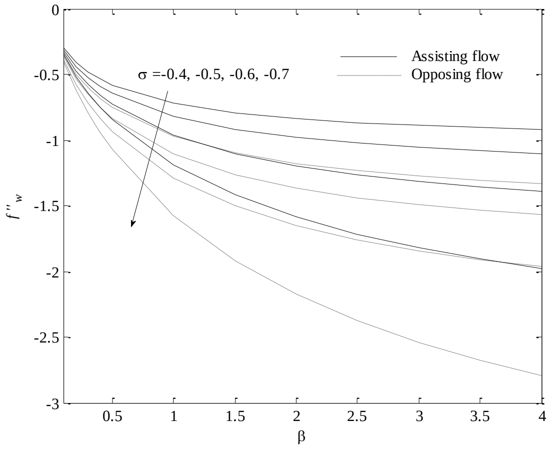

Fig. 20 Variation of $f''(0)$ versus β for various values of second order slip parameter

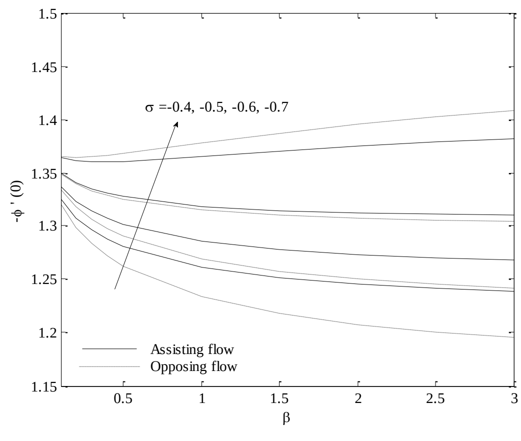

Fig. 21 Variation of $\phi'(0)$ versus β for various values of second order slip parameter

On the other hand, the rheological equation of state for an isotropic and incompressible flow of a Casson fluid as [11]

$$\tau_{ij} = \begin{cases} 2\left(\mu_B + \dfrac{P_y}{\sqrt{2\pi}}\right)e_{ij}, & \pi > \pi_c \\ 2\left(\mu_B + \dfrac{P_y}{\sqrt{2\pi_c}}\right)e_{ij}, & \pi < \pi_c \end{cases}$$

where, τ_{ij} is the (i,j)-th component of the stress tensor, $\pi = e_{ij}e_{ij}$ and e_{ij} are the (i,j)-th component of the deformation rate, π is the product of the component of deformation rate with itself, π_c is a critical value of this product based on the non-Newtonian model, μ_B is plastic dynamic viscosity of the non-Newtonian fluid, and p_y is the yield stress of the fluid. So, if a shear stress less than the yield stress is applied to the fluid, it behaves like a solid, whereas if a shear stress greater than yield stress is applied, it starts to move. Considering the balance laws of mass, linear momentum and energy and with the help of Boussinesq's approximation the equations governing this flow can be written in the usual form as

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad (1)$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\left(1 + \frac{1}{\beta}\right)\frac{\partial^2 u}{\partial y^2} \pm g\beta^*(C - C_\infty) \qquad (2)$$

$$u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = D\frac{\partial^2 C}{\partial y^2} - k(C - C_\infty) \qquad (3)$$

where, (u,v) are the velocity components in (x,y) directions, respectively, ν is the kinematic viscosity, C is the concentration of the fluid inside the thermal boundary layer, whereas C_∞ is the ambient concentration, β is the Casson parameter. β^* is the thermal expansion coefficient and k is the reaction rate of the solute, D is the diffusion coefficient. The last term on the right-hand side of equation (2) represents the influence of the diffusion of chemically reactive species on the flow field, with "+" sign corresponding to the flow assisting region and "-" sign corresponding to the flow opposing region.
Applying the following boundary conditions for the governing equations

$$y = 0, \quad u = ax + U_{\text{slip}}, \quad v = v_w, \quad C = C_w$$
$$y \to \infty, \quad u \to 0, \quad C \to C_\infty \qquad (4)$$

The model of U_{slip} is taken as Wu [22]

$$U_{\text{slip}} = \frac{2}{3}\left(\frac{3 - \alpha l^3}{\alpha} - \frac{3}{2}\frac{1 - l^2}{K_n}\right)\lambda\frac{\partial u}{\partial y} - \frac{1}{4}\left(l^4 + \frac{2}{K_n^2}(1 - l^2)\right)\lambda^2\frac{\partial^2 u}{\partial y^2} = A\frac{\partial u}{\partial y} + B\frac{\partial^2 u}{\partial y^2}$$

where $l = \min\left(\frac{1}{K_n}, 1\right)$, α with is the momentum accommodation coefficient with $0 \le \alpha \le 1$, and $\lambda (> 0)$ is the molecular mean free path. Based on the definition of l, it is seen that, for any given value of Kundsen number K_n we have $0 \le l \le 1$. Since the molecular mean free path λ is always positive it results in that A is positive and B is a negative number. ax is the velocity of the stretching surface, a being a positive constant. v_w is a prescribed distribution of suction ($v_w < 0$) or blowing ($v_w > 0$). A majority of the existing exact solutions in fluid mechanics are similarity solutions which reduce the number of independent variables by one or more. The methods for generating similarity transformations for equations of physical interest are discussed by Ames [23]. Similarity solutions are often asymptotic solutions to a given problem and may have utility in this area of limiting solutions. Similarity solutions may be used to gain physical insight into these details of complex fluid flows and these solutions exhibit most of the characteristic as well as the influence of the physical and thermal parameters of the actual problem. In order to get a similarity solution of the problem we define the following transformations

$$\eta = \sqrt{\frac{a}{\nu}}\,y, \quad \psi = \sqrt{a\nu}\,x f(\eta), \quad \phi(\eta) = \frac{C - C_\infty}{C_w - C_\infty} \qquad (5)$$

Substituting Eq. (5) into Eqs. (2) and (3) we get the following ordinary governing differential equations:

$$\left(\frac{1 + \beta}{\beta}\right)f''' + ff'' - f'^2 + \lambda\phi = 0 \qquad (6)$$

$$\frac{1}{Sc}\phi'' + f\phi' - (f' + \gamma)\phi = 0 \qquad (7)$$

The boundary conditions (4) then turn into

$$f(0) = S, \quad f'(0) = 1 + \delta f''(0) + \sigma f'''(0), \quad \phi(0) = 1$$
$$f'(\infty) \to 0, \quad \phi(\infty) \to 0 \qquad (8)$$

In the above equations, the prime denotes ordinary differentiation with respect to the similarity variable η, where, $\lambda = \pm Gr/\mathrm{Re}^2$ is the diffusion parameter, with $Gr = \left(g\beta^*(C_w - C_\infty)x^3\right)/\nu^2$ is the local Grashof number and $\mathrm{Re} = ax^2/\nu$ is the local Reynolds number. Further, $(\lambda > 0)$ corresponding to flow assisting region, whereas $\lambda < 0$ corresponding to flow opposing region. $Sc = \nu/D$, $\gamma = k/a$ are the Schmidt number and reaction rate parameter, respectively. The mass transfer parameter $S = -v_w/\sqrt{a\nu}$, $S > 0$ (i. e. $v_w < 0$) corresponding to suction and $S < 0$ (i. e. $v_w > 0$) corresponding to blowing. $\delta = A\sqrt{a/\nu}$ is the first-order slip flow parameter and $\sigma = Ba/\nu$ is the second-order slip flow parameter. In addition, the exact analytical solution of Eq. (6) when $\lambda = 0$ and slip is absent is given as

$$f(\eta) = \sqrt{1 + \beta^{-1}}\left(1 - e^{-\frac{\eta}{\sqrt{1+\beta^{-1}}}}\right),$$

$$f'(\eta) = e^{-\frac{\eta}{\sqrt{1+\beta^{-1}}}}, \quad f''(\eta) = -\frac{1}{\sqrt{1 + \beta^{-1}}}e^{-\frac{\eta}{\sqrt{1+\beta^{-1}}}} \qquad (9)$$

The local mass flux may be written by Fourier's Law as

$$q_m = -D\left.\frac{\partial C}{\partial y}\right|_{y=0} = -D\sqrt{\frac{a}{\nu}}(C_w - C_\infty)\phi'(0)$$

The local mass transfer coefficient is given by

$$h = \frac{q_m}{C_w - C_\infty}$$

In practical applications, the quantity of physical interest in our case is the local Sherwood number Sh, which may be written in non-dimensional form as

$$Sh = \frac{hx}{D} = -\sqrt{\mathrm{Re}}\, f'(0) \qquad (10)$$

Results and discussion

The system of ordinary differential equations (6) and (7) subject to the boundary conditions (8) was solved numerically using the function bvp4c from Matlab for different values of the Casson parameter , the diffusion parameter λ , mass transfer S , the Schmidt number Sc , first-order slip parameter δ and second-order slip parameter σ , reaction rate parameter γ . The relative tolerance was set to 10^{-10}. In this method, we have chosen a suitable finite value of $\eta \to \infty$ namely $\eta = \eta_\infty = 20$. The guess should satisfy the boundary conditions and reveal the behavior of the solution. However, it is difficult to come up with a sufficiently good guess for the solution of the system of the ordinary differential equations (6) and (7) in the case of opposing flow. To overcome this difficulty, we start with a set of parameter values for which the problem is easy to be solved. Then, we use the obtained result as initial guess for the solution of the problem with small variation of the parameters. This is repeated until the right values of the parameters are reached. The results are given to carry out a parametric study showing influences of several non-dimensional parameters. For the validation of the numerical results obtained in this investigation, the case when the Casson parameter approaches to infinity ($\beta \to \infty$, i.e. Newtonian fluid case) has been considered and compared with the previously published results. In Table 1 we compare our obtained results for various values of first order slip parameter with the available results of Andersson [2] for skin-friction coefficient $f''(0)$ when slip is absent, which show an excellent agreement.

Figs. 2 and 3 illustrate the influence of Casson parameter β on velocity and concentration distributions respectively, considering the two cases of assisting and opposing flow (i.e. $\lambda = -0.5, 0.5$) when $Sc = 0.63, S = 0.5, \gamma = 0.7, \delta = 0.4$ and $\sigma = -0.6$. The increasing values of the Casson parameter i.e. the decreasing yield stress (the fluid behaves as Newtonian fluid as Casson parameter becomes large) suppress the velocity field. The effect of increasing values of β is to reduce the rate of transport, and hence, the boundary layer thickness decreases. It is observed that $f'(\eta)$ and the associated boundary layer thickness are decreasing function of β. The effect of increasing β leads to enhance the concentration field for steady motion Figs. 3. It can also be seen from Fig. 2 that the momentum boundary layer thickness decreases as β increases and hence induces an increase in the absolute value of the velocity gradient at the surface. On the other hand, first the peak of the velocity at the surface increasing with β until critical value of η (0.2287) then the velocity decreases. The chemical reaction parameter affects the species concentration distribution and this can be verified from Fig. 4, considering the two cases of assisting and opposing flow (i.e. $\lambda = -0.5, 0.5$). From the figure it reveals that the values of the concentration profiles decrease with increasing chemical reaction parameter i.e. the chemical reaction opposes the diffusion of species concentration distribution. Fig. 5 depicts the effect of the Schmidt number on concentration distribution. The value of the concentration profile at a particular point decreases significantly with increasing Sc and consequently, the concentration boundary layer thickness becomes thinner. This decrease in the solute concentration causes a reduction in the solutal buoyancy effects resulting in less induced flow along the surface.

Table 1. The skin-friction coefficient $f''(0)$ for various values of δ when $\lambda = S = 0$ and $\beta \to \infty$ (Newtonian fluid)

δ	Andersson [2]	Present
0.0	-1.0000	-1.00000
0.1	-0.8721	-0.87243
0.2	-0.7764	-0.77632
0.5	-0.5912	-0.59092
1.0	-0.4302	-0.43103
2.0	-0.2840	-0.28321
5.0	-0.1448	-0.14479
10.0	-0.0812	-0.08120
20.0	-0.0438	-0.04376
50.0	-0.0186	-0.01858
100.0	-0.0095	-0.00946

The variation of velocity field and reactive concentration distribution for several values of the diffusion parameter λ are obtained. Figs. 6 and 7 plotted the dimensionless velocity and concentration profiles. With increasing values of λ the velocity at a fixed point increases, but converse effect is observed for species concentration profiles i.e. $f(\eta)$ at a fixed decreases with λ . This effect of diffusion of reactive species on the flow field is very significant in physical and practical point of view. In addition, Figs. 8 and 9 display the effects of suction/blowing parameter. With increasing suction/blowing parameter, fluid velocity and species concentration distribution are found to decrease. That is, the effect of S is to decrease the fluid velocity in the boundary-layer and in turn, the wall shear stress decreases. The increase S causes thinning of the boundary layer. However, species concentration at a point is found to decrease with increasing S . This causes a decrease in the rate of mass transfer. Figs. 10 and 11 display the effect of partial first order slip parameter on the velocity and concentration profiles. From Fig. 10 we have found a special point $\eta = 1.82$, before and after that point the behavior of velocity profiles change completely. The velocity decreases with the increasing values of first order slip parameter up to that point whereas increases to some extent after this. Consequently, with increase of first order slip parameter the thickness of boundary layer increases except very large values of δ, because if we increase δ to a value tending to infinity then the boundary layer structure will disappear. In addition, from Fig. 11 it is observed that the value of concentration profile at a point increases with δ. Fig. 12 depicts that with the increase in second order slip parameter σ , the fluid velocity decreases, the higher the value of σ , the lower is the fluid velocity but with the increase in η, the velocity profiles intersect and the behavior changes. Fig. 13 shows that with the increase in σ , the species concentration increases. Figs. 14 and 15 demonstrate the effect of reaction rate parameter and Schmidt number on variation of mass transfer coefficient $-f'(0)$, as it is shown for the increase of Sc and γ the values $-f'(0)$ increase. Variations of skin-friction coefficient and the mass transfer coefficient for various values of diffusion parameter are displayed in Figs. 16 and 17. It is clear that both of $f''(0)$ and $-f'(0)$ increase with increasing λ . It can be seen in Fig. 18, when only the first order slip parameter δ is considered, the reduced skin friction $f''(0)$ increases with the increase of the parameter δ, but the mass transfer rate

$-f''(0)$ decreases, Fig. 19. The same, observations are seen for the second order slip parameter s from Figs. 20 and 21.

Conclusions

A numerical analysis of steady two-dimensional boundary layer flow with a second-order slip of non-Newtonian Casson fluid over a stretching or shrinking sheet with first-order chemical reaction effect has been performed. It is shown in this paper how the first and second-order slip parameters, reaction rate parameter, diffusion parameter and Casson parameter affect the species concentration distribution, velocity and the mass transfer coefficient. We can conclude that:

- The effect of increasing values of the Casson parameter b is to suppress the velocity field, whereas the species concentration is enhanced with increasing Casson parameter.
- Increasing values of the diffusion parameter l tends to suppress the species concentration field (as the reaction rate parameter), but the velocity field is enhanced with increasing diffusion parameter.
- Surface mass transfer rate S influences the flow and concentration fields. Suction at the surface produces higher entrainment velocities, whereas injection makes the velocity and concentration distributions more linear.
- First and second-order slip parameters make the velocity decreases and species concentration increases.

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