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Separation cordial labeling of graphs

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ABSTRACT

This paper introduces a new type of labeling called separation cordial labeling. A separation cordial labeling of graph $\mathbf{G}$ is a bijection $\mathbf{f}$ from $\mathbf{V}$ to $\{1, 2, \ldots, |V|\}$ such that each edge uv is assigned the label 1 if $\mathbf{f(u) + f(v)}$ is an odd number and label 0 if $\mathbf{f(u) + f(v)}$ is an even number. Then the number of edges labeled 0 and the number of edges labeled 1 differ by at most 1. If a graph has a separation cordial labeling, then it is called separation cordial graph. Here, the class $\mathbf{Pl_n}$ ($n \geq 5$), $\mathbf{Pl_{m,n}}$ $(\mathbf{m},\ \mathbf{n} \geq \mathbf{3})$ of planar graphs, full binary tree, the star graph $\mathbf{K_{1,q}}$, the complete bipartite graph $\mathbf{K_{m,n}}$, path $\mathbf{P_n}$, the cycle $\mathbf{C_n}$, are discussed and found to be separation cordial. Also, found that complete graph $\mathbf{K_n}$ is not separation cordial for, $\mathbf{n} \geq \mathbf{4}$.

1. Introduction

Let $G = (V, E)$, be a finite, undirected, connected graph with no loop or multiple edges. The order and size of G are denoted by 'p' and 'q' respectively ($p = |V|$ and $q = |E|$). For basic graph theoretic terminology, we refer to **[1], [2]** and **[3]**. Graph labeling **[4]** is a strong relation between numbers and structure of graphs **[3]** A graph labeling is a bijection 'f' from a subset of the elements of a graph to the set of positive integers. The domain of 'f' is the set of vertices, for vertex labeling and for edge labeling the domain of 'f' is the set of edges. A useful survey to know about the numerous graph labeling methods is given by J. A. Gallian **[5].** The origin of labeling can be attributed to A. Rosa **[6]** or R.L. Graham and N.J.A. Sloane **[7].** A vertex labeling **[4]** of a graph is an assignment f of labels to the vertices of G that induces for each edge uv a label depending on the vertex label $f(u)$ and $f(v)$. The two important labeling methods are called graceful and harmonious labelings. Cordial labeling is a variation of both graceful and harmonious labeling **[8].** The concept of cordial labeling was introduced by I. Cahit **[8].**

Definition 1.1: Let $G = (V, E)$ be a graph. A mapping $f\colon V(G) \to \{0, 1\}$ is called *binary vertex labeling* of G and $f(v)$ is called the label of the vertex v of G under f

For an edge $e = uv$, the induced edge labeling $f*\colon E(G) \to \{0, 1\}$ is given by $f*(e) = |f(u) - f(v)|$. Let $v_f(0)$ and $v_f(1)$ be the number of vertices of G having labels 0 and 1 respectively under f. Let $e_f(0)$ and $e_f(1)$ be the number of edges having labels 0 and 1 respectively under $f*$.

Definition 1.2: A binary vertex labeling of a graph G is called a *cordial labeling*, if $|v_f(0) - v_f(1)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$. A graph G is cordial, if it admits cordial labeling.

2. Separation Cordial Labeling

The concept of cordial labeling, motivated to introduce a new special type of cordial labeling called separation cordial labeling as follows:

Definition 2.1: A *separation cordial labeling* of a graph G with vertex set V is a bijection f from V to $\{1, 2, \ldots, p\}$ such that if each edge uv assigned the label 1 if $f(u) + f(v)$ is an odd number and label 0 if $f(u) + f(v)$ is an even number then the number of edges labeled 0 and the number of edges labeled 1 differ by at most 1.

A graph G is separation cordial, if it admits separation cordial labeling.

Theorem 2.1: *Given a positive integer n, then there is a separation cordial graph G which has n vertices.*

Proof: The positive integer n is divided into four cases.

Case (i): $n \equiv 0\ (mod\ 4)$

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Construct a path containing $\frac{n}{2}$ vertices $v_1, v_2, \ldots, v_{\frac{n}{2}}$, which are labeled as 1, 2, .. , $\frac{n}{2}$ respectively. The edges $v_i v_{i+1}$, for $1 \leq i \leq \frac{n}{2} - 1$, have the label 1. Add $\frac{n}{2}$ vertices $v_{\frac{n}{2}+1}\, v_{\frac{n}{2}+2}, \ldots, v_n$, below (above), parallel to the path constructed earlier, which are labeled as $\frac{n}{2}+1, \frac{n}{2}+2, \ldots, n$ respectively to the above (below) vertices $v_1, v_2, \ldots, v_{\frac{n}{2}}$, and then join by a path as $v_i\, v_{\frac{n}{2}+i}$, for $1 \leq i \leq \frac{n}{2}$. We see that the labels of the edges $v_i\, v_{\frac{n}{2}+i}$, for $1 \leq i \leq \frac{n}{2}$ are all 0.

So, we have $e_f(0) = \frac{n}{2}$, $e_f(1) = \frac{n}{2} - 1$ and hence $|e_f(0) - e_f(1)| = 1$. Thus, the resultant graph G is separation cordial.

Case (*ii*): $n \equiv 1$ (*mod* 4)

As above construct a path containing $\frac{n+1}{2}$ vertices $v_1, v_2, \ldots, v_{\frac{n+1}{2}}$, which are labeled as 1, 2, … , $\frac{n+1}{2}$ respectively. The edges $v_i v_{i+1}$, for $1 \leq i \leq \frac{n-1}{2}$, have the label 1. Add $\frac{n-1}{2}$ vertices $v_{\frac{n+1}{2}+1}$, $v_{\frac{n+1}{2}+2}, \ldots, v_n$ below (above), parallel to the path constructed earlier, which are labeled as $\frac{n+1}{2}+1, \frac{n+1}{2}+2, \ldots, n$ respectively to the above (below) vertices $v_2, v_3, \ldots, v_{\frac{n+1}{2}}$ and then join by a path as $v_{1+i}\; v_{\frac{n+1}{2}+i}$ for $1 \leq i \leq \frac{n+1}{2} - 1$. The labels of the edges $v_{i+i}\; v_{\frac{n+1}{2}+i}$, for $1 \leq i \leq \frac{n+1}{2} - 1$ are all 0.

Also, $e_f(0) = \frac{n+1}{2} - 1$, $e_f(1) = \frac{n+1}{2} - 1$ and hence $|e_f(0) - e_f(1)| = 0$. Therefore, the resultant graph G is separation cordial.

Case (*iii*): $n \equiv 2$ (*mod* 4)

Construct a path containing $\frac{n}{2}$ vertices $v_1, v_2, \ldots, v_{\frac{n}{2}}$, which are labeled as 1, 2, .. , $\frac{n}{2}$ respectively. The edges $v_i v_{i+1}$, for $1 \leq i \leq \frac{n}{2} - 1$, have the label 1 each. Add $\frac{n}{2}$ vertices $v_n, v_{\frac{n}{2}+1}, v_{\frac{n}{2}+2}, \ldots, v_{n-1}$ below (above), parallel to the path constructed earlier, which are labeled as $n, \frac{n}{2}+1, \frac{n}{2}+2, \ldots, n-1$ respectively to the above (below) vertices $v_1, v_2, \ldots, v_{\frac{n}{2}}$, and then join by a path as $v_{1+i}\; v_{\frac{n}{2}+i}$, for $1 \leq i \leq \frac{n}{2} - 1$. Join the vertex v_1 and v_n. The label of the edge $v_1 v_n$ is also 1. The labels of the edges $v_{1+i}\; v_{\frac{n}{2}+i}$, for $1 \leq i \leq \frac{n}{2} - 1$ are all 0.

Here, $e_f(0) = \frac{n}{2} - 1$, $e_f(1) = \frac{n}{2} - 1 + 1$ and hence $|e_f(0) - e_f(1)| = 1$. Thus the constructed graph G is separation cordial.

Case (*iv*): $n \equiv 3$ (*mod* 4)

Construct a path containing $\frac{n+1}{2}$ vertices $v_1, v_2, \ldots, v_{\frac{n+1}{2}}$ which are labeled as $1, 2, \ldots, \frac{n+1}{2}$ respectively. The edges $v_i v_{i+1}$, for $1 \leq i \leq \frac{n-1}{2}$, have the label 1. Add $\frac{n-1}{2}$ vertices $v_{\frac{n+1}{2}+1}, v_{\frac{n+1}{2}+2}, \ldots, v_n$, below (above), parallel to the path constructed earlier, which are labeled as $\frac{n+1}{2}+1, \frac{n+1}{2}+2, \ldots, n$ respectively to the above (below) vertices $v_1, v_2, \ldots, v_{\frac{n+1}{2}-1}$ and then join by a path as $v_i v_{\frac{n+1}{2}+i}$, for $1 \leq i \leq \frac{n+1}{2} - 1$. The labels of the edges $v_i v_{\frac{n+1}{2}+i}$, for $1 \leq i \leq \frac{n+1}{2} - 1$ are 0 each.

So, we have $e_f(0) = \frac{n+1}{2} - 1$, $e_f(1) = \frac{n+1}{2} - 1$ and hence $|e_f(0) - e_f(1)| = 0$. Therefore, the resultant graph G is separation cordial. Hence the proof.

The Class Pl_n of Planar Graphs

J. Baskar Babujee **[9]** defined a class of planar graphs obtained by removing some edges from the complete graphs K_n. The class of planar graphs on *n* vertices so obtained having maximum number of edges possible is denoted by Pl_n

Definition 2.1[9]: Let K_n be the complete graph on '*n*' vertices $V_n = \{1, 2, \ldots, n\}$. The class of graphs Pl_n has the vertex set V_n and the edge set $E_n = E(K_n / \{(k, l) : 3 \leq k \leq n-2, k+2 \leq l \leq n\}$.

We use K_n to denote a complete graph with '*n'* vertices and with all possible edges. A graph is said to be embedded in a surface *S* when it is drawn in *S* so that two edges don't intersect. A graph is *planar* if it can be embedded in the plane. The complete graph K_n $(1 \leq n \leq 4)$ are planar. For $n \geq 5$, K_n is non planar.

We construct planar graphs from K_n, $(n \geq 5)$.

Construction:

Place the vertices v_1 and v_2 as the end points of a horizontal line segment as shown in Figure. 1. Now place the vertices $v_3, v_4, \ldots, v_{n-1}, v_n$ along a vertical line (perpendicular to the line segment used for placing v_1 and v_2) with v_3 at the top and v_n at the bottom so that v_1, v_2 *and* v_3 form a triangular face. Join the vertices $v_4, v_5, .., v_n$ to v_1 and v_2 to form $2n - 5$ interior faces in this graph Pl_n The edges of the graph Pl_n can now be drawn without any crossings. All the faces of this graph are of size 3.

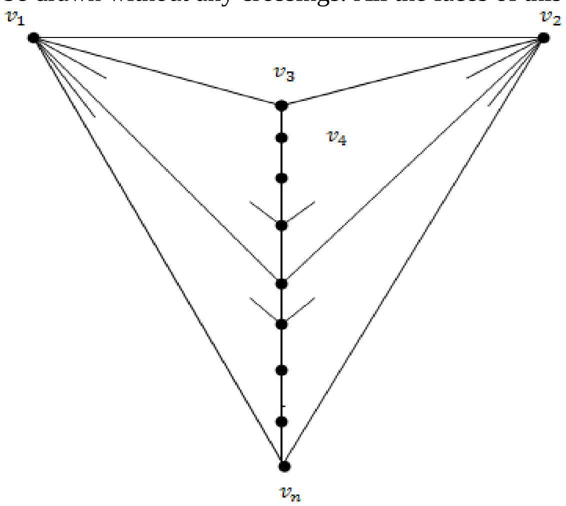

Figure 1: The class Pl_n

Theorem 2.2: *The class* Pl_n *of planar graphs is separation cordial, for $n \geq 5$.*

Proof: Consider the planar class $Pl_n(V, E)$ with n vertices $v_1, v_2, \dots, v_n$ and $3n - 6$ edges. Define a bijection f from V to $\{1, 2, \dots, p\}$. as follows.
The labeling of the vertices are:

$$f(v_i) = i, for \begin{cases} i = 1, 2 \\ i = 3k,\ 1 \leq k \leq \left[\frac{n}{3}\right] \\ i \equiv 1\ (mod\ 3)\ and\ i = |V| \end{cases}$$

$$f(v_i) = i + 1, for\ i = 3k + 1, 1 \leq k \leq \left[\frac{n}{3}\right] - 1$$

$$f(v_i) = i - 1, for\ i = 3k + 2, 1 \leq k \leq \left[\frac{n}{3}\right] - 1$$

For even values of n, $e_f(0) = e_f(1)$
For odd values of n and not a multiple of 3, $e_f(1) = e_f(0) + 1$ and for odd values of n and a multiple of 3, $e_f(0) = e_f(1) + 1$.

Hence, in both case, $|e_f(0) - e_f(1)| \leq 1$

Therefore, class Pl_n of planar graphs is separation cordial, for $n \geq 5$.

The class $Pl_{m,n}$ of Bipartite Planar Graphs

Here, we introduce a class of planar graphs denoted by $Pl_{m,n}$ obtained from the complete bipartite graphs $K_{m,n}$ $(m, n \geq 3)$ by removing certain edges.

Definition 2.3: Let $K_{m,n}$ $(V_m\ U_n)$be the complete bipartite graph on $V_m = \{v_1, v_2, \dots, v_m\}$ and $U_n = \{u_1, u_2, \dots, u_n\}$. The class of graphs $Pl_{m,n}$ (V, E) has the vertex set $V_m \cup U_n$ and the edge set $E = E(K_{m,n}(V_m\ U_n) / \{(v_x, u_y) : 3 \leq x \leq m \text{ and } 2 \leq y \leq n - 1\}$.

The complete bipartite graphs $K_{1,n}$ $(n \geq 1)$ and $K_{2,n}$ $(n \geq 2)$ are planar.

But for $m, n \geq 3$, $K_{m,n}$ is non planar.

We construct planar graphs from $K_{m,n}$ $(m, n \geq 3)$

Construction:

Place the vertices $u_1, u_2, \dots, u_n$ in that order along a horizontal line segment with $u_1$as the left end -point and u_n as the right end-point. Place the vertices $v_m, v_{m-1}, \dots, v_3, v_2$ in that order along a vertical line segment with v_m as the bottom end-point and v_2 as the top end-point so that these entire line segment is below the horizontal line segment, where $u_1, u_2, \dots, u_n$ are placed. At last place v_1 above the horizontal line segment so that the vertices v_1, u_i, v_2*and* u_{i+1} form a face of length 4, for $1 \leq i \leq n - 1$. Join the vertices

$\boldsymbol{v_3, v_4, .., v_m}$ to $\boldsymbol{u_1}$ and $\boldsymbol{u_n}$ to form $\boldsymbol{m+n-3}$ interior faces in this graph $\boldsymbol{Pl_{m,n}}$ as shown in Figure 2. All the faces of this graph are of size 4

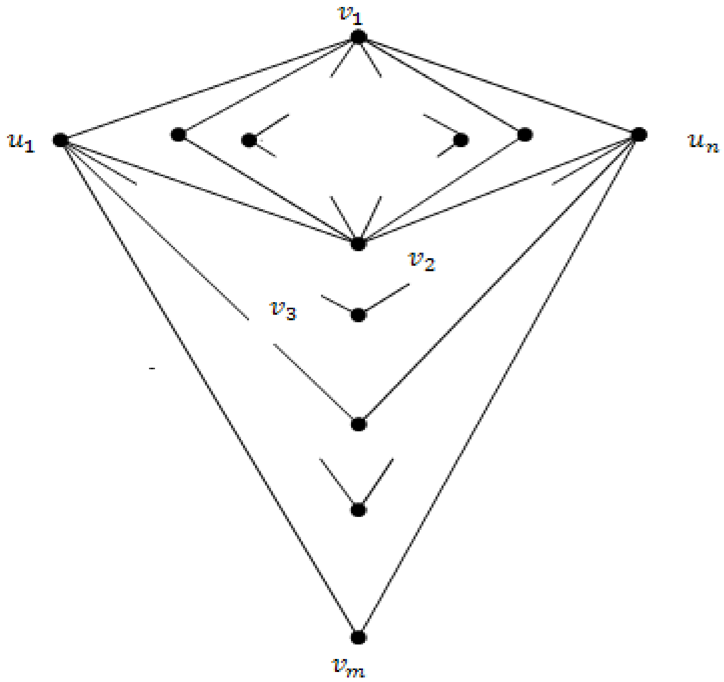

Figure 2: The class $\boldsymbol{Pl_{m,n}}$

Theorem 2.3: *The class $\boldsymbol{Pl_{m,n}}$ of planar graphs is separation cordial, for m, n ≥ 3 and at least one of m or n is even.*

Proof: Consider the planar class $\boldsymbol{Pl_{m,n}}$ $(\boldsymbol{V, E})$ with $m + n$ vertices $\boldsymbol{v_1, v_2, \ldots, v_m, u_1, u_2, \ldots, u_n}$ and $2m + 2n - 4$ edges. Define a bijection $\boldsymbol{f}$ from $\boldsymbol{V}$ to $\{\boldsymbol{1, 2, \ldots, p}\}$ as follows.
The labeling of the vertices are:

$\boldsymbol{f(u_i) = i\,,\ for\ 1 \le i \le n}$
$\boldsymbol{f(v_i) = n + i\,,\ for\ 1 \le i \le m}$
Here, $\boldsymbol{e_f(0)} = \boldsymbol{m + n - 2} = \boldsymbol{e_f(1)}$ for m, $n \geq 3$ and at least one of them is even.
Hence, $|\boldsymbol{e_f(0) - e_f(1)}| \le \boldsymbol{1}$
Therefore, the class $\boldsymbol{Pl_{m,n}}$ of planar graphs is separation cordial, for m, $n \geq 3$ and at least one of them is even.

Theorem 2.4: *If G is a separation cordial graph of even size, then G – e is also separation cordial, for all e ∈ E (G).*

Proof: Let the size of the separation cordial graph be q. Then it follows that $\boldsymbol{e_f(0) = e_f(1)} = \boldsymbol{\frac{q}{2}}$. Let e be any edge in G which is labeled as either 0 or 1. Then in $G – e$, clearly, $\boldsymbol{e_f(0)} = \boldsymbol{e_f(1) + 1}$ or $\boldsymbol{e_f(1)} = \boldsymbol{e_f(0) + 1}$ and therefore $|\boldsymbol{e_f(0) - e_f(1)}| \le \boldsymbol{1}$. Thus $G – e$ is separation cordial, for all $e \in E\,(G)$.

Theorem 2.5: *If G is a separation cordial graph of odd size, then G – e is also separation cordial for some e ∈ E (G).*

Proof: Let the size of the separation cordial graph be q. Then it follows that $\boldsymbol{e_f(0) = e_f(1) + 1}$ or $\boldsymbol{e_f(1) = e_f(0) + 1}$. If $\boldsymbol{e_f(0)} = \boldsymbol{e_f(1)} + 1$, then removing the edge $\boldsymbol{e}$ which is labeled as 0 is also separation cordial. If $\boldsymbol{e_f(1) = e_f(0) + 1}$, then removing the edge $\boldsymbol{e}$ which is labeled as 1 is also separation cordial. Then, in both case, $\boldsymbol{e_f(0) = e_f(1)}$. But if we remove the edge $\boldsymbol{e}$ which is labeled as 1 in $\boldsymbol{e_f(0)} = \boldsymbol{e_f(1)} + 1$, changes the graph $\boldsymbol{G - e}$ into a non separation cordial graph. Similarly the latter case also. Therefore, $\boldsymbol{G - e}$ is also separation cordial for some $\in \boldsymbol{E\,(G)}$.

Definition 2.4: An *ordered rooted tree* is a binary tree if each vertex has at most two children.

Definition 2.5: A *full binary tree* is a binary tree in which each vertex has exactly two children.

Theorem 2.6: *Every full binary tree is separation cordial.*

Proof: We have every full binary tree has odd number of vertices and hence has even number of edges. Let $\boldsymbol{T}$ be a full binary tree and let $\boldsymbol{v}$ be a root of $\boldsymbol{T}$ which is called zero level vertex. Also, the $\boldsymbol{i^{th}}$ level of $\boldsymbol{T}$ has $\boldsymbol{2^i}$ vertices. If $\boldsymbol{T}$ has $\boldsymbol{n}$ levels, then number of vertices of $\boldsymbol{T}$ is $\boldsymbol{2^{n+1} - 1}$ and the number of edges is $\boldsymbol{2^{n+1} - 2.}$

Now, assign the label 1 to the root $\boldsymbol{v}$. Next, we assign the labels $\mathbf{2^i}$, $\mathbf{2^i+1},\ldots,$ $\mathbf{2^{i+1}-1}$ to the $\boldsymbol{i^{th}}$ level vertices for $\mathbf{1} \le \boldsymbol{i} \le \boldsymbol{n}$. Then, upto the $\boldsymbol{i^{th}}$ level edges, $\boldsymbol{e_f(0)} = \mathbf{2^i-1}$ and $\boldsymbol{e_f(1)} = \mathbf{2^i-1}$, for $\mathbf{1} \le \boldsymbol{i} \le \boldsymbol{n}$.
Thus $|\boldsymbol{e_f(0)} - \boldsymbol{e_f(1)}| \le \mathbf{1}$ and therefore $\boldsymbol{T}$ is separation cordial.

Theorem 2.7: *The star graph $K_{1,q}$ is separation cordial.*

Proof: The star graph $K_{1,q}$ has $q+1$ vertices and q edges. Now assign the label 1 to the centre of the star graph. Next we assign the labels 2 to $\boldsymbol{q}+\mathbf{1}$ to the other $\boldsymbol{q}$ vertices in any order. If $\boldsymbol{q}$ is even $e_f(0) = e_f(1) = \frac{q}{2}$ and if q is odd, $e_f(1) = \frac{q+1}{2}$ and $e_f(0) = \frac{q-1}{2}$. Thus $|\boldsymbol{e_f(0)} - \boldsymbol{e_f(1)}| \le \mathbf{1}$ always. Hence $K_{1,q}$ is separation cordial.

Theorem 2.8: *The complete graph K_n is not separation cordial, for $n \ge 4$.*

Proof: The complete graph $\mathbf{K_n}$ has n vertices $\mathbf{v_1, v_2, \ldots, v_n}$ and $\frac{\mathbf{n\,(n-1)}}{\mathbf{2}}$ edges. Define a bijection $\boldsymbol{f}$ from $\boldsymbol{V}$ to $\{\mathbf{1,2,\ldots,\boldsymbol{p}}\}$ as follows.

The labeling of the vertices are: $\boldsymbol{f(v_i) = i\,,\ for\ 1 \le i \le n}$.

Case (i) suppose n is even

First calculate $e_f(1)$. The edges $\boldsymbol{v_1v_{2i}}$, for $\mathbf{1} \le \boldsymbol{i} \le \frac{n}{2}$ gives $e_f(1) = \frac{n}{2}$.
The edges $\boldsymbol{v_2v_{1+2i}}$, for $\mathbf{1} \le \boldsymbol{i} \le \frac{n}{2}-\mathbf{1}$ gives $e_f(1) = \frac{n}{2}-\mathbf{1}$. The edges $\boldsymbol{v_3v_{2+2i}}$, for $\mathbf{1} \le \boldsymbol{i} \le \frac{n}{2}-\mathbf{1}$ gives $e_f(1) = \frac{n}{2}-\mathbf{1}$. The edges $\boldsymbol{v_4v_{3+2i}}$, for $\mathbf{1} \le \boldsymbol{i} \le \frac{n}{2}-\mathbf{2}$ gives $e_f(1) = \frac{n}{2}-\mathbf{2}$ and so on. At last the edges $\boldsymbol{v_{n-1}v_{n-2+2i}}$, for $\mathbf{1} \le \boldsymbol{i} \le \frac{n}{2}-(\frac{n}{2}-\mathbf{1})$ gives $e_f(1) = \frac{n}{2}-(\frac{n}{2}-\mathbf{1})$.

Adding all the values of $e_f(1)$, we get,

$$e_f(1) = \frac{n}{2} + (\frac{n}{2}-\mathbf{1}) + (\frac{n}{2}-\mathbf{1}) + (\frac{n}{2}-\mathbf{2}) + \left(\frac{n}{2}-\mathbf{2}\right) + \ \ldots + \left[\frac{n}{2} - (\frac{n}{2}-\mathbf{1})\right]$$

$$= (\boldsymbol{n}-\mathbf{1})\frac{n}{2} - \left[\mathbf{0+1+1+\ 2+2}+\cdots+\left(\frac{n}{2}-\mathbf{1}\right)+\left(\frac{n}{2}-\mathbf{1}\right)\right]$$

$$= (\boldsymbol{n}-\mathbf{1})\frac{n}{2} - \left[\mathbf{2}\ (\mathbf{1+2}+\cdots+\left(\frac{n}{2}-\mathbf{1}\right)\right]$$

$$= (\boldsymbol{n}-\mathbf{1})\frac{n}{2} - \left[\mathbf{2}\left[\frac{(\frac{n}{2}-\mathbf{1})\frac{n}{2}}{\mathbf{2}}\right]\right]$$

$$= (\boldsymbol{n}-\mathbf{1})\frac{n}{2} - \left[(\frac{n}{2}-\mathbf{1})\frac{n}{2}\right]$$

$e_f(1) = \frac{n^2}{4}$
We have, $e_f(0) + e_f(1) = \frac{n(n-1)}{2}$

$$e_f(0) = \frac{n(n-1)}{2} - e_f(1) = \frac{n(n-1)}{2} - \frac{n^2}{4} = \frac{n^2}{4} - \frac{n}{2}.$$

Thus, $|\boldsymbol{e_f(0)} - \boldsymbol{e_f(1)}| \ge \mathbf{2}$, for $n \ge 4$.

Therefore, the complete graph K_n is not separation cordial, for even values of $n \ge 4$

Case (ii) suppose n is odd.

As above, calculate $e_f(1)$. The edges $\boldsymbol{v_1v_{2i}}$, for $\mathbf{1} \le \boldsymbol{i} \le \frac{n-1}{2}$ gives $e_f(1) = \frac{n-1}{2}$. The edges $\boldsymbol{v_2v_{1+2i}}$, for $\mathbf{1} \le \boldsymbol{i} \le \frac{n-1}{2}$ gives $e_f(1) = \frac{n-1}{2}$. The edges $\boldsymbol{v_3v_{2+2i}}$, for $\mathbf{1} \le \boldsymbol{i} \le \frac{n-1}{2}-\mathbf{1}$ gives $e_f(1) = \frac{n-1}{2}-\mathbf{1}$. The edges $\boldsymbol{v_4v_{3+2i}}$, for $\mathbf{1} \le \boldsymbol{i} \le \frac{n-1}{2}-\mathbf{1}$ gives $e_f(1) = \frac{n-1}{2}-\mathbf{1}$ and so on. At last the edges $\boldsymbol{v_{n-1}v_{n-2+2i}}$, for $\mathbf{1} \le \boldsymbol{i} \le \frac{n-1}{2}-(\frac{n-1}{2}-\mathbf{1})$ gives $e_f(1) = \frac{n-1}{2}-(\frac{n-1}{2}-\mathbf{1})$.

Adding all the values of $e_f(1)$, we get,

$$e_f(1) = \frac{n-1}{2} + \frac{n-1}{2} + (\frac{n-1}{2}-\mathbf{1}) + (\frac{n-1}{2}-\mathbf{1}) + \cdots, + \left[\frac{n-1}{2} - (\frac{n-1}{2}-\mathbf{1})\right]$$

$$= (\boldsymbol{n}-\mathbf{1})\ (\frac{n-1}{2}) - \left[\mathbf{0+0+1}+\cdots+\left(\frac{n-1}{2}-\mathbf{1}\right)+\left(\frac{n-1}{2}-\mathbf{1}\right)\right]$$

$$= (n-1)\left(\frac{n-1}{2}\right) - \left[2\left(1+2+\cdots+\left(\frac{n-1}{2}-1\right)\right)\right]$$

$$= (n-1)\left(\frac{n-1}{2}\right) - \left[2\left[\frac{\left(\frac{n-1}{2}-1\right)\left(\frac{n-1}{2}\right)}{2}\right]\right]$$

$$= (n-1)\left(\frac{n-1}{2}\right) - \left[\left(\frac{n-1}{2}-1\right)\left(\frac{n-1}{2}\right)\right]$$

$e_f(1) = \frac{n^2-1}{4}$

We have, $e_f(0) + e_f(1) = \frac{n(n-1)}{2}$

$$e_f(0) = \frac{n(n-1)}{2} - e_f(1) = \frac{n(n-1)}{2} - \left(\frac{n^2-1}{4}\right) = \frac{n^2+1}{4} - \frac{n}{2}.$$

Thus, $|e_f(0) - e_f(1)| \geq 2$, for $n \geq 4$.

Therefore, the complete graph K_n is not separation cordial, for odd values of $n \geq 4$.

Hence, the complete graph K_n is not separation cordial, for $n \geq 4$.

Theorem 2.9: *The complete bipartite graph $K_{m,n}$ is separation cordial.*

Proof: The complete bipartite graph $K_{m,n}$ has $m+n$ vertices $v_1, v_2, \ldots, v_m$, u_1 $u_2, \ldots, u_n$, and mn edges. Define a bijection f from V to $\{1, 2, \ldots, p\}$ as follows.

The labeling of the vertices are:

$f(v_i) = i,\ for\ 1 \leq i \leq m$
$f(u_i) = m + i,\ for\ 1 \leq i \leq n.$

Here, $e_f(0) = e_f(1) = \frac{mn}{2}$, for at least one of m or n is even and $e_f(0) = \frac{mn-1}{2}$, $e_f(1) = \frac{mn+1}{2}$, for both m and n are odd. Hence, $|e_f(0) - e_f(1)| \leq 1$, for all values of m and n. Therefore, the complete bipartite graph $K_{m,n}$ is separation cordial.

Definition 2.6: The number of edges in a path is called the *length of the path*. A path of length n is called n – path and is denoted by P_{n+1}.

Theorem 2.10: *The path P_n is separation cordial.*

Proof: The path P_n has n vertices $v_1, v_2, \ldots, v_n$ and $n - 1$ edges. Define a bijection f from V to $\{1, 2, \ldots, p\}$. The path P_1 and P_2 are separation cordial trivially. In P_3, by labeling the vertices v_1, v_2 and v_3 as 1, 3 and 2, it is clear the separation cordiality. Place the vertices $v_1, v_2, \ldots, v_n$ in such a way that $v_i v_{i+1}$, for $1 \leq i \leq n - 1$.

The labeling of the vertices for $n \geq 4$ are

$$f(v_i)\begin{cases} i, for\ i \equiv 0, 1\ (mod\ 4) \\ i, for\ i \equiv 2\ (mod\ 4) and\ i = |v| \\ i+1, for\ i \equiv 2\ (mod\ 4) and\ i < |v| \\ i-1, for\ i \equiv 3\ (mod\ 4) \end{cases}$$

Here, $e_f(0) = \frac{n}{2}$, $e_f(1) = \frac{n}{2} - 1$, for $n \equiv 0\ (mod\ 4)$
$e_f(0) = \frac{n-1}{2}$, $e_f(1) = \frac{n-1}{2}$, for $n \equiv 1, 3\ (mod\ 4)$
$e_f(0) = \frac{n}{2} - 1$, $e_f(1) = \frac{n}{2}$, for $n \equiv 2\ (mod\ 4)$

Thus, $|e_f(0) - e_f(1)| \leq 1$

Therefore, the path graph P_n is separation cordial.

Theorem 2.11: *The cycle C_n $(n \geq 3)$ is separation cordial, where n is not congruent to 2 mod 4.*

Proof: The cycle C_n has n vertices $v_1, v_2, \ldots, v_n$ and n edges. Define a bijection f from V to $\{1, 2, \ldots, p\}$. Place the vertices $v_1, v_2, \ldots, v_n$ in such a way that $v_i v_{i+1}$, for $1 \le i \le n-1$ and $v_1 v_n$ are adjacent. The labeling of the vertices for $n = 3$ are $f(v_i) = i$, $for\ 1 \le i \le 3$. Therefore, $|e_f(0) - e_f(1)| \le 1$.

The labeling of the vertices for $n \ge 4$ are

$f(v_1) = 1$.

$$f(v_i)\begin{cases} i, for\ i \equiv 0, 1\ (mod\ 4) \\ i+1, for\ i \equiv 2\ (mod\ 4) and\ i < |v| \\ i-1, for\ i \equiv 3\ (mod\ 4) \end{cases}$$

Here, $e_f(0) = \frac{n}{2}$, $e_f(1) = \frac{n}{2}$, for $n \equiv 0\ (mod\ 4)$

$e_f(0) = \frac{n+1}{2}$, $e_f(1) = \frac{n-1}{2}$, for $n \equiv 1\ (mod\ 4)$

$e_f(0) = \frac{n-1}{2}$, $e_f(1) = \frac{n+1}{2}$, for $n \equiv 3\ (mod\ 4)$

If $n \equiv 2\ (mod\ 4)$ $|e_f(0) - e_f(1)| \ge 2$.

Therefore, the cycle C_n $(n \ge 3)$ is separation cordial,where n is not congruent to 2 mod 4.

Definition 2.7: Consider t copies of stars namely ${K^{(1)}}_{1,n}, {K^{(2)}}_{1,n}, \ldots, {K^{(t)}}_{1,n}$. Then G = $\langle {K^{(1)}}_{1,n}, {K^{(2)}}_{1,n}, \ldots, {K^{(t)}}_{1,n} \rangle$ is the graph obtained by joining apex (central) vertices of each ${K^{(i)}}_{1,n}$ and ${K^{(i+1)}}_{1,n}$ to new vertex x_i, where $1 \le i \le t-1$.

Theorem 2.12: *The graph* G = $\langle {K^{(1)}}_{1,n}, {K^{(2)}}_{1,n}, \ldots, {K^{(t)}}_{1,n} \rangle$ *is separation cordial.*

Proof: The graph G has $t(n+2) - 1$ vertices $v_1, v_2, \ldots, v_{t(n+2)-1}$ and $t(n+2) - 2$ edges. Define a bijection f from V to $\{1, 2, \ldots, p\}$. Let v_i be the apex (central) vertices of ${K^{(i)}}_{1,n}$, for $1 \le i \le t$. Then place $v_{t+(m-1)(n+1)+i}$, $1 \le i \le n$, $1 \le m \le t$ be the vertices of ${K^{(m)}}_{1,n}$. Also, ${K^{(i)}}_{1,n}$ and ${K^{(i+1)}}_{1,n}$ are adjacent to the common vertex $v_{t+(n+1)i}$, for $1 \le i \le t-1$.

The labeling of the vertices, $f(v_i) = i$, for $1 \le i \le |V|$.

Here, $e_f(0) = e_f(1) = \frac{t(n+2)-2}{2}$, if at least one of t or n is even.

$e_f(0) = \frac{t(n+2)-3}{2}$ and $e_f(1) = \frac{t(n+2)-1}{2}$ if both t and n are odd.

Therefore, $|e_f(0) - e_f(1)| \le 1$ in both case. Hence, graph G is separation cordial.

3. Observations

Observation 1: The class Pl_n *(n ≥ 5)* of planar graphs are P_4- packable [9], but not randomly **[10].**

Observation 2: The class $Pl_{m,n}$ *(m, n* ≥ 4 and even*)* of bipartite planar graphs are randomly C_4- packable **[10]**.

Observation 3: The complete graph K_n is separation cordial only for n = 1, 2 and 3

4. Conclusion

The class Pl_n *(n ≥ 5),* $Pl_{m,n}$ *(m, n ≥ 3, at least one of m or n is even), full binary tree, the star graph* $K_{1,q}$*, the complete bipartite graph* $K_{m,n}$*, path graph* P_n*, are separation cordial. The cycle graph* C_n*, where n is not congruent to 2 mod 4 are separation cordial under certain conditions. But the complete graph* K_n *(n ≥ 4) is not separation cordial.*

5. References

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