The Distribution of Mixed Sum of Independent Random Variables Pertaining to Srivastava's Polynomials and Aleph-Function

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ABSTRACT

The aim of the present paper is to obtain the distribution of mixed sum of two independent random variables with different probability density functions. One with probability density function defined in finite range and the other with probability density function defined in infinite range and associated with product of Srivastava's polynomials and Aleph-function. We use the Laplace transform and its inverse to obtain our main result. The result obtained here is quite general in nature and is capable of yielding a large number of corresponding new and known results merely by specializing the parameters involved therein. To illustrate, some special cases of our main result are also given.

Introduction

In recent years, the distribution of sum of random variables has gained great importance in many areas of science and engineering. For example, sums of independent gamma random variables have application in problems of queuing theory such as determination of total waiting time, in civil engineering such as determination of the total excess water flow in a dam. They also appear in obtaining the inter arrival time of drought events which is the sum of the drought duration and the successive non drought duration. In last several years many authors notably Linhart [9], Jackson [8] and Grice and Bain [5] have studied the applications of distribution of sum of random variables. The distribution of the sum of two independent random variables has been obtained by many research workers, particularly when both the variates come from the same family of distribution. In this context the works of Albert [1] for uniform variates, Holm and Alouini [7], Moschopoulos [11] and Provost [13] for gamma variates, Van-Dorp and Kotz [21] for triangular variats and Loaiciga and Leipnik [10] for Gumbel variates are worth mentioning.

Furthermore, Nason [12] has obtained the distribution of the sum of t and Gaussian random variables and pointed out its application in Bayesian wavelet shrinkage. Very recently, Singh and Kumar [16], Chaurasia and Singh [2] and Gupta [6] have studied the distribution of mixed sum of two independent random variables with different probability density functions. We know that the distribution of sum of several independent random variables when each random variable is of simply infinite or doubly infinite range can easily be obtained by means of characteristic function or moment generating function. But, when the random variables are distributed over finite range, these methods are not much useful and the power of integral transform method comes sharply into focus. In the present paper, we obtain the distribution of sum of two independent random variables, X_1 and X_2, where X_1 possess finite uniform probability density function and X_2 follows infinite probability density function involving the product of Srivastava's polynomials and $\aleph$-function, given by the equations (1) and (2) respectively. Thus

$$f_1(x_1) = \begin{cases} \dfrac{1}{a}, & 0 \leq x_1 \leq a \\ 0, & \text{otherwise} \end{cases} \quad a > 0 \qquad \dots(1)$$

and

$$f_2(x_2) = \begin{cases} C\, x_2^{\lambda-1} e^{-\mu x_2} S_t^v[W x_2^\alpha] \aleph_{p_i,q_i,\tau_i;r}^{m,n}\left[z x_2^\gamma \,\middle|\, \begin{matrix} (a_j,A_j)_{1,n}, [\tau_i[a_{ji},A_{ji})]_{n+1,p_i;r} \\ (b_j,B_j)_{1,m}, [\tau_i(b_{ji},B_{ji})]_{m+1,q_i;r} \end{matrix} \right], & x_2 \geq 0 \\ 0, & \text{otherwise} \end{cases} \qquad \dots(2)$$

where

$$C^{-1} = \mu^{-\lambda} \sum_{k=0}^{[t/v]} \frac{(-t)_{vk}}{k!} A_{t,k} w^k \mu^{-\alpha k}$$

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$$\times \aleph^{m,\,n+1}_{p_i+1,q_i,\tau_i;r}\left[z\mu^{\gamma}\left|{}^{[1-\lambda-\alpha k,\gamma],(a_j,A_j)_{1,n},[\tau_i(a_{ji},A_{ji})]_{n+1,p_i;r}}_{(b_j,B_j)_{1,m},[\tau_i(b_{ji},B_{ji})]_{m+1,q_i;r}}\right.\right], \qquad \ldots(3)$$

and the following conditions are satisfied:

$$\gamma>0,\mu>0,\lambda+\alpha k-\gamma\min_{1\le j\le m}\left(\frac{b_j}{B_j}\right)>0,$$

(i)

$$\varphi_\ell>0, |\arg(z)|<\frac{\pi}{2}\varphi_\ell, \ell=1,...,r\,;$$

(ii)

$$\varphi_\ell\ge0, |\arg(z)|<\frac{\pi}{2}\varphi_\ell \text{ and } R\{\zeta_\ell\}+1<0,$$

where

$$\varphi_\ell=\sum_{j=1}^{n}A_j+\sum_{j=1}^{m}B_j-\tau_\ell\left(\sum_{j=n+1}^{p_\ell}A_{j\ell}+\sum_{j=m+1}^{q_\ell}B_{j\ell}\right)$$

and

$$\zeta_\ell=\sum_{j=1}^{m}b_j-\sum_{j=1}^{n}a_j+\tau_\ell\left(\sum_{j=m+1}^{q_\ell}b_{j\ell}-\sum_{j=n+1}^{p_\ell}a_{j\ell}\right)+\frac{1}{2}(p_\ell-q_\ell)$$

(iii) The parameters of $\aleph$-function and Srivastava's polynomials are real and so restricted that $f_2(x_2)$ remains non-negative.

In the study of fractional driftless Fokker-Planck equations with power law diffusion coefficients, there arises naturally a special function, which is a special case of the $\aleph$, that is Aleph-function. The $\aleph$-function is a generalization of the familiar H-function and I-function. The idea to introduce Aleph-function belongs to Südland et al. [19], however the notation and complete definition is presented here in the following manner in terms of the Mellin-Barnes type integrals [also see 20]:

$$\aleph[z]=\aleph^{m,n}_{p_i,q_i,\tau_i;r}[z]=\aleph^{m,n}_{p_i,q_i,\tau_i;r}\left[z\left|{}^{(a_j,A_j)_{1,n},[\tau_i(a_{ji},A_{ji})]_{n+1,p_i;r}}_{(b_j,B_j)_{1,m},[\tau_i(b_{ji},B_{ji})]_{m+1,q_i;r}}\right.\right],$$

$$=\frac{1}{2\pi i}\int_L\Omega^{m,n}_{p_i,q_i,\tau_i;r}(\xi)\,z^{-\xi}\,d\xi, \qquad \ldots(4)$$

$$i=\sqrt{-1}$$

for all $z\ne0$, where   and

$$\Omega^{m,n}_{p_i,q_i,\tau_i;r}(\xi)=\frac{\displaystyle\prod_{j=1}^{m}\Gamma(b_j+B_j\xi)\prod_{j=1}^{n}\Gamma(1-a_j-A_j\xi)}{\displaystyle\sum_{i=1}^{r}\tau_i\left\{\prod_{j=n+1}^{p_i}\Gamma(a_{ji}+A_{ji}\xi)\prod_{j=m+1}^{q_i}\Gamma(1-b_{ji}-B_{ji}\xi)\right\}}, \qquad \ldots(5)$$

$$L=L_{i\gamma\infty},\gamma\in R, \text{ extends from } \gamma-i\infty \text{ to } \gamma+i\infty,$$

the integration path   and is such that the poles, assumed to be simple of $\Gamma(1-a_j-A_j\xi), j=1,...,n$ do not coincide with the poles of $\Gamma(b_j+B_j\xi), j=1,...,m.$. The parameter p_i, q_i are non-negative integers satisfying $0\le n\le p_i, 1\le m\le q_i, \tau_i>0$ for $i=1,...,r.$

The parameter A_j, B_j, A_{ji}, $B_{ji}>0$ and a_j, b_j, a_{ji}, $b_{ji}\in C$. The empty product in (5) is interpreted as unity. For the existence conditions and other details of the Aleph-function see Südland et al [19,20].

The Srivastava's polynomials [17] is defined as

$$S_t^v[w]=\sum_{k=0}^{[t/v]}\frac{(-t)_{vk}}{k!}A_{t,k}\,w^k\,, t=0,1,2,...\qquad \ldots(6)$$

where v is an arbitrary positive integer, the coefficients $A_{t,k}(t,k\ge0)$ are arbitrary constants, real or complex.

Distribution of the mixed sum of two independent random variables

Theorem 1. If X_1 and X_2 are two independent random variables having the probability density function defined by (1) and (2) respectively. Then the probability density function of

$$Y = X_1 + X_2 \qquad \qquad \qquad \ldots(7)$$

is given by

$$g(y) = g_1(y), \qquad 0 \le y \le a$$

$$= g_1(y) - g_2(y), \qquad a < y < \infty \qquad \qquad \ldots(8)$$

where

$$g_1(y) = \frac{C}{a} \sum_{k=0}^{[t/v]} \sum_{n=0}^{\infty} \frac{(-t)_{vk}}{k!} A_{t,k}\, w^k\, y^{\lambda+\alpha k}\, \frac{(-\mu y)^n}{n!}$$

$$\times \aleph_{p_i+1,q_i+1,\tau_i;r}^{m,n+1} \left[z\, y^{-\gamma} \left| \begin{array}{l} [1-\lambda-\alpha k-n,\gamma],(a_j,A_j)_{1,n},[\tau_i(a_{ji},A_{ji})]_{n+1,p_i;r} \\ (b_j,B_j)_{1,m},[\tau_i(b_{ji},B_{ji})]_{m+1,q_i;r},[-\lambda-\alpha k-n,\gamma] \end{array} \right. \right], y \ge 0 \qquad \ldots(9)$$

and

$$g_2(y) = \frac{C}{a} \sum_{k=0}^{[t/v]} \sum_{n=0}^{\infty} \frac{(-t)_{vk}}{k!} A_{t,k}\, w^k\, (y-a)^{\lambda+\alpha k}\, \frac{\{-\mu(y-a)\}^n}{n!}$$

$$\times \aleph_{p_i+1,q_i+1,\tau_i;r}^{m,n+1} \left[z\, (y-a)^{-\gamma} \left| \begin{array}{l} [1-\lambda-\alpha k-n,\gamma],(a_j,A_j)_{1,n},[\tau_i(a_{ji},A_{ji})]_{n+1,p_i;r} \\ (b_j,B_j)_{1,m},[\tau_i(b_{ji},B_{ji})]_{m+1,q_i;r},[-\lambda-\alpha k-n,\gamma] \end{array} \right. \right], y \ge a \qquad \ldots(10)$$

C is given by (3) and the following conditions are satisfied:

(i)
$$\gamma > 0, \mu > 0, \lambda + \alpha k - \gamma \min_{1 \le j \le m} \left(\frac{b_j}{B_j} \right) > 0,$$

(ii)
$$\varphi_\ell > 0, |\arg(z)| < \frac{\pi}{2}\varphi_\ell, \ell = 1,\ldots, r\,;$$

$$\varphi_\ell \ge 0, |\arg(z)| < \frac{\pi}{2}\varphi_\ell \text{ and } R\{\zeta_\ell\} + 1 < 0,$$

where

$$\varphi_\ell = \sum_{j=1}^{n} A_j + \sum_{j=1}^{m} B_j - \tau_\ell \left(\sum_{j=n+1}^{p_\ell} A_{j\ell} + \sum_{j=m+1}^{q_\ell} B_{j\ell} \right)$$

and

$$\zeta_\ell = \sum_{j=1}^{m} b_j - \sum_{j=1}^{n} a_j + \tau_\ell \left(\sum_{j=m+1}^{q_\ell} b_{j\ell} - \sum_{j=n+1}^{p_\ell} a_{j\ell} \right) + \frac{1}{2}(p_\ell - q_\ell)$$

(iii)   The parameters of $\aleph$-function and Srivastava's polynomials are real and so restricted that $g_1(y)$ and $g_2(y)$ remains non-negative.

Proof. Let the Laplace transform of Y be denoted by $\phi_y(s)$, then

$$\phi_y(s) = L\{f_1(x_1);s\}L\{f_2(x_2);s\}. \qquad \qquad \ldots(11)$$

The Laplace transform of $f_1(x_1)$ is a simple integral so it can easily be evaluated and for the Laplace transform of $f_2(x_2)$, we express the $\aleph$-function in terms of Mellin-Barnes type contour integral (4) and the Srivastava's polynomials in form of series (6). Further, we interchange the order of summation, x_2- and ξ-integrals and evaluate x_2-integral as gamma integral to get

$$\phi_y(s) = \frac{C}{a} \frac{(1-e^{-as})}{s} \sum_{k=0}^{[t/v]} \frac{(-t)_{vk}}{k!} A_{t,k}\, w^k\, (s+\mu)^{-(\lambda+\alpha k)}$$

$$\times \aleph^{m,n+1}_{p_i+1,q_i,\tau_i;r}\left[z\,(s+\mu)^{\gamma}\begin{array}{|l}[1-\lambda-\alpha k,\gamma],(a_j,A_j)_{1,n},[\tau_i(a_{ji},A_{ji})]_{n+1,p_i;r}\\(b_j,B_j)_{1,m},[\tau_i(b_{ji},B_{ji})]_{m+1,q_i;r}\end{array}\right].$$

...(12)

Now, we break the above expressions in two parts, as follows

$$\phi_y(s)=\frac{C}{a}\sum_{k=0}^{[t/v]}\frac{(-t)_{vk}}{k!}A_{t,k}\,w^k\,\frac{(s+\mu)^{-(\lambda+\alpha k)}}{s}$$

$$\times \aleph^{m,n+1}_{p_i+1,q_i,\tau_i;r}\left[z\,(s+\mu)^{\gamma}\begin{array}{|l}[1-\lambda-\alpha k,\gamma],(a_j,A_j)_{1,n},[\tau_i(a_{ji},A_{ji})]_{n+1,p_i;r}\\(b_j,B_j)_{1,m},[\tau_i(b_{ji},B_{ji})]_{m+1,q_i;r}\end{array}\right]$$

$$-\frac{C}{a}\sum_{k=0}^{[t/v]}\frac{(-t)_{vk}}{k!}A_{t,k}\,w^k e^{-as}\,\frac{(s+\mu)^{-(\lambda+\alpha k)}}{s}$$

$$\times \aleph^{m,n+1}_{p_i+1,q_i+1,\tau_i;r}\left[z\,(s+\mu)^{\gamma}\begin{array}{|l}[1-\lambda-\alpha k,\gamma],(a_j,A_j)_{1,n},[\tau_i(a_{ji},A_{ji})]_{n+1,p_i;r}\\(b_j,B_j)_{1,m},[\tau_i(b_{ji},B_{ji})]_{m+1,q_i;r}\end{array}\right].$$

...(13)

To obtain the inverse Laplace transform of first term of equation (13), we express the $\aleph$-function in contour integral, collect the terms involving 's' and take its inverse Laplace transform and use the known result (Erdélyi [3], p.238, eq.9). Further, writing the confluent hypergeometric function thus obtained in series form and interpreting the result by equation (4), we get the value of $g_1(y)$. The inverse Laplace transform of second term easily follows by the value of $g_1(y)$ and shifting property for Laplace transform.

Special Cases

As Aleph function and Srivastava's polynomials are the most generalized special function, numerous special cases can be deduced by making suitable changes in the parameters. But, for the sake of brevity, some interesting special case of Theorem 1 are given below.

(i)   If we take $\tau_i=1,\,\forall\,i=1,...,r$ in the main Theorem, then the Aleph function reduces to an I-function [14] and there holds the following result

$$f_2(x_2)=\begin{cases}C_1\,x_2^{\lambda-1}\,e^{-\mu x_2}\,S_t^v[W\,x_2^{\alpha}]I^{m,n}_{p_i,q_i;r}\left[zx_2^{\gamma}\begin{array}{|l}(a_j,A_j)_{1,n},[a_{ji},A_{ji}]_{n+1,p_i;r}\\(b_j,B_j)_{1,m},[b_{ji},B_{ji}]_{m+1,q_i;r}\end{array}\right],x_2\geq 0\\[3mm]0,\quad\text{otherwise}\end{cases}$$

...(14)

where

$$C_1^{-1}=\mu^{-\lambda}\sum_{k=0}^{[t/v]}\frac{(-t)_{vk}}{k!}A_{t,k}\,w^k\,\mu^{-\alpha k}$$

$$\times I^{m,n+1}_{p_i+1,q_i;r}\left[z\mu^{\gamma}\begin{array}{|l}[1-\lambda-\alpha k,\gamma],(a_j,A_j)_{1,n},[a_{ji},A_{ji}]_{n+1,p_i;r}\\(b_j,B_j)_{1,m},[b_{ji},B_{ji}]_{m+1,q_i;r}\end{array}\right],$$

...(15)

and the corresponding pdf of Y as obtained from the equation (8) is given by

$$h(y)=h_1(y),\quad 0\leq y\leq a$$

$$=h_1(y)-h_2(y),\quad a<y<\infty$$

...(16)

where

$$h_1(y)=\frac{C_1}{a}\sum_{k=0}^{[t/v]}\sum_{n=0}^{\infty}\frac{(-t)_{vk}}{k!}A_{t,k}\,w^k\,y^{\lambda+\alpha k}\,\frac{(-\mu y)^n}{n!}$$

$$\times I^{m,n+1}_{p_i+1,q_i+1;r}\left[z\,y^{-\gamma}\begin{array}{|l}[1-\lambda-\alpha k-n,\gamma],(a_j,A_j)_{1,n},[a_{ji},A_{ji}]_{n+1,p_i;r}\\(b_j,B_j)_{1,m},[b_{ji},B_{ji}]_{m+1,q_i;r},[-\lambda-\alpha k-n,\gamma]\end{array}\right],\quad y\geq 0$$

...(17)

and

$$h_2(y) = \frac{C_1}{a} \sum_{k=0}^{[t/v]} \sum_{n=0}^{\infty} \frac{(-t)_{vk}}{k!} A_{t,k} \, w^k \, (y-a)^{\lambda+\alpha k} \frac{\{-\mu(y-a)\}^n}{n!}$$

$$\times I_{p_i+1,q_i+1;r}^{m,n+1} \left[z(y-a)^{-\gamma} \left| \begin{array}{l} [1-\lambda-\alpha k-n,\gamma],(a_j,A_j)_{1,n},[a_{ji},A_{ji}]_{n+1,p_i;r} \\ (b_j,B_j)_{1,m},[b_{ji},B_{ji}]_{m+1,q_i;r},[-\lambda-\alpha k-n,\gamma] \end{array} \right], \quad y \geq a$$

...(18)

where C_1 is given by (15) and the same conditions are satisfied as given in Theorem 1.

(ii)   Further, if we set $\tau_i = 1, \forall i = 1,..., r$ and r = 1 in the main theorem, then the Aleph function reduces to the familiar H-function of Fox [4], we arrive at the results recently obtained by Chaurasia and Singh [2].

(iii)   By applying Theorem 1 to the case of Hermite polynomials (Srivastava and Singh [16] and Sezgö [15]) and by setting

$$S_t^2[w] \to w^{t/2} H_t\left[\frac{1}{2\sqrt{w}} \right],$$

in which case v = 2, $A_{t,k} = (-1)^k,$ the pdf $f_2(x_2)$ assumes the following form

$$f_2(x_2) = \begin{cases} C_2 \, x_2^{\lambda-1} \, e^{-\mu x_2} \, S_t^2[w \, x_2^{\alpha}] \aleph_{p_i,q_i,\tau_i;r}^{m,n} \left[zx_2^{\gamma} \left| \begin{array}{l} (a_j,A_j)_{1,n},[\tau_i[a_{ji},A_{ji}]]_{n+1,p_i;r} \\ (b_j,B_j)_{1,m},[\tau_i(b_{ji},B_{ji})]_{m+1,q_i;r} \end{array} \right], x_2 \geq 0 \\[4mm] 0, \qquad \text{otherwise} \end{cases}$$

...(19)

where

$$C_2^{-1} = \mu^{-\lambda} \sum_{k=0}^{[t/2]} \frac{(-t)_{2k}}{k!} (-1)^k \, w^k \, \mu^{-\alpha k}$$

$$\times \aleph_{p_i+1,q_i,\tau_i;r}^{m,n+1} \left[z\mu^{\gamma} \left| \begin{array}{l} [1-\lambda-\alpha k,\gamma],(a_j,A_j)_{1,n},[\tau_i(a_{ji},A_{ji})]_{n+1,p_i;r} \\ (b_j,B_j)_{1,m},[\tau_i(b_{ji},B_{ji})]_{m+1,q_i;r} \end{array} \right],$$

...(20)

and the corresponding pdf of Y as obtained from the equation (8) is given by

$$u(y) = u_1(y), \quad 0 \leq y \leq a$$

$$= u_1(y) - u_2(y), \quad a < y < \infty$$

where

$$u_1(y) = \frac{C_2}{a} \sum_{k=0}^{[t/2]} \sum_{n=0}^{\infty} \frac{(-t)_{2k}}{k!} (-1)^k \, w^k \, y^{\lambda+\alpha k} \frac{(-\mu y)^n}{n!}$$

$$\times \aleph_{p_i+1,q_i+1,\tau_i;r}^{m,n+1} \left[z \, y^{-\gamma} \left| \begin{array}{l} [1-\lambda-\alpha k-n,\gamma],(a_j,A_j)_{1,n},[\tau_i(a_{ji},A_{ji})]_{n+1,p_i;r} \\ (b_j,B_j)_{1,m},[\tau_i(b_{ji},B_{ji})]_{m+1,q_i;r},[-\lambda-\alpha k-n,\gamma] \end{array} \right], y \geq 0$$

...(21)

and

$$u_2(y) = \frac{C_2}{a} \sum_{k=0}^{[t/2]} \sum_{n=0}^{\infty} \frac{(-t)_{2k}}{k!} (-1)^k \, w^k \, (y-a)^{\lambda+\alpha k} \frac{\{-\mu(y-a)\}^n}{n!}$$

$$\times \aleph_{p_i+1,q_i+1,\tau_i;r}^{m,n+1} \left[z(y-a)^{-\gamma} \left| \begin{array}{l} [1-\lambda-\alpha k-n,\gamma],(a_j,A_j)_{1,n},[\tau_i(a_{ji},A_{ji})]_{n+1,p_i;r} \\ (b_j,B_j)_{1,m},[\tau_i(b_{ji},B_{ji})]_{m+1,q_i;r},[-\lambda-\alpha k-n,\gamma] \end{array} \right], y \geq a$$

...(22)

where C_2 is given by (20) and the same conditions are satisfied as given in Theorem 1.

The importance of our result lies in its manifold generality. In view of the generality of the Aleph-function and Srivastava's polynomials, on specializing the various parameters in the Aleph-function and the Srivastava's polynomials, we obtain, from our results, several pdfs such as the gamma pdf, beta pdf, Rayleigh pds, Weibull pdf, Chi-Squared pdf, one-sided exponential pdf, half-

Cauchy pdf etc. and their distribution functions. Thus, the results presented in this paper would at once yield a very large number of pdfs occurring in the problems of science and engineering.

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