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Radiation effect on mixed convection boundary layer flow of dusty fluid over a stretching porous surface

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ABSTRACT

The present study focused on the numerical solution of boundary layer flow of a radiating dusty fluid over a stretching porous surface. The governing boundary layer equations of the problem are formulated and transformed into ordinary differential equation by using similarity transformation. The resulting equations are then solved numerically using Runge Kutta fourth order scheme along with shooting technique. It has been observed that the temperature of the particle phase increases with the increase of radiation parameter but the radiation parameter has no significant effect on fluid phase temperature.

Nomenclature

E_c Eckert number
q_r radiation heat flux
q_{r_p} radiation heat flux of particle phase
F_r Froud number
G_r Grashof number
Re Reynolds number
P_r Prandtl number
T_∞ temperature at large distance from the wall.
T_p temperature of particle phase.
T_w wall temperature
$U_w(x)$ stretching sheet velocity
c_p specific heat of fluid
c_s specific heat of particles
k_s thermal conductivity of particle
u_p , v_p velocity component of the particle along x-axis and y-axis
A constant
Ra thermal radiation
c stretching rate
f_0 suction parameter
g acceleration due to gravity
k thermal conductivity of fluid
l characterstic length
T temperature of fluid phase.
u,v velocity component of fluid along x-axis and y-axis
x,y cartesian coordinate
K* Mean absorption co-efficient

Greek Symbols :

φ volume fraction
β fluid particle interaction parameter
β^* volumetric coefficient of thermal expansion
σ^* the Stefan Bolzman constant
ρ density of the fluid

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ρ_p density of the particle phase
ρ_s material density
η similarity variable
λ mixed convection facter
θ fluid phase temperature
θ_p dust phase temperature
μ dynamic viscosity of fluid
ν kinematic viscosity of fluid
γ ratio of specific heat
τ relaxation time of particle phase
τ_T thermal relaxation time i.e. the time required by the dust particle to adjust its temperature relative to the fluid.
τ_p velocity relaxation time i.e. the time required by the dust particle to adjust its velocity relative to the fluid.
ε diffusion parameter
ω density ratio

Introduction

The laminar boundary layer flow on a moving surface are important for both practical as well as theoretical point of view because of their wide applications in the aerodynamic extrusion of plastic sheet, glass blowing, cooling or drying of papers, drawing plastic films, extrusion of polymer melt-spinning process and heat treated materials traveling on conveyer belt etc.

The study of the boundary layer flow over a stretched surface moving with a constant velocity was initiated by Sakiadis B.C.[16]in1961 .Then many researchers extended the above study with the effect of Heat Transfer .Tsou et.al [20] studied the effect of Heat Transfer and experimentally confirmed the numerical result of Sakiadis .Grubka et.al[10] investigated the temperature field in the flow over a stretching surface when subject to uniform heat flux .Anderson [9] discussed a new similarity solution for the temperature fields . L.J.Crane,[13] has studied flow past a stretching plate. Sharidan et.al [17] presented similarity solutions for unsteady boundary layer flow and heat Transfer due to stretching sheet Ali,et.al[2].have studied Unsteady flow and heat transfer past an ax symmetric permeable shrinking sheet with radiation effect. Chen [18] investigated mixed convection of a power law fluid past a stretching surface in presence of thermal radiation and magnetic field .Chakrabarti et.al[12] have studied note on boundary layer in a dusty gas. B.J.Gireesha et.al[5] have studied the effect of radiation on boundary layer flow and heat Transfer of a dusty fluid over a stretching sheet in the presence of a free stream velocity .They have examined the Heat Transfer characteristics for two type of boundary conditions namely variable wall temperature and variable Heat flux..G.K.Ramesh et.al [9] have investigated the momentum and heat transfer characteristics in hydrodynamic flow of dusty fluid over an inclined stretching sheet with non uniform heat source/sink .B.J. Gireesh et.al [3] also studied the mixed convective flow a dusty fluid over a stretching sheet in presence of thermal radiation, space dependent heat source/sink. S.Manjunatha,et.al [22] have studied "Effect of thermal radiation on boundary layer flow and heat transfer of dusty fluid over an unsteady stretching sheet .Prasad, et.al[21] have studied radiation effect on MHD Unsteady free convection flow with a mass transfer past a vertical plate with variable surface temperature and concentration. Tasawar Hayat, et.al.[19] have studied radiation and mass transfer effect on the magnetohydrodyanamic unsteady flow induced by a stretching sheet. Zheng et.al [25] have studied analytical solutions of unsteady boundary flow and heat transfer on a permeable surface with non – uniform heat source/ sink.

In the present paper, the behavior of incompressible, laminar boundary- layer flows of a dusty fluid over a semi infinite stretching sheet along the whole length of the plate is studied using a similarity transformation .The terms related to the heat added to the system due to slip-energy flux, heat due to conduction and viscous dissipation, in the energy equation of particle phase, the momentum equation for particulate phase in normal direction, have been considered for better understanding of the boundary layer characteristics. The effects of volume fraction on skin friction, heat transfer and other boundary layer characteristics also have been studied.

Flow Analysis of the Problem and Solution:

Consider an unsteady two dimensional laminar boundary layer flow of an incompressible viscous dusty fluid over a semi-infinite stretching sheet in the region $y > 0$.The flow is generated by the action of two equal and opposite forces along the x-axis and y-axis being normal to the flow .The sheet being stretched with the velocity $U_w(x)$ along the x-axis, keeping the origin fixed in the fluid of ambient temperature T_∞ . Both the fluid and the dust particle clouds are suppose to be static at the beginning. The dust particles are assumed to be spherical in shape and uniform in size.

The governing equations of unsteady two dimensional boundary layer incompressible flows of dusty fluids are given by

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$\frac{\partial}{\partial t}\rho_p+\frac{\partial}{\partial x}\left(\rho_p u_p\right)+\frac{\partial}{\partial y}\left(\rho_p v_p\right)=0 \tag{2}$$

$$\rho\left[\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right]=(1-\varphi)\mu\frac{\partial^2 u}{\partial y^2}-\frac{1}{\tau_p}\varphi\rho_s\left(u-u_p\right)+(1-\varphi)\rho g\beta^*\left(T-T_\infty\right) \tag{3}$$

$$\varphi\rho_s\left(\frac{\partial u_p}{\partial t}+u_p\frac{\partial u_p}{\partial x}+v_p\frac{\partial u_p}{\partial y}\right)=\frac{\partial}{\partial y}\left(\varphi\mu_s\frac{\partial u_p}{\partial y}\right)+\frac{1}{\tau_p}\varphi\rho_s\left(u-u_p\right)+\varphi\left(\rho_s-\rho\right)g \tag{4}$$

$$\varphi\rho_s\left(\frac{\partial v_p}{\partial t}+u_p\frac{\partial v_p}{\partial x}+v_p\frac{\partial v_p}{\partial y}\right)=\frac{\partial}{\partial y}\left(\varphi\mu_s\frac{\partial v_p}{\partial y}\right)+\frac{1}{\tau_p}\varphi\rho_s\left(v-v_p\right) \tag{5}$$

$$(1-\varphi)\rho c_p\left[\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right]=(1-\varphi)k\frac{\partial^2 T}{\partial y^2}+\frac{1}{\tau_T}\varphi\rho_s c_s\left(T_p-T\right)+\frac{1}{\tau_p}\varphi\rho_s\left(u-u_p\right)^2+(1-\varphi)\mu\left(\frac{\partial u}{\partial y}\right)^2-(1-\varphi)\frac{\partial q_r}{\partial y} \tag{6}$$

$$\varphi\rho_s c_s\left[\frac{\partial T_p}{\partial t}+u_p\frac{\partial T_p}{\partial x}+v_p\frac{\partial T_p}{\partial y}\right]=\frac{\partial}{\partial y}\left(\varphi k_s\frac{\partial T_p}{\partial y}\right)-\frac{1}{\tau_p}\varphi\rho_s c_s\left(T_p-T\right)-\frac{1}{\tau_p}\varphi\rho_s\left(u-u_p\right)^2+\varphi\mu_s\left[u_p\frac{\partial^2 u_p}{\partial y^2}+\left(\frac{\partial u_p}{\partial y}\right)^2\right]-\varphi\frac{\partial q_{r_p}}{\partial y} \tag{7}$$

With boundary conditions $u = U_w(x,t) = \frac{cx}{1-at}$, $v = V_w(x,t) = -\frac{v_0}{\sqrt{1-at}}$ $at\ y = 0$
$\rho_p = \omega\rho, u = 0, u_p = 0, v_p \to v \quad as\ y \to \infty$ (8)

Where ω is the density ratio in the main stream.
In order to solve (6) and (7), we consider non –dimensional temperature boundary conditions as follows

$$T = T_w = T_\infty + T_0 \frac{cx^2}{\nu(1-at)^2}\ at\ y = 0 \text{ and } T \to T_\infty\ , T_p \to T_\infty\ as\ y \to \infty \quad (9)$$

Using the Rosseland approximation for radiation heat flux is simplified as where σ^* and K^* are the Stefan Bolzman constant and the mean absorption co-efficient respectfully.

$$q_r = -\frac{4\sigma^*}{3K^*}\frac{\partial T^4}{\partial y} \quad (10)$$

Assuming that the temperature differences with in the flow such that term T^4 may be expressed as a linear function of the temperature. We expand T^4 in a Taylor series about T_∞ and neglecting the higher order terms beyond the first degree in $(T - T_\infty)$we get

$$T^4 \cong 4T_\infty^3 T - 3T_\infty^4 \quad (11)$$

For most of the gases $\tau_p \approx \tau_T$, $k_s = k\frac{c_s}{c_p}\frac{\mu_s}{\mu}$ if $\frac{c_s}{c_p} = \frac{2}{3P_r}$

Introducing the following non dimensional variables in equation (1) to (7)

$$u = \frac{cx}{1-at}f'(\eta)\ ,\ v = -\sqrt{\frac{c\nu}{1-at}}f(\eta)\ ,\ \frac{\varphi\rho_s}{\rho} = \frac{\rho_p}{\rho} = \rho_r = H(\eta)$$

$$u_p = \frac{cx}{1-at}F(\eta)\ ,\ v_P = \sqrt{\frac{c\nu}{1-at}}\ G(\eta)\ ,\ \eta = \sqrt{\frac{c}{\nu(1-at)}}\ y$$

$$P_r = \frac{\mu c_p}{k}\ ,\ \beta = \frac{1-at}{c\tau_p}\ ,\ \epsilon = \frac{\nu_s}{\nu}\ ,\varphi = \frac{\rho_p}{\rho_s}, \mathrm{A} = \frac{a}{c}\ , E_c = \frac{c\nu}{C_pT_0}$$

$$G_r = \frac{g\beta^*(T_w - T_\infty)x^3}{\nu^2}\ , F_r = \frac{c^2x^2}{(1-at)^2gx} = \frac{U^2}{lg},\ \gamma = \frac{\rho_s}{\rho}\ , \nu = \frac{\mu}{\rho}\ , f_0 = -\frac{v_0}{\sqrt{c\nu}}, R_a = \frac{16\sigma^*T_\infty^3}{3K^*k}$$

$Re^2 = \frac{c^2x^4}{(1-at)^2\nu^2}$, $\lambda = \frac{Gr}{Re^2} = \frac{g\beta^*(T-T_\infty)(1-at)^2}{c^2x}$ where λ is the mixed convection factor.

$$\theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}\ ,\ \theta_p(\eta) = \frac{T_p - T_\infty}{T_w - T_\infty}\ ,$$

Where $T - T_\infty = T_0 \frac{cx^2}{\nu(1-at)^2}\theta$, $T_p - T_\infty = T_0 \frac{cx^2}{\nu(1-at)^2}\theta_p$

$$\frac{\partial q_r}{\partial y} = -\frac{16T_\infty^3\sigma^*}{3K^*}\frac{\partial^2 T}{\partial y^2}\ ,\ \frac{\partial q_{r_p}}{\partial y} = -\frac{16T_\infty^3\sigma^*}{3K^*}\frac{\partial^2 T_p}{\partial y^2}$$

a is the positive constant which measures the unsteadiness with boundary condition
c is the stretching rate and being a positive constant.
c_p is the specific heat of fluid phase.
k is the thermal conductivity.
β is the fluid particle interaction parameter.
A is the positive constant.
The equations (1) to (7) become

$$H'(\eta) = -\big(H(\eta)F(\eta) + H(\eta)G'(\eta)\big)/\left(A\frac{\eta}{2} + G(\eta)\right) \quad (12)$$

$$f'''(\eta) = -f(\eta)f''(\eta) + [f'(\eta)]^2 + A\left[f'(\eta) + \frac{\eta}{2}f''(\eta)\right] - \frac{1}{(1-\varphi)}\beta H(\eta)[F(\eta) - f'(\eta)] - Gr\theta(\eta) \quad (13)$$

$$A\left[\frac{\eta}{2}F'(\eta) + F(\eta)\right] + \big(F(\eta)\big)^2 + G(\eta)F'(\eta) - \epsilon F''(\eta) + \beta\big(F(\eta) - f'(\eta)\big) - \frac{1}{F_r}\left(1 - \frac{1}{\gamma}\right) = 0 \quad (14)$$

$$\frac{A}{2}[\eta G'(\eta) + G(\eta)] + G(\eta)G'(\eta) = \epsilon G''(\eta) - \beta[f(\eta) + G(\eta)] \quad (15)$$

$$\theta'' = \left(Pr(2f'\theta - f\theta') - \frac{2}{3}\frac{\beta}{1-\varphi}H[\theta_p - \theta] - \frac{1}{1-\varphi}PrE_c\beta H[F - f']^2 - PrE_c(f'')^2 + \frac{A}{2}Pr\big(\eta\theta'(\eta) + 4\theta(\eta)\big)\right)/(1 + R_a) \quad (16)$$

$$\theta_p'' = \frac{P_r}{\epsilon}\left[\frac{A}{2}\left(\theta_p'(\eta)\eta + 4\theta_p(\eta)\right) + 2F(\eta)\theta_p + G(\eta)\theta_p'(\eta) + \beta\left(\theta_p(\eta) - \theta(\eta)\right) + \frac{3}{2}E_cP_r\beta\big(f'(\eta) - F(\eta)\big)^2 - \frac{3}{2}\epsilon E_cP_r\left(F(\eta)F''(\eta) + \big(F'(\eta)\big)^2\right)\right]/\left(\frac{\epsilon}{P_r} + \frac{1.5R_a}{\gamma}\right) \quad (17)$$

With boundary conditions

$$\left.\begin{array}{l} G'(\eta) = 0, f(\eta) = f_0, f'(\eta) = 1, F'(\eta) = 0, \theta(\eta) = 1, \theta_p' = 0\ \ as\ \eta \to 0 \\ f'(\eta) = \lambda, F(\eta) = \lambda, G(\eta) = -f(\eta), H(\eta) = \omega, \theta(\eta) = 0, \theta_p = 0\ \ as\ \eta \to \infty \end{array}\right\} \quad (18)$$

The physical quantities of interest are the skin friction coefficient C_f and the local Nusselt number Nu_x which defined as

$C_f = \frac{\tau_w}{\rho U_w^2}$, $Nu_x = \frac{xq_w}{X(T_w - T_\infty)^\iota}$ where the surface shear stress $\tau_w$and surface heat flux q_w are given by

$$\tau_w = \mu\left(\frac{\partial u}{\partial y}\right)_{y=0}, q_w = -k\left(\frac{\partial T}{\partial y}\right)_{y=0}$$

Using the non dimensional variables, we obtained

$$C_f\, Re_x^{1/2} = f^{u}(0), \frac{Nu_x}{Re_x^{1/2}} = -\theta^{\iota}(0)$$

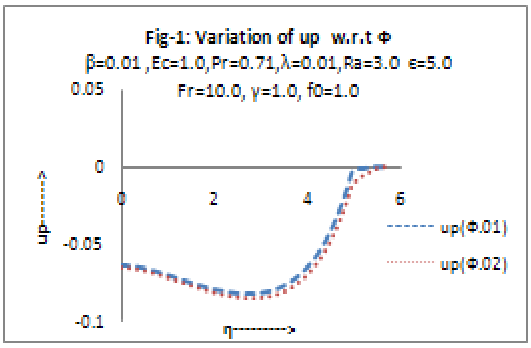
Solution of the problem:

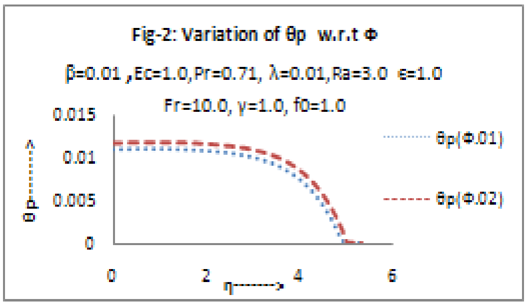
Here in this problem the value of $f''(0), F(0), G(0), H(0), \theta'(0), \theta_p(0)$ are not known but $f'(\infty) = 0, F(\infty) = 0, G(\infty) = -f(\infty), H(\infty) = \omega, \theta(\infty) = 0, \theta_p(\infty) = 0$ are given. We use Shooting method to determine the value of $f''(0), F(0), G(0), H(0), \theta'(0), \theta_p(0)$ .We have supplied $f''(0) = \propto_0$ and $f''(0) = \propto_1$. The improved value of $f''(0) = \propto_2$ is determined by utilizing linear interpolation formula. Then the value of $f'(\propto_2, \infty)$ is determined by using Runge-Kutta method. If $f'(\propto_2, \infty)$ is equal to $f'(\infty)$ up to a certain decimal accuracy, then $\propto_2$ i.e $f''(0)$ is determined, otherwise the above procedure is repeated with $\propto_0 = \propto_1$ *and* $\propto_1 = \propto_2$ until a correct $\propto_2$ is obtained. The same procedure described above is adopted to determine the correct values of $F(0), G(0), H(0), \theta'(0), \theta_p(0)$.

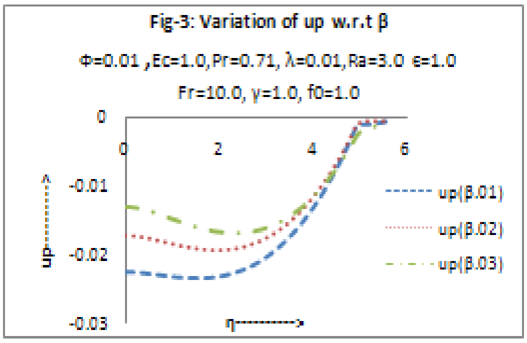
The essence of Shooting technique to solve a boundary value problem is to convert the boundary value problem into initial value problem. In this problem the missing value of $\theta'(0)$ and $f''(0)$ for different set of values of parameter are chosen on hit and trial basis such that the boundary condition at other end i.e. the boundary condition at infinity (η_∞)are satisfied.A study was conducting to examine the effect of step size as the appropriate values of step size $\Delta\eta$ was not known to compare the initial values of $\theta'(0)$ and $f''(0)$ .If they agreed to about 6 significant digits, the last value of η_∞used was considered the appropriate value; otherwise the procedure was repeated until further change in η_∞did not lead to any more change in the value of $\theta'(0)$ and $f''(0)$ .The step size $\Delta\eta$ =0.0625 has been found to ensure to be the satisfactory convergence criterion of 1×10^{-6}.The solution of the present problem is obtained by numerical computation after finding the infinite value for η. It has been observed from the numerical result that the approximation to $\theta'(0)$ and $f''(0)$ are improved by increasing the infinite value of η which is finally determined as η = 5.0 with a step length of 0.0625 beginning from η = 0. Depending upon the initial guess and number of steps N. the values of $f''(0)$ and $\theta'(0)$ are obtained from numerical computations which are given in table – 1 for different parameters.

Results and discussion:

The equation (12) to (17) subjected to boundary conditions (18) were solved numerically, by shooting method using the Runge-Kutta fourth order algorithm. The computations were done by the computer language FORTRAN-77.The results of heat transfer and skin friction coefficient characteristics are shown in Table-1, which shows that it is a close agreement with the existing literature. The effect of various parameters on the velocity profiles and temperature profiles also demonstrated graphically.







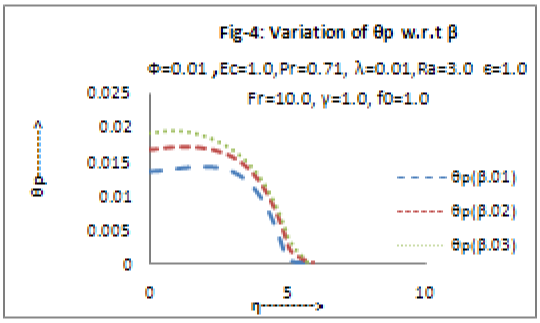


Fig-4: Variation of θp w.r.t β
Φ=0.01 ,Ec=1.0,Pr=0.71, λ=0.01,Ra=3.0 ϵ=1.0
Fr=10.0, γ=1.0, f0=1.0
θp------->
η---------->
θp(β.01)
θp(β.02)
θp(β.03)

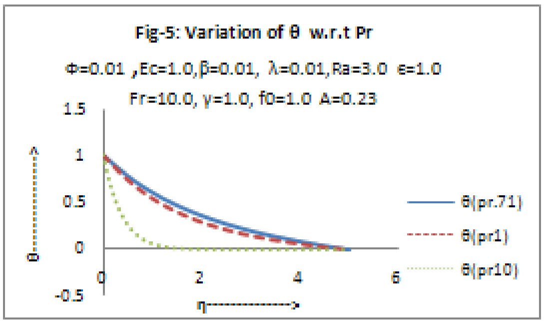


Fig-5: Variation of θ w.r.t Pr
Φ=0.01 ,Ec=1.0,β=0.01, λ=0.01,Ra=3.0 ϵ=1.0
Fr=10.0, γ=1.0, f0=1.0 A=0.23
θ------------->
η-------------->
θ(pr.71)
θ(pr1)
θ(pr10)

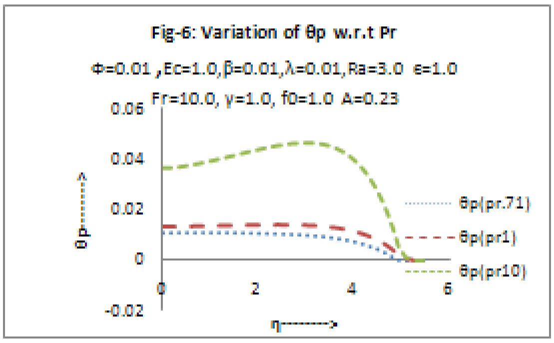


Fig-6: Variation of θp w.r.t Pr
Φ=0.01 ,Ec=1.0,β=0.01,λ=0.01,Ra=3.0 ϵ=1.0
Fr=10.0, γ=1.0, f0=1.0 A=0.23
θp--------->
η-------->
θp(pr.71)
θp(pr1)
θp(pr10)

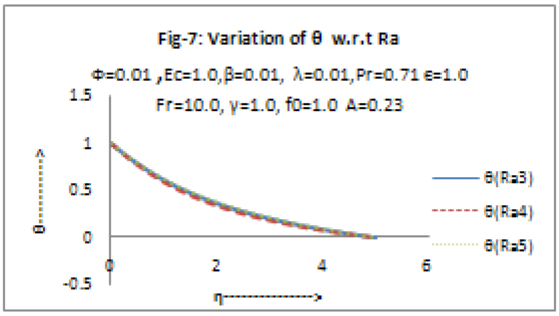


Fig-7: Variation of θ w.r.t Ra
Φ=0.01 ,Ec=1.0,β=0.01, λ=0.01,Pr=0.71 ϵ=1.0
Fr=10.0, γ=1.0, f0=1.0 A=0.23
θ---------->
η--------------->
θ(Ra3)
θ(Ra4)
θ(Ra5)

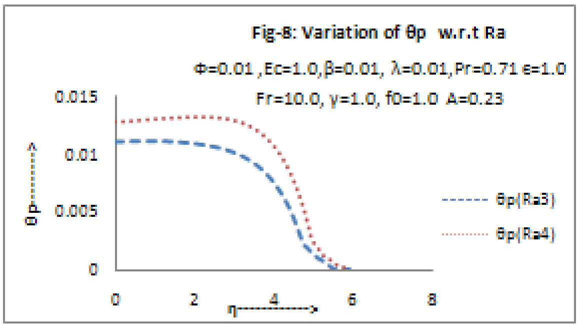


Fig-8: Variation of θp w.r.t Ra
Φ=0.01 ,Ec=1.0,β=0.01, λ=0.01,Pr=0.71 ϵ=1.0
Fr=10.0, γ=1.0, f0=1.0 A=0.23
θp--------->
η----------->
θp(Ra3)
θp(Ra4)

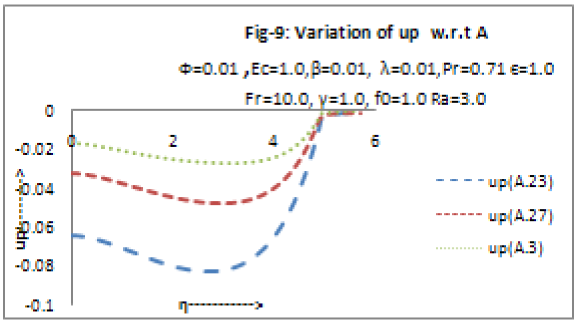


Fig-9: Variation of up w.r.t A

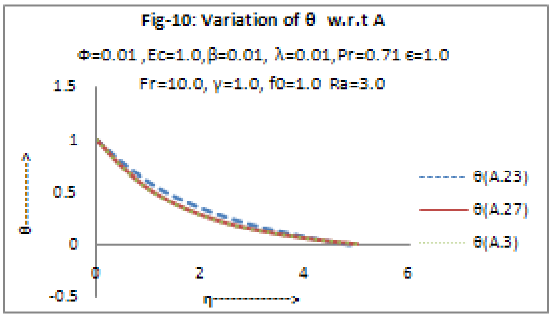


Fig-10: Variation of θ w.r.t A

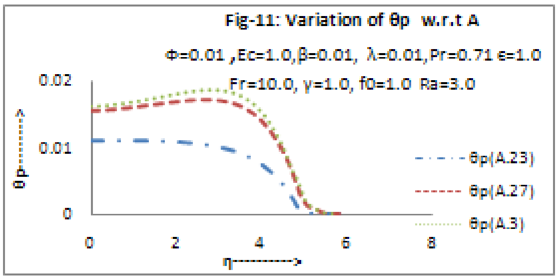


Fig-11: Variation of θp w.r.t A

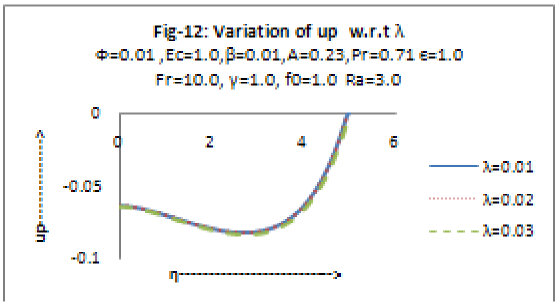


Fig-12: Variation of up w.r.t λ

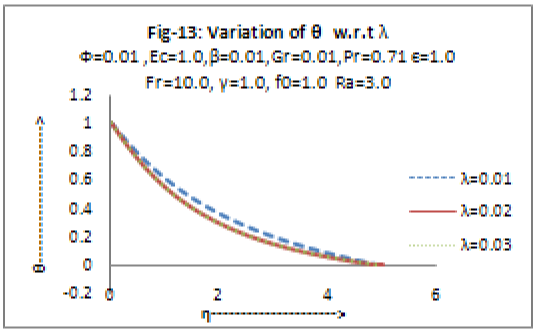


Fig-13: Variation of θ w.r.t λ

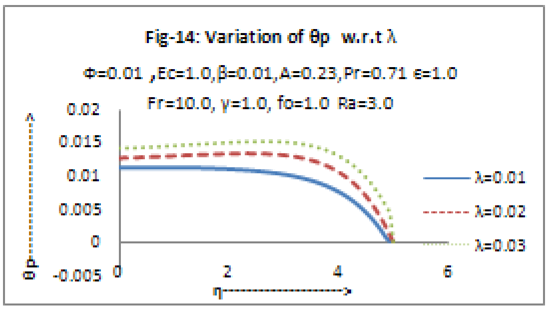


Fig-14: Variation of θp w.r.t λ

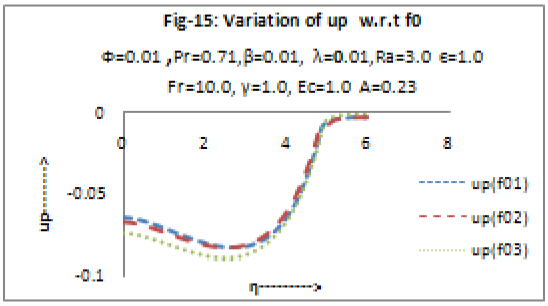


Fig-15: Variation of up w.r.t f0
Φ=0.01 ,Pr=0.71,β=0.01, λ=0.01,Ra=3.0 ε=1.0
Fr=10.0, γ=1.0, Ec=1.0 A=0.23
up------>
η---------->
up(f01)
up(f02)
up(f03)

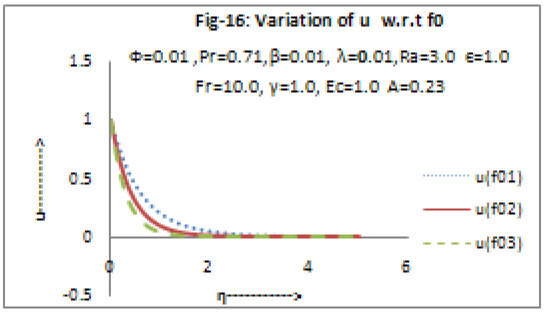


Fig-16: Variation of u w.r.t f0
Φ=0.01 ,Pr=0.71,β=0.01, λ=0.01,Ra=3.0 ε=1.0
Fr=10.0, γ=1.0, Ec=1.0 A=0.23
u--------->
η------------>
u(f01)
u(f02)
u(f03)

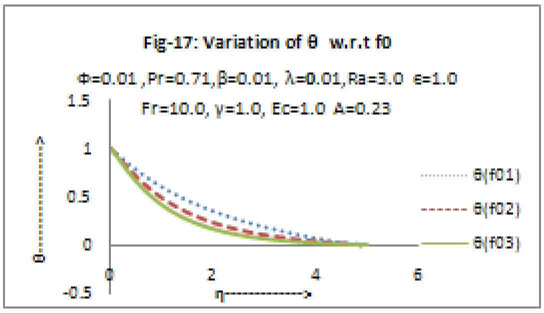


Fig-17: Variation of θ w.r.t f0
Φ=0.01 ,Pr=0.71,β=0.01, λ=0.01,Ra=3.0 ε=1.0
Fr=10.0, γ=1.0, Ec=1.0 A=0.23
θ------------->
η-------------->
θ(f01)
θ(f02)
θ(f03)

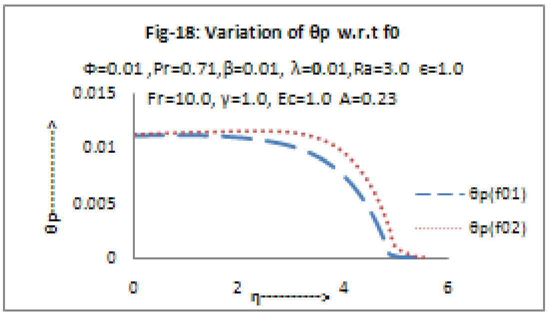


Fig-18: Variation of θp w.r.t f0
Φ=0.01 ,Pr=0.71,β=0.01, λ=0.01,Ra=3.0 ε=1.0
Fr=10.0, γ=1.0, Ec=1.0 A=0.23
θp------------>
η----------->
θp(f01)
θp(f02)

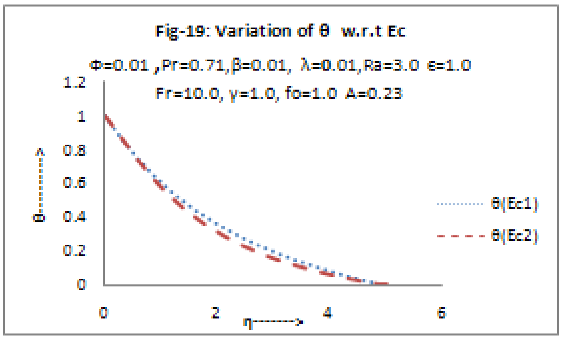


Fig-19: Variation of θ w.r.t Ec
Φ=0.01 ,Pr=0.71,β=0.01, λ=0.01,Ra=3.0 ε=1.0
Fr=10.0, γ=1.0, fo=1.0 A=0.23
θ---------->
η-------->
θ(Ec1)
θ(Ec2)

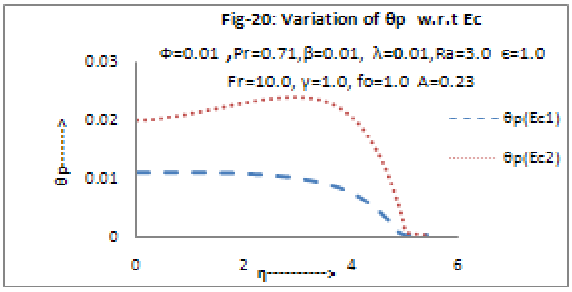


Fig-20: Variation of θp w.r.t Ec
Φ=0.01 ,Pr=0.71,β=0.01, λ=0.01,Ra=3.0 ε=1.0
Fr=10.0, γ=1.0, fo=1.0 A=0.23
θp------->
η----------->
θp(Ec1)
θp(Ec2)

Fig -1, illustrates the variation of velocity profiles up with η for various values of φ. It is observed that the effect of increasing the values of φ is to increase the velocity ׀up׀ of particle phase.Fig -2, illustrates the variation of temperature up with η for various values of φ. It is observed that the effect of increasing the values of φ is to decrease the temperature θp of particle phase.Fig -3, depicts velocity profile up versus η, for different values of β. It is observed that the effect of increasing values of β is to increase velocity ׀up׀ of particle phase.Figure-4 ,depict temperature profiles θp versus η, for different values of β. It is observed that the effect of increasing values of β is to increase of particle phase temperature θ .Figure-5 indicates the temperature profiles θ versus η, for the effect of Prandtl number Pr on fluid phase. It is observed temperature profiles θ decrease for increasing Pr.Figure-6 indicates the temperature profiles θp versus η, for the effect of Prandtl number Pr. It is observed temperature profiles θp increase for increasing Pr on particle phase.Figure-7 illustrates temperature profiles θ versus η for the effect of thermal radiation parameter Ra. There is no effect in temperature for increasing Ra on fluid phase.Figure-8 illustrates temperature profiles θp versus η for the effect of thermal radiation parameter Ra. It is observed temperature profiles θp increase for increasing Ra on particle phase.Fig -9, illustrates the variation of velocity profiles up with η for the effect of unsteadiness parameter A. It is observed that the effect of increasing the values of A is to increase the velocity ׀up׀ of particle phase.Figure-10 illustrates temperature profiles θ versus η for the effect of unsteadiness parameter A. It is observed that the increasing the values of A is to decrease temperature profiles θ on fluid phase.In fig.-11 illustrates temperature profiles θp versus η for different values of unsteadiness parameter A.It is observed that the increase in A produces significant increases in the thickness of the thermal boundary layer on particle phase.Figure-12 illustrates the variation of velocity profiles up with η for various values of λ. It is observed that the effect of increasing values of λ is to decrease the velocity ׀up׀ of particle phase.Figure-13 depicts temperature profiles θ versus η, for different values of λ. It illustrates temperature θ decrease for increasing λ on fluid phase.Figure-14 depicts temperature profiles θp versus η, for different values of λ. It illustrates temperature θp increase for increasing λ on particle phase.Figure-15represent the velocity profiles u versus η for the effect of suction parameter f0. It is observed velocity u decreases with increase f0 on fluid phase.Figure-16 represent the velocity profiles up versus η for the effect of suction parameter f0. It is observed velocity ׀up׀ decreases with increase f0 on particle phase.Figure-17 represent the temperature profiles θ versus η for the effect of suction parameter f0. It illustrates temperature profiles θ decreases for increasing f0 on fluid phase.Figure-18 represent the temperature profiles θp versus η for the effect of suction parameter f0. It is observed that the increasing of f0 produces significant increases in the thickness of the thermal boundary layer on particle phase.Figure-19 is the graph for temperature profile θ versus η for the effect of Eckert number Ec . It is evident that temperature profile θ decreases for increasing Ec in fluid phase.Figure-20 is the graph for temperature profiles θp versus η for the effect of Eckert number Ec . It is evident that temperature profile θp increases for increasing Ec on particle phase.

Table-1 Values of wall velocity gradient -$f^{u}(0)$, wall temperature gradient -$\theta^{\iota}(0)$ and other values are given below for different values of β, E_c , λ, P_r, Ra , f0 and A .

β	φ	E_c	λ	P_r	Ra	f_0	A	$-f^{u}(0)$	-F(0)	-G(0)	H(0)	-$\theta^{\iota}(0)$	Θp(0)
0.01	0.01	1	0.01	0.71	3	1.0	0.23	1.672081	.02269	.78042	.230414	.561978	.013725
0.02								1.67117	.01716	.794267	.225673	.564371	.016925
0.03								1.671532	.01295	.807724	.217674	.564997	.014175
0.01	0.01	1	0.01	0.71	3	1.0	0.23	1.672037	.063982	.415259	.546421	467922	.011086
	0.02							1.611232	.06527	.413377	.568218	.466463	.011587
	0.03							1.671431	.06112	.414245	.519191	.467934	.01173
0.01	0.01	1	0.01	0.71	3	1.0	0.23	1.672037	.063982	.415259	.546421	467922	.011086
		2						1.67191	.06355	.413917	.523459	.382705	.020019
		3						1.671531	.06447	414362	.557769	.199247	.02046
0.01	0.01	1	0.01	0.71	3	1.0	0.23	1.672037	.063982	.415259	.546421	467922	.011086
			0.02					1.665604	.06346	.414578	.546057	.567770	.012588
			0.03					1.655247	.06483	.414702	.579376	.571649	0.01417
0.01	0.01	1	0.01	0.71	3	1.0	0.23	1.672037	.063982	.415259	.546421	467922	.011086
				1.0				1.672138	.0632	.414284	.548276	.564387	.013237
				10.0				1.677637	.06374	.414327	.544709	2.49950	.036708
0.01	0.01	1	0.01	0.71	3	1.0	0.23	1.672037	.063982	.415259	.546421	467922	.011086
					4			1.671399	.06162	.414082	.506570	.497159	.012751
					5			1.671162	.06186	.413267	.504678	.452601	.010856
0.01	0.01	1	0.01	0.71	3	1.0	0.23	1.672037	.063982	.415259	.546421	467922	.011086
						2.0		2.450775	.06631	.459479	.478429	.580961	.011231
						3.0		3.328022	.0735	.510153	.438228	.627981	.00792
0.01	0.01	1	0.01	0.71	3	1.0	0.23	1.672037	.063982	.415259	.546421	467922	.011086
							0.27	1.68168	.01694	.366489	.441300	.580487	.016167
							0.30	1.690126	.03179	.335124	.437326	.591897	.015593

Conclusion

In this study, numerical analysis is presented to investigate the mixed convective heat transfer of a dusty fluid over a stretching porous surface. Thermal radiation terms have been included in the energy equations in unsteady dusty fluid. Velocity and temperature profiles are presented graphically and analyzed. Influence of physical parameters found to effect the problem under consideration are the fluid particle interaction parameter , mixed convective parameter, suction parameter ,radiation parameter ,Prandtl number and Eckert number. On this basis of the above study we have the following observations:

1. Velocity of fluid phase decreases but velocity ׀up׀ of particle phase decreases as f0 increases.
2. Temperature of fluid phase decreases and dust phase increases as f0 increases.
3. Effect of unsteady parameter A increases the velocity profiles ׀up׀ of particle phase.
4. Effect of unsteady parameter A decreases the temperature of fluid but increases temperature of particle phase.
5. Velocity ׀up׀ of particle phase decreases and also temperature of fluid phase decreases as λ increases.

6. Temperature of particle phase increases with the increase of β,Pr,Ra,λ and Ec.
7. The radiation parameter Ra has no significant effect on Temperature of fluid phase .
8. Velocity of particle phase| up| increases with the increase of β and φ.
9. Fluid phase temperature θ decreases with increase of Pr and Ec.

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