



Soret and Dufour effects on Darcy–Forchheimer MHD mixed convection in a fluid saturated porous media with viscous dissipation and thermophoresis

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ABSTRACT

An analysis is presented to investigate the Dufour, Soret and thermophoresis parameter effects on MHD Mixed convection, heat and mass transfer about an isothermal vertical flat plate embedded in a fluid-saturated porous medium in the presence of viscous dissipation. The similarity solution is used to transform the problem under consideration into a boundary value problem of coupled ordinary differential equations, which are solved numerically by using the finite difference method. Numerical computations are carried out for the non-dimensional physical parameter. The results are analyzed for the effect of different physical parameters such as Dufour number, Soret number, thermophoretic, MHD, mixed convection, Eckert number, inertia parameter, buoyancy ratio, and Schmid number on the flow, heat, and mass transfer characteristics.

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Introduction

The problems of mixed convective MHD flows are of prime importance in a number of industrial applications in Geophysical and Astro-Physical situations. The problem of convective MHD flows has wide range of publications in emerging fields viz granular insulation, geothermal systems in heating and cooling chambers, fossil fuel, combustion, energy process, solar energy and space vehicle re entry. Some examples in living organisms are fluid transport mechanism among which the blood flow in circulatory system, with flow in airways. A classical example is in nuclear power station where the separation of uranium U235 from U238 by gases diffusion takes place. Over the past two decades, studies in aerosol particle deposition due to thermophoresis have gained importance for engineering applications. The technological problems include particle deposition onto wafers in the microelectronics industry, particle surfaces produced by condensing vapor–gas mixtures, particles impacting the blade surface of gas turbines, and others such as filtration in gas cleaning and nuclear reactor safety. In engineering particle, usually more than one mechanism can act simultaneously and their interactions need to be considered for accurate prediction of deposition rates. In this work, the mechanism of particle deposition onto a vertical surface by the coupled effects of viscous dissipation, mixed convection, and thermophoresis is examined. Most of the research efforts [1], [2], [3] and [4] concerned free convection using Darcy's law, which states the volume-averaged velocity is proportional to the pressure gradient.

The Darcy model is shown to be valid under conditions of low velocities and small porosity. In many practical situations, the porous medium is bounded by an impermeable wall, has high flow rates, and reveals non uniform porosity distribution in the near-wall region, thereby making Darcy's law inapplicable. To model the real physical situations better, it is therefore necessary

to include the non-Darcian effects in the analysis of convective transport in a porous medium. Small particles, such as dust, when suspended in a gaseous medium possessing a temperature gradient, it will move in the direction opposite to the temperature gradient. This motion is known as thermophoresis, occurs because gas molecules colliding on one side of a particle have different average velocities from those on the other side due to the temperature gradient. This phenomenon has been the subject of considerable study in the past. In optical fiber synthesis, thermophoresis has been identified as the principal mechanism of mass transfer as used in the technique of modified chemical vapor deposition (MCVD) [5]. Thermophoresis a gaseous mixture of reactive precursors is directed over a heated substrate where solid film deposits are located. In particular, the mathematical modeling of the deposition of silicon thin films using MCVD methods has been accelerated by the quality control measures enforced by the micro-electronics industry. Such topics involve a variety of complex fluid dynamical processes including thermophoretic transport of particles deposits, heterogeneous/homogeneous chemical reactions, homogeneous particulate nucleation and coupled heat and energy transfer. The mass transfer caused by the temperature gradient is called Soret effect, while the heat transfer caused by the concentration gradient is called Dufour effect. Eckert and Drake [6] presented several cases of Dufour effect. Weaver and Viskanta [7] have pointed out that when the differences of the temperature and the concentration are large or when the difference of the molecular mass of the two elements in a binary mixture is great, the coupled interaction is significant. A primary discussion on the effect of the cross-coupled diffusion in a system with horizontal temperature and concentration a gradient was made by Malashev and Gaikad [8].

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Due to the importance of Soret (thermal-diffusion) and Dufour (diffusion-thermo) effects for the fluids with very light molecular weight as well as medium molecular weight many investigators have studied and reported results for these flows of whom the names are Dursunkaya and Worek [9], Anghel and Takhar [10], Postelnicu [11] are worth mentioning. Very recently, Alam and Rahman [12] studied the Dufour and Soret effects on steady MHD free convective heat and mass transfer flowpast a semi-infinite vertical porous plate embedded in a porous medium. Combined heat and mass transfer problems with chemical reaction are of importance in many processes and have, therefore, received a considerable amount of attention in recent years. Yih[13] presented an analysis of the forced convection boundary layer flow over a wedge with uniform suction/blowing, whereas Watanabe [14] investigated the behaviour of the boundary layer over a wedge with suction injection in forced flow. Anjali Devi and Kandasamy [15] studied effects of chemical reaction, heat and mass transfer on non –linear MHD laminar boundary layer flow over a wedge with suction and injection. The problem of Darcy–Forchheimer mixed convection heat and mass transfer in fluid-saturated porous media was studied by Rami et al. [16]. Goren [17] was one of the first to study the role of thermophoresis in the laminar flow of a viscous and incompressible fluid. He used the classical problem of flow over a flat plate to calculate deposition rates and showed that substantial changes in surface deposition can be obtained by increasing the difference between the surface and free stream temperatures. This was later followed by the effect of thermophoresis on particle deposition from a mixed convection flow onto a vertical plate by Chang et al. [18] and Jayaraj et al. [19]. Also, Tsai [20] obtained the effect of wall suction and thermophoresis on aerosol particle deposition from a laminar flow over a flat plate.

Hence, in this paper we aim at analyzing the influence of MHD on mixed convection flow, heat and mass transfer about an isothermal vertical plate embedded in a fluid saturated porous medium and the effects of viscous dissipation and thermophoresis in both aiding and opposing flows. Thermophoresis is also a key mechanism of study in semiconductor technology, especially controlled high-quality wafer production as well as in radioactive particle deposition in nuclear reactors safety simulations and MHD energy generation system operations. A number of analytical and experimental papers in thermophoretic heat and mass transfer have been communicated. The thermophoretic flow of larger diameter particles was investigated by Kanki et al. [21]. Talbot et al. [23] presented a seminal study, considering boundary layer flow with thermophoretic effects, which has become a benchmark for subsequent studies. Recently, N.Kishan et al.[24], studied the influence of viscous dissipation and thermophoresis in Darcy-forcheimer MHD mixed convection heat and mass transfer in fluid –saturated porous media.

Mathematical formulation:

Consider the steady mixed convection boundary layer over a vertical flat plate of constant temperature T_w and concentration C_w , which is embedded in a fluid-saturated porous medium of ambient temperature T_∞ and concentration C_∞ , respectively. The x-coordinate is measured along the plate from its leading edge and the y-coordinate normal to it. Allowing for both Brownian motion of particles and thermophoretic transport, the governing boundary layer equations are

$$\frac{\partial u + \partial v}{\partial x \partial y} = 0 \tag{1}$$

$$[1 + \frac{\sigma B_0^2 K}{\rho v}] \frac{\partial u}{\partial y} + c_f \sqrt{K_1} \frac{\partial u^2}{\partial y} = \mp g \left(\frac{K_1}{v} \right) \left[\beta_T \frac{\partial T}{\partial y} + \beta_C \frac{\partial C}{\partial y} \right] \tag{2}$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{\lambda_g}{\rho c_p} \frac{\partial^2 T}{\partial y^2} + \frac{v}{c_p} \left(\frac{\partial u}{\partial y} \right)^2 + \frac{D_m K_T}{C_s C_p} \left(\frac{\partial^2 C}{\partial y^2} \right) \tag{3}$$

$$u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D \frac{\partial^2 C}{\partial y^2} - \frac{\partial}{\partial y} (V_T C) + \frac{D_m K_T}{T_m} \left(\frac{\partial^2 T}{\partial y^2} \right) \tag{4}$$

The boundary conditions are given by $y=0, v = 0, T = T_w, C = C_w, y \rightarrow \infty, u = u_\infty, T = T_\infty, C = C_\infty$ (5)

where u, v are velocity components along x, y coordinates, respectively, T and C are, respectively, the temperature and concentration, c_f is the Forchheimer coefficient, K_1 is the Darcy permeability, g is the acceleration due to gravity, v is the kinematic viscosity, β_T is the coefficient of thermal expansion, β_C is the coefficient of concentration expansion, C_p is the specific heat of the fluid at constant pressure, q_r is the radiative heat flux, and D is the mass diffusivity. In Eq. (2), the plus sign corresponds to the case where the buoyancy force has a component “aiding” the forced flow and the minus signs refer to the “opposing” case.

In Eq. (4), the thermophoretic velocity V_T was given by [22]

$$V_T = -k v \frac{\Delta T}{T} = -k v \frac{\partial T}{\partial y} \tag{6}$$

where k is the thermophoretic coefficient, which is given by [23] as

$$k = \frac{2 C_s (\lambda_g / \lambda_p + C_t K_n) C_1}{(1 + 3 C_m K_n) (1 + 2 \lambda_g / \lambda_p + 2 C_t K_n)} \tag{7}$$

Where $C_m, C_s,$ and C_t are constants and λ_g and λ_p are the thermal conductivities of the fluid and diffused particles, respectively. C_1 is the Cunningham correction factor and K_n is the Knudsen number.

Now we define the following dimensionless variables for mixed convection

$$\eta = \frac{\gamma P}{x} e_x^{1/2}, \psi = \alpha P e_x^{1/2} f(\eta), \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}, \varphi(\eta) = \frac{C - C_w}{C_w - C_\infty} \tag{8}$$

where ψ is the stream function that satisfies the continuity equation and η is the dimensionless similarity variable. With these changes of variables, Eq. (1) is identically satisfied and Eqs. (2), (3) and (4) are transformed to

$$(1 + Ha^2) f'' + 2 \Lambda f' f'' = \mp \frac{Ra_x}{Pe_x} (\theta' + N \varphi') \tag{9}$$

$$\theta'' + \frac{1}{2} f \theta' + Pr Ec (f'')^2 + D_f \varphi'' = 0 \tag{10}$$

$$\frac{1}{Sc} \varphi'' - \tau (\varphi \theta'' + \theta' \varphi') + \frac{1}{2 Pr} f \varphi' + Sr \theta'' = 0 \tag{11}$$

The corresponding boundary conditions take the form $f(0)=0, \theta(0)=1, \varphi(0)=1, f'(\infty)=1, \theta(\infty)=0, \varphi(\infty)=0$ (12)

where the primes denote differentiation with respect to $\eta, \Lambda = C_f \sqrt{K_1} u_\infty / v$ is the inertia parameter, $Ra_x = (K_1 g \beta_T) \frac{(T_w - T_\infty) x}{\alpha v}$ is the thermal Rayleigh number, $Pe_x = \frac{u_\infty x}{\alpha}$ is the local Peclet number,

$N = \frac{\beta_C (C_w - C_\infty)}{\beta_T (T_w - T_\infty)}$ is the buoyancy ratio,

$\tau = -K \frac{(T_w - T_\infty)}{T}$ is the thermophoretic parameter, and

$Ec = u_\infty^2 / C_p (T_w - T_\infty)$ is the Eckert number.

$D_f = \frac{D_m K_T}{C_s C_p}$ is the Dufour number, $Sr = \frac{D_m K_T}{T_m}$ is the Soret number

The important physical quantities of our interest are the Nusselt number Nu_x and Sherwood number Sh_x . These can be defined as follows:

$$Nu_x = \left(\frac{q_w x}{(T_w - T_\infty)k} \right) = -1/2 P e_x^{1/2} \theta'(0), q_w = -k \left(\frac{\partial T}{\partial y} \right)_{y=0} \quad (13)$$

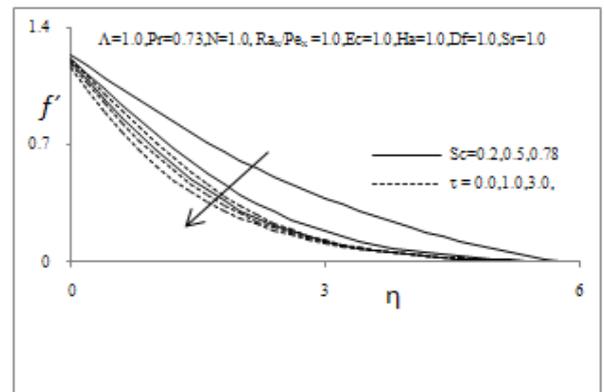
$$Sh_x = \left(\frac{J_w x}{(C_w - C_\infty)D} \right) = -P e_x^{1/2} \varphi'(0), J_w = -D \left(\frac{\partial C}{\partial y} \right)_{y=0} \quad (14)$$

The governing non dimensional system of equations (9) to (11) along with the boundary conditions (12) are solved using a numerical perturbation method referred to as the method of successive linearization Technique Bellman R.E et al. [25].

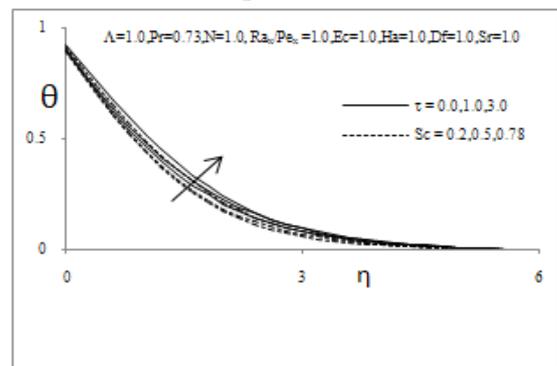
Results and discussion:

The set of non-linear ordinary differential equations (9)-(11) with boundary conditions in(12) have been solved numerically by using the implicit finite difference scheme and analyzed graphically. In this paper computations are carried out for different flow parameters such as inertia parameter Λ , Thermal Rayleigh number Ra_x , local polet number Pe_x , buoyance ratio N , Schmidt number Sc , thermophoretic parameter τ , Eckert number Ec , Soret number Sr , Dufour number D_f and Magnetic parameter Ha . In addition the boundary condition $\eta \rightarrow \infty$ is approximated by $\eta_{max}=6$; which is sufficiently large for the velocity to approach the relevant stream velocity. Figures 1-5 represent typical velocity, temperature and concentration profiles for various values of different flow parameters. Fig.1 (a)-1(c) demonstrates the effects of thermophoretic parameter τ and Schmidt number Sc on velocity, temperature and concentration profiles. The Schmidt number Sc values are chosen as $Sc = 0.2$ (Hydrogen), $Sc = 0.6$ (water weeper), $Sc = 0.78$ (Ammonia), which represents diffusing chemical species of most common interest in air at 20°C and one atmosphere pressure. From fig.1 (a) an increasing of thermophoretic parameter τ and Schmidt number Sc lead to a decreasing in the velocity profiles. The influence of thermophoretic parameter τ on temperature profiles is shown in fig.1 (b). It is noticed from the figure that increasing thermophoretic parameter τ leads to slidely increase fluid temperature and a rise in the Schmidt number Sc from 0.2 – 0.78 leads to increases in temperature profiles. The influence of thermophoretic parameter τ and the Schmidt number Sc are represented in fig 1(c). From this figure the effect of thermophoretic parameter τ and the Schmidt number Sc are decelerate the concentration profiles. Figures 2(a)-2(c) depicts the influence of mixed convection parameter Ra_x/Pe_x on the velocity, temperature and concentration profiles. The mixed convection parameter Ra_x/Pe_x on the velocity profile is shown in fig.2(a) and it is observed that the velocity profile increases with the increasing of mixed convection parameter Ra_x/Pe_x . The temperature profile decreases with the increase of mixed convection parameter Ra_x/Pe_x is noticed in fig.2(b).From the fig.2(c), it is noticed that the effect of mixed convection parameter Ra_x/Pe_x is to increases the concentration profiles. The effect of inertia parameter Λ and buoyance parameter N on velocity, temperature and concentration profiles are shown in figure 3(a)-3(c). The velocity profiles increases with the increase of buoynce parameter N and inertia parameter Λ is noticed from the fig. 3(a). The temperature profiles are drawn for the different values of buoynce parameter N and inertia parameter Λ in fig.3 (b). It is observed that the effect of byounce ratio parameter N is leads to decrease the temperature profiles and that an increase in the inertia parameter Λ is leads to increase the fluid temperature. The influence of inertia parameter Λ and the influence of buoynce parameter N is represented in fig. 3 (c) . From this figure the effect of buoynce parameter N is decelerates the concentration profiles. and the concentration profiles increases with increase of inertia parameter Λ . The influence of viscous dispersion and magnetic parameter Ha effects on velocity profiles are shown in fig.4(a)-4(c). From the figure 4(a), An increase in the viscous dispersion parameter results in an

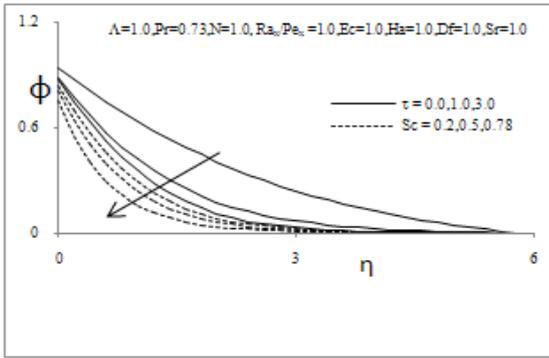
increasing of the velocity profiles is observed and an increase the magnetic parameter Ha , decreases the Hydro magnetic boundary layer causing the reduced the fluid velocity. The viscous dispersion and magnetic parameter effects on temperature profiles are shown in fig.4(b). With the effect of viscous dispersion and Maganetic parameter Ha there is a significant increase in the temperature profiles is noticed. From figure 4(c) same phenomenon it is observed. Fig.5 (a)-5(c). Shows the effect of Dufour number D_f and soret number Sr . Fig.5(a) and 5(b) it is observed that an increasing of Dufour number D_f and soret number Sr leads to decreasing in the velocity profiles and temperature profiles and reverse phenomenon observed in figure 5(c). The heat and mass transfer results in terms of $Nu_x / Pe_x^{1/2}$ and $Sh_x / Pe_x^{1/2}$ as functions of the mixed convection parameter Ra_x / Pe_x with and without viscous dissipation effect at different values of thermophoretic parameter are displayed in figures 6 and 7. While τ increasing $Nu_x / Pe_x^{1/2}$ decreasing and $Sh_x / Pe_x^{1/2}$ increasing in the presence and absence Ec . It is worth pointing out that at $Ec \neq 0$, the effect of τ on $Nu_x / Pe_x^{1/2}$ is more than in the case of $Ec = 0$ and the opposing phenomena occurs with $Sh_x / Pe_x^{1/2}$. Also, as Ec increases, $Nu_x / Pe_x^{1/2}$ and $Sh_x / Pe_x^{1/2}$ decrease greatly. On the other hand, when Ra_x / Pe_x increases $Nu_x / Pe_x^{1/2}$ and $Sh_x / Pe_x^{1/2}$ decrease in the presence of Ec , but it increases in the absence of Ec , as shown in figures 6 and 7. This is because increasing Ra_x / Pe_x increases the momentum transport in the boundary layer and more heat and mass species are carried out of the surface, thus decreasing the thickness of the thermal and concentration boundary layer and hence increasing the heat and mass transfer rates.



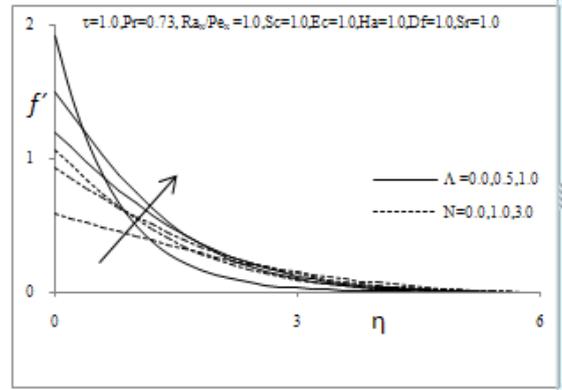
Graph 1a. Effects of Sc, τ on non-dimensional velocity profiles



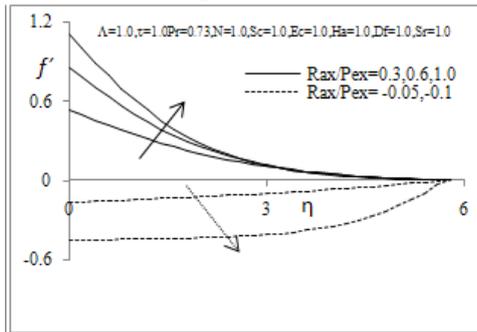
Graph 1b. Effects of Sc, τ on no-dimensional Temperature profiles



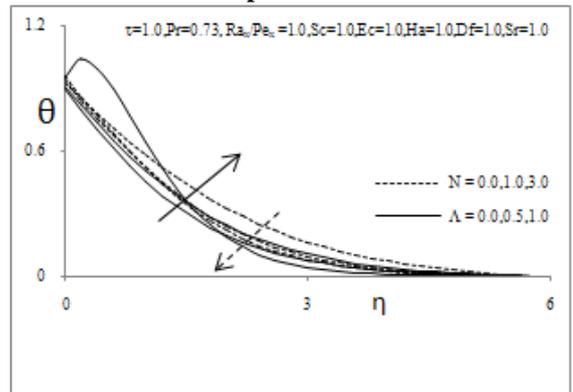
Graph 1c. Effects of Sc, τ on non-dimensional Concentration profiles



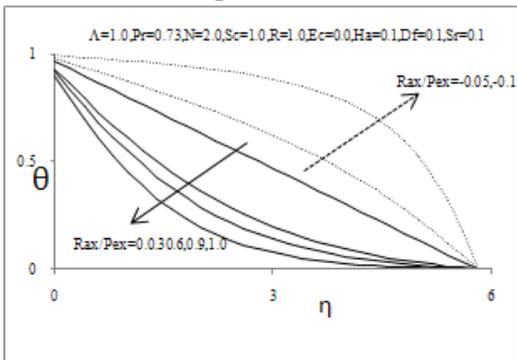
Graph 3a. Effects of Λ, N on non-dimensional velocity profiles



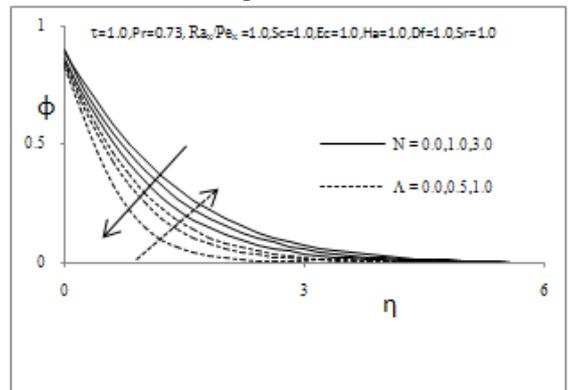
Graph 2a. Effect of Rax/PeX on non-dimensional velocity profiles



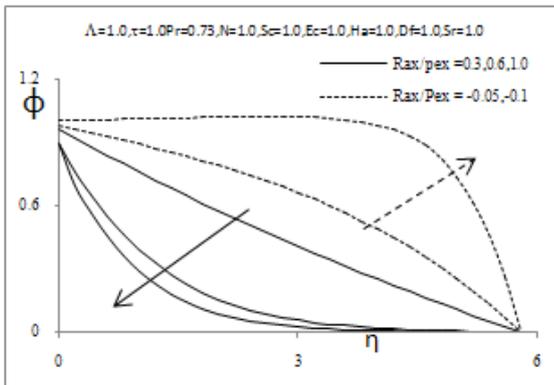
Graph 3b. Effects of N, Λ on non-dimensional Temperature profiles



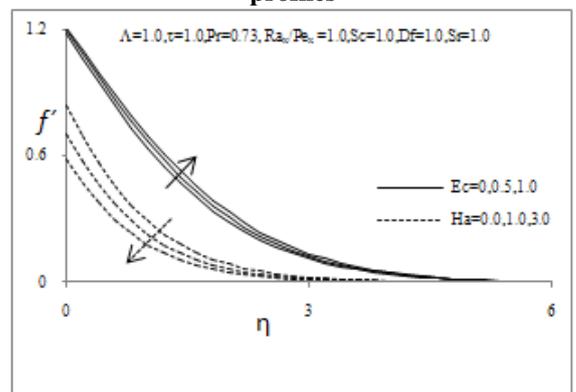
Graph 2b. Effect of Rax/PeX on non-dimensional Temperature profiles



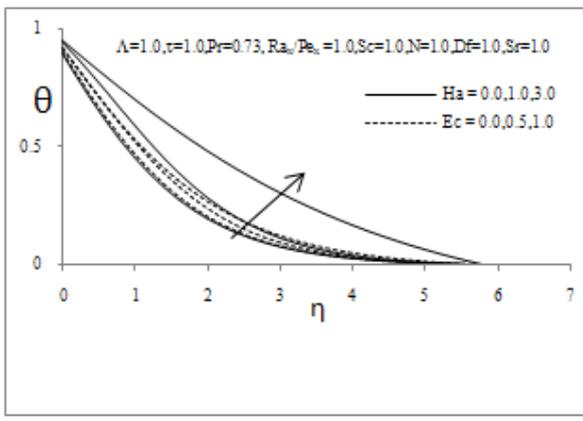
Graph 3c. Effects of N, Λ on non-dimensional Concentration profiles



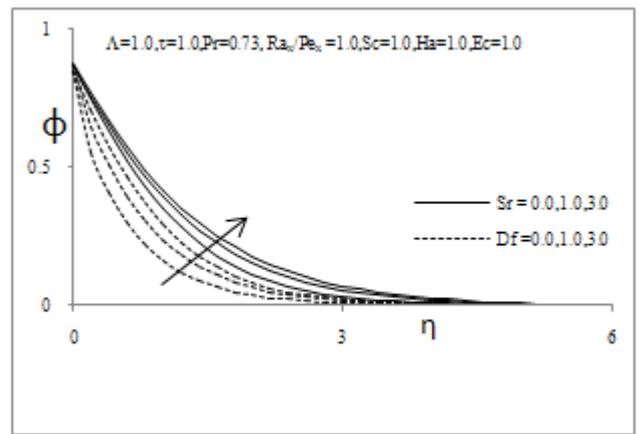
Graph 2c. Effect of Rax/PeX on non-dimensional Concentration profiles



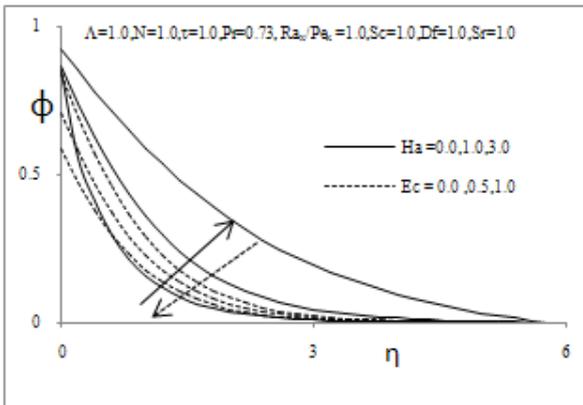
Graph 4a. Effect of Ha, Ec on non-dimensional velocity profiles



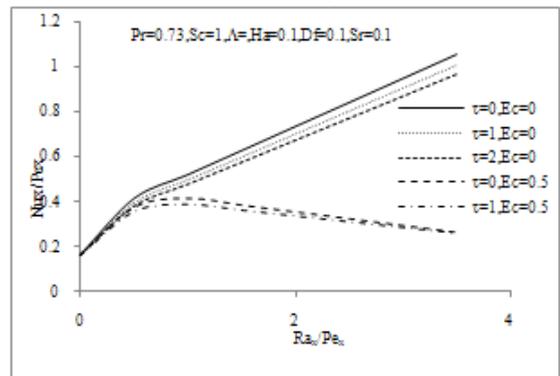
Graph 4b. Effects of Ha, Ec on non-dimensional Temperature profiles



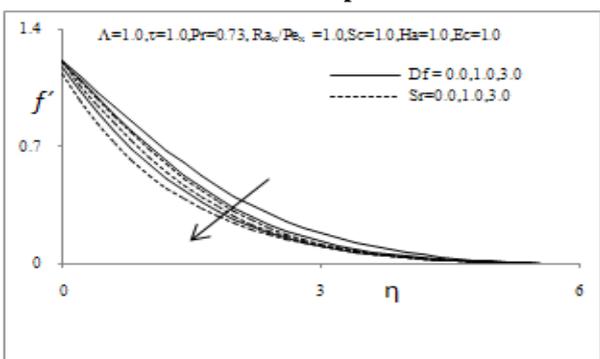
Graph 5c. Effects of Df, Sr on non-dimensional Concentration profiles



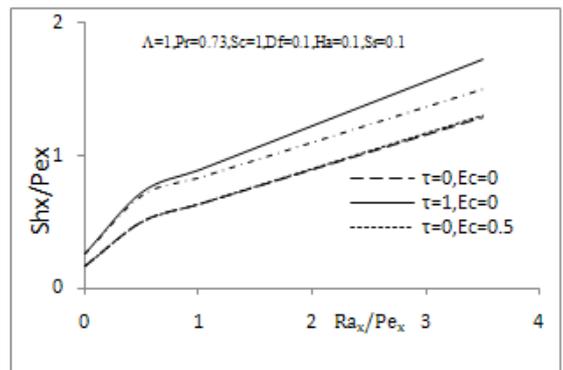
Graph 4c. Effects of Ha, Ec on non-dimensional Concentration profiles



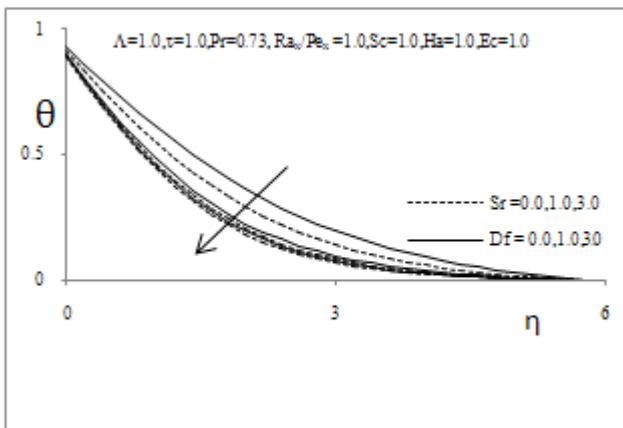
Graph 6. Local Nusselt number as a function of Ra_x/Pe_x for various values of τ and Ec



Graph 5a. Effects of Df, Sr on non-dimensional velocity profiles



Graph 7. Local Sherwood number as a function of Ra_x/Pe_x for various values of τ and Ec



Graph 5(b). Effects of Sr, Df on non-dimensional Temperature profiles

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