

# On Evaluating the Performance of Students Using Fuzzy Techniques 

Vinodhini.S and Sophia Porchelvi.R<br>Department of Mathematics, A.D.M College for Women (Autonomous), Nagapattinam, Tamilnadu, India.

## ARTICLE INFO

## Article history:

Received: 5 April 2015;
Received in revised form:
17 May 2015;
Accepted: 22 May 2015;

## Keywords

Fuzzy logic,
ATD matrix,
RTD matrix.


#### Abstract

Evaluation of student's performance is one of the most important responsibilities of every educational institute. The marks obtained by the students in examination should not be the only criteria to decide academic excellence. In this paper, aims to discuss the study of the student's performance by including the Intelligence quotient (IQ). Here the fuzzy matrices are used to find the level of performance of the students. The fuzzy logic is used for finding the academically excellent student.


© 2015 Elixir All rights reserved.

## Introduction

Completing effective performance evaluation serves many important functions within educational organization and doing so is essential in virtually every educational institute. An effective student performance evaluation system can play an important role; motivating peak individual performance and improving institutional quality. Intelligence quotient (IQ) is an age-related measure of intelligence level and is described as 100 times the mental age. The word 'quotient' means the result of dividing one quantity by another and a definition of intelligence is mental ability or quickness of mind.

In this paper the performance of 50 students is evaluated including the IQ using the Average Time Dependent (ATD) matrix and Refined Time Dependent Data matrix (RTD). And fuzzy logic is used to find the academically excellent student among the 50 students. This method based on this evaluation is followed by the one given by [4] and [11].

## Basic Definitions

## Definition

The characteristic function can be generalized such that the values assigned to the elements of the universal set fall within a specified range and indicate the membership grade of these elements in the set. Larger values denote higher degree of set membership. Such a function is called a membership function and the set defined by it a Fuzzy set.

## Definition

An $\alpha$-cut of the fuzzy set $A$ is a crisp set $A_{\alpha}$ that contains all the elements of the universal set $X$ that have a membership grade in A greater than or equal to the specified value of $\alpha$. It can be written as
$\mathrm{A}_{\alpha}=\left\{\mathrm{x} \boldsymbol{\epsilon} \mathrm{X} \mid \mu_{\mathrm{A}}(\mathrm{x}) \geq \alpha\right\}$

## Definition

When a fuzzy set A is a finite support its Fuzzy cardinality $|\tilde{A}|$ is a fuzzy set on $N$ whose membership function is defined by $\mu_{|\tilde{A}|}\left(\left|\mathrm{A}_{\alpha}\right|\right)=\alpha$ for all $\alpha$ in the level set of A .

## Definition

The Intersection of fuzzy set $A$ and $B$ is a fuzzy set $A \cap B$ such that $\mu_{A \Pi B}(x)=\min \left[\mu_{A}(x), \mu_{B}(x)\right]$ for every $x \in X$. Here the membership grade of an element $x$ in fuzzy set $A \cap B$ is the smaller of its membership grades in set $A$ and set $B$.

## Definition

Fuzzy logic is a form of many-valued logic which deals with reasoning that is approximate rather than fixed and exact compared to traditional binary sets fuzzy logic variables may have a truth value that ranges in degree between 0 and 1 .

## Description of simple Fuzzy Matrix Model

Here we describe a simple fuzzy matrix model when we have a raw data in hand. The raw data is taken as a Average Time Dependent Data matrix (ATD). At the second stage, the average or mean and the Standard Deviation (S.D) of every column in the ATD matrix, are determined. Using the average $\mu_{\mathrm{j}}$ of each $\mathrm{j}^{\text {th }}$ column and $\sigma_{\mathrm{j}}$ the S.D of each $\mathrm{j}^{\text {th }}$ column, a parameter $\alpha$ from the interval $[0,1]$ is chosen and the Refined Time Dependent Data matrix (RTD matrix) ( $\mathrm{e}_{\mathrm{ij}}$ ) is formed using the formula:
If $\mathrm{a}_{\mathrm{ij}} \leq\left(\mathrm{u}_{\mathrm{j}}-\alpha^{*} \sigma_{\mathrm{j}}\right)$ then $\mathrm{e}_{\mathrm{ij}}=-1$
else if $a_{i j} \in\left(u_{j}-\alpha * \sigma_{j}, u_{j}+\alpha * \sigma_{j}\right)$ then $e_{i j}=0$
else if $\mathrm{a}_{\mathrm{ij}} \geq\left(\mathrm{u}_{\mathrm{j}}+\alpha^{*} \sigma_{\mathrm{j}}\right)$ then $\mathrm{e}_{\mathrm{ij}}=1$.
where $\mathrm{a}_{\mathrm{ij}}$ 's are the entries of the ATD matrix.
The ATD matrix is thus converted into the Refined Time Dependent Data Matrix. This matrix is also at times termed as the fuzzy matrix as the entries are 1,0 , and -1 . Now, the row sum of this matrix gives the performance level of the students.
The following parameters are used here:

1. Marks in standard X (M.I.X)
2. Marks in standard XII (M.I.XII)
3. Marks in graduation (DP)
4. Performance in Intelligence Quotient tests (IQ)
5. Extra Talents (ET)
6. Estimation of student's performance using fuzzy matrices

Estimation of ATD matrix of students in age:
$\left.\begin{array}{ccccccc}\text { M.I.X } & \text { M.I.XII } & \text { DP } & \text { IQ } & E T & & \\ & A 1 & 72 & 68 & 70 & 44 & 1 \\ & A 2 & 87 & 81 & 90 & 56 & 1 \\ & A 3 & 78 & 76 & 85 & 60 & 1 \\ & A 4 & 70 & 62 & 64 & 56 & 0 \\ & A 5 & 86 & 76 & 61 & 56 & 1 \\ & A 6 & 76 & 67 & 81 & 56 & 0 \\ & A 7 & 83 & 68 & 71 & 42 & 0 \\ & A 8 & 97 & 92 & 94 & 72 & 1 \\ & A 9 & 77 & 67 & 72 & 60 & 0 \\ & A 10 & 70 & 63 & 51 & 50 & 0\end{array}\right]$
else if $\mathrm{a}_{\mathrm{ij}} \geq\left(\mathrm{u}_{\mathrm{j}}+\alpha * \sigma_{\mathrm{j}}\right)$ then $\mathrm{e}_{\mathrm{ij}}=1$.
where $a_{i j}$ 's are the entries of the ATD matrix.
The ATD matrix is thus converted into the Refined Time Dependent Data Matrix. This matrix is also at times termed as the fuzzy matrix as the entries are 1,0 , and -1 . Now, the row sum of this matrix gives the performance level of the students.
The following parameters are used here:

1. Marks in standard X (M.I.X)
2. Marks in standard XII (M.I.XII)
3. Marks in graduation (DP)
4. Performance in Intelligence Quotient tests (IQ)
5. Extra Talents (ET)
6. Estimation of student's performance using fuzzy matrices Estimation of ATD matrix of students in age:
$\left.\begin{array}{cccccc}\text { M.I.X } & \text { M.I.XII } & D P & I Q & E T & \\ & A 1 & 72 & 68 & 70 & 44 \\ & A 2 & 87 & 81 & 90 & 56 \\ & A 3 & 78 & 76 & 85 & 60 \\ & A 4 & 70 & 62 & 64 & 56 \\ & A 5 & 86 & 76 & 61 & 56 \\ & A 6 & 76 & 67 & 81 & 56 \\ & A 7 & 83 & 68 & 71 & 42 \\ & A 8 & 97 & 92 & 94 & 72 \\ & A 9 & 77 & 67 & 72 & 60 \\ & A 10 & 70 & 63 & 51 & 50 \\ & & & & \\ & & & & & \end{array}\right]$

RTD matrix for $\alpha=0.15$
$\left[\begin{array}{rrrrr}-1 & -1 & -1 & -1 & 1 \\ 1 & 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 & 1 \\ -1 & -1 & -1 & 0 & 0 \\ 1 & 1 & -1 & 0 & 1 \\ -1 & -1 & 1 & 0 & 0 \\ 1 & -1 & -1 & -1 & 0 \\ 1 & 1 & 1 & 1 & 1 \\ -1 & -1 & -1 & 1 & 0 \\ -1 & -1 & -1 & -1 & 0\end{array}\right] \quad\left[\begin{array}{r}-3 \\ 4 \\ 4 \\ -3 \\ 2 \\ -1 \\ -2 \\ 5 \\ -2 \\ -4\end{array}\right]$


Estimation of ATD matrix for the students in age 18:

| M.I.X |  |  |  |  | M.I.XII |
| :---: | :---: | :---: | :---: | :---: | :---: |
| B1 | $\left[\begin{array}{ccccc}87 & 76 & 73 & 60 & \text { IQ } \\ B 2 & 88 & 70 & 84 & 68 \\ B 3 & 1 \\ B 3 & 90 & 71 & 83 & 72 \\ B 4 & 92 & 75 & 89 & 52 \\ B 5 & 77 & 66 & 67 & 56 \\ B 6 & 90 & 83 & 88 & 54 \\ B 7 & 50 \\ B & 51 & 58 & 48 & 1 \\ B 8 & 89 & 85 & 90 & 60 \\ B 9 & 90 & 90 & 79 & 60 \\ B 10 & 88 & 77 & 77 & 50 \\ \hline\end{array}\right]$ |  |  |  |  |

RTD matrix for $\alpha=0.15$

$$
\left[\begin{array}{rrrrr}
-1 & -1 & -1 & -1 & 1 \\
1 & 1 & 1 & 0 & 1 \\
0 & 1 & 1 & 1 & 1 \\
-1 & -1 & -1 & 0 & 0 \\
1 & 1 & -1 & 0 & 1 \\
-1 & -1 & 1 & 0 & 0 \\
1 & -1 & -1 & -1 & 0 \\
1 & 1 & 1 & 1 & 1 \\
-1 & -1 & -1 & 1 & 0 \\
-1 & -1 & -1 & -1 & 0
\end{array}\right] \quad\left[\begin{array}{r}
-3 \\
4 \\
4 \\
-3 \\
2 \\
-1 \\
-2 \\
5 \\
-2 \\
-4
\end{array}\right]
$$

## Graph depicting the students performance at age

 17

Estimation of ATD matrix for the students in age 18:

|  |  | M.I.XII | DP | IQ | ET |
| :---: | :---: | :---: | :---: | :---: | :---: |
| B1 | [87 | 76 | 73 | 60 | 0 |
| B2 | 88 | 70 | 84 | 68 | 1 |
| B3 | 90 | 71 | 83 | 72 | 1 |
| B4 | 92 | 75 | 89 | 52 | 1 |
| B5 | 77 | 66 | 67 | 56 | 1 |
| B6 | 90 | 83 | 88 | 54 | 0 |
| B7 | 50 | 51 | 58 | 48 | 1 |
| B8 | 89 | 85 | 90 | 60 | 1 |
| $B 9$ | 90 | 90 | 79 | 60 | 1 |
| B10 | 88 | 77 | 77 | 50 | 0 |

RTD matrix for $\alpha=0.15$

$$
\left[\begin{array}{rrrrr}
1 & 1 & -1 & 1 & 0 \\
1 & -1 & 1 & 1 & 1 \\
1 & -1 & 1 & 1 & 1 \\
1 & 0 & 1 & -1 & 1 \\
-1 & -1 & -1 & -1 & 1 \\
1 & 1 & 1 & -1 & 0 \\
-1 & -1 & -1 & -1 & 1 \\
1 & 1 & 1 & 1 & 1 \\
1 & 1 & 0 & 1 & 1 \\
1 & 1 & 0 & -1 & 0
\end{array}\right] \quad\left[\begin{array}{r}
2 \\
3 \\
3 \\
2 \\
-3 \\
2 \\
-3 \\
5 \\
4 \\
1
\end{array}\right]
$$



Estimation of average and standard deviation (SD)

| Average | 79.6 | 72 | 73.9 | 55.2 |
| :--- | :--- | :--- | :--- | :--- |
| SD | 9.84 | 8.54 | 12.85 | 8.10 |

## Estimation of average and standard deviation (SD)

| Average | 79.6 | 72 | 73.9 | 55.2 |
| :--- | :--- | :--- | :--- | :--- |
| SD | 9.84 | 8.54 | 12.85 | 8.10 |

Average and standard deviation for ATD matrix

| Average | 84.1 | 74.4 | 78.8 | 58 |
| :---: | :--- | :--- | :--- | :--- |
| SD | 12.01 | 10.43 | 9.83 | 7.26 |

The average and standard deviation for ATD matrix

| Average | 85.9 | 73.8 | 80.3 | 49.6 |
| :--- | :--- | :--- | :--- | :--- |
| SD | 4.67 | 10.88 | 5.91 | 5.85 |

The average and standard deviation of the ATD matrix

| Average | 82.6 | 74.5 | 79.3 | 54 |
| :--- | :--- | :--- | :--- | :--- |
| SD | 8.75 | 9.42 | 8.08 | 6.6 |

The average and standard deviation for ATD matrix

| Average | 81.3 | 72.6 | 77.1 | 63.2 |
| :--- | :--- | :--- | :--- | :--- |
| SD | 7.62 | 5.53 | 8.22 | 6.07 |

Table 4.1 Details of topper students

| Students | M.I.X | M.I.XII | DP | IQ | ET |
| :--- | :--- | :--- | :--- | :--- | :--- |
| A8 | 1 | 1 | 1 | 0.7 | 1 |
| B8 | 1 | 1 | 1 | 0.4 | 1 |
| C5 | 1 | 1 | 1 | 0.3 | 1 |
| C6 | 1 | 1 | 1 | 0.2 | 1 |
| D1 | 1 | 1 | 1 | 0.3 | 0 |
| D7 | 1 | 1 | 1 | 0.4 | 0 |
| E4 | 1 | 1 | 1 | 0.7 | 0 |

Table 4.2 Final calculation of gradation about each parameter

| Students | M.I.X | M.I.XII | DP | IQ | ET |
| :--- | :--- | :--- | :--- | :--- | :--- |
| A8 | 97 | 92 | 94 | 72 | 1 |
| B8 | 89 | 85 | 90 | 60 | 1 |
| C5 | 90 | 85 | 84 | 56 | 1 |
| C6 | 90 | 80 | 88 | 52 | 1 |
| D1 | 90 | 82 | 93 | 56 | 0 |
| D7 | 92 | 88 | 89 | 60 | 0 |
| E4 | 92 | 80 | 78 | 70 | 0 |

Estimation of ATD matrix for students in age 19:

| M.I.X | M.I.XII | DP | IQ | ET |
| :---: | :---: | :---: | :---: | :---: |
| C1 ${ }^{90}$ | 80 | 88 | 40 | 1 |
| C2 82 | 74 | 76 | 56 | 1 |
| C3 85 | 60 | 78 | 56 | 1 |
| C4 86 | 80 | 81 | 48 | 1 |
| C5 90 | 85 | 84 | 56 | 1 |
| C6 90 | 80 | 88 | 52 | 1 |
| C7 91 | 74 | 77 | 52 | 0 |
| C8 76 | 65 | 72 | 44 | 0 |
| C9 88 | 88 | 87 | 48 | 1 |
| C10-81 | 52 | 72 | 44 | 1 |

RTD matrix for $\alpha=0.15$

$$
\left[\begin{array}{rrrrr}
1 & 1 & 1 & -1 & 1 \\
-1 & 0 & -1 & 1 & 1 \\
1 & -1 & -1 & 1 & 1 \\
0 & 1 & 0 & -1 & 1 \\
1 & 1 & 1 & 1 & 1 \\
1 & 1 & 1 & 1 & 1 \\
1 & 0 & -1 & 1 & 0 \\
-1 & -1 & -1 & -1 & 0 \\
1 & 1 & 1 & -1 & 1 \\
-1 & -1 & -1 & -1 & 1
\end{array}\right] \quad\left[\begin{array}{r}
3 \\
0 \\
1 \\
1 \\
5 \\
5 \\
1 \\
-4 \\
3 \\
-3
\end{array}\right]
$$



Estimation of ATD matrix for students in age 21:

|  | M.I.X | M.I.XII | DP | IQ | ET |
| :---: | :---: | :---: | :---: | :---: | :---: |
| D1 | ${ }^{90}$ | 82 | 93 | 56 | ${ }^{0}$ |
| D2 | 86 | 80 | 75 | 60 | 1 |
| D3 | 86 | 74 | 72 | 64 | 1 |
| D4 | 60 | 65 | 70 | 56 | 1 |
| D5 | 88 | 66 | 78 | 60 | 1 |
| D6 | 75 | 55 | 76 | 60 | 1 |
| D7 | 92 | 88 | 89 | 60 | 0 |
| D8 | 83 | 79 | 80 | 56 | 0 |
| D9 | 82 | 82 | 90 | 68 | 0 |
| D10 | 84 | 74 | 70 | 60 | 0 |

RTD matrix for $\alpha=0.15$ Row sum matrix
$\left[\begin{array}{rrrrr}1 & 1 & 1 & 1 & 0 \\ 1 & 1 & -1 & 1 & 1 \\ 1 & 0 & -1 & 1 & 1 \\ -1 & -1 & -1 & 1 & 1 \\ -1 & -1 & 0 & 1 & 1 \\ -1 & -1 & -1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & -1 & 1 & 0\end{array}\right] \quad\left[\begin{array}{r}4 \\ 3 \\ 2 \\ 1 \\ 0 \\ -1 \\ 4 \\ 2 \\ 3 \\ 0\end{array}\right]$


Estimation of ATD matrix for age 22:

| M.I.X | M.I.XII | DP | IQ | ET |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | E1 [82 | 72 | 78 | 68 | 17 |
|  | E2 82 | 78 | 76 | 64 | 0 |
|  | E3 84 | 71 | 75 | 56 | 0 |
|  | E4 92 | 80 | 78 | 70 | 0 |
|  | E5 79 | 66 | 78 | 72 | 0 |
|  | E6 95 | 76 | 88 | 60 | 0 |
|  | E7 69 | 61 | 59 | 60 | 0 |
|  | E8 82 | 72 | 69 | 56 | 0 |
|  | E9 72 | 78 | 89 | 70 | 0 |
|  | E10 76 | 72 | 81 | 56 | 1 |

RTD matrix for $\alpha=0.15$

$$
\left[\begin{array}{rrrrr}
0 & 0 & 0 & 1 & 1 \\
0 & 1 & 0 & 0 & 0 \\
1 & -1 & -1 & -1 & 0 \\
1 & 1 & 0 & 1 & 0 \\
-1 & -1 & 0 & 1 & 0 \\
1 & 1 & 1 & -1 & 0 \\
-1 & -1 & -1 & -1 & 0 \\
0 & 0 & -1 & -1 & 0 \\
-1 & 1 & 1 & 1 & 0 \\
-1 & 0 & 1 & -1 & 1
\end{array}\right]
$$

Row sum matrix
$\left[\begin{array}{r}2 \\ 1 \\ -2 \\ 3 \\ -1 \\ 2 \\ -4 \\ -2 \\ 2 \\ 0\end{array}\right]$


The following academically excellent students are chosen from the above 5 various RTD matrices,
Age 17 - student A8
Age 18 - student B8
Age 19 - students C5, C6
Age 21 - students D1, D7
Age 22 - students E4
Finding the Best Student Using Fuzzy Logic
We have to make the following formulation to decide the gradations of each parameter. Fuzzy trapezoidal numbers are used for this analysis.
For marks in standard X and XII

0
(percentage marks -50 ) / 30
1
For marks in degree 0
(percentage marks -50) / $20 \quad 50 \leq$ percentage $<70$
1
For parameter corresponding to IQ

| 0 | $\mathrm{IQ}<45$ |
| :--- | :---: |
| $(\mathrm{IQ}-45) / 35$ | $45 \leq \mathrm{IQ}<80$ |
| 1 | $\mathrm{IQ} \geq 80$ |

Table indicates the marks details of the students
Forming fuzzy sets corresponding to each parameters
M.I.X : 1/A8 + 1/B8 + 1/C5 + 1/C6 + 1/D1 + 1/D7 + 1/E4
M.I.XII: $1 / \mathrm{A} 8+1 / \mathrm{B} 8+1 / \mathrm{C} 5+1 / \mathrm{C} 6+1 / \mathrm{D} 1+1 / \mathrm{D} 7+1 / \mathrm{E} 4$

DP $: 1 / \mathrm{A} 8+1 / \mathrm{B} 8+1 / \mathrm{C} 5+1 / \mathrm{C} 6+1 / \mathrm{D} 1+1 / \mathrm{D} 7+1 / \mathrm{E} 4$
IQ $\quad: 0.7 / \mathrm{A} 8+0.4 / \mathrm{B} 8+0.3 / \mathrm{C} 5+0.2 / \mathrm{C} 6+0.3 / \mathrm{D} 1+0.4 / \mathrm{D} 7$ + 0.7/E4
ET $\quad: 1 / \mathrm{A} 8+1 / \mathrm{B} 8+1 / \mathrm{C} 5+1 / \mathrm{C} 6$
Intersection of fuzzy sets
$\{1 / \mathrm{A} 8+1 / \mathrm{B} 8+1 / \mathrm{C} 5+1 / \mathrm{C} 6+1 / \mathrm{D} 1+1 / \mathrm{D} 7+1 / \mathrm{E} 4\}$
Here the fuzzy intersection is taken without considering the
Here the fuzzy intersection is taken without considering the
students intelligence quotient and extra- curricular activities. So we can't choose best student from this intersection.
Now the intersection taken by considering the parameters ET and IQ
M.I.X $\cap$ M.I.XII $\cap$ DP $\cap$ IQ $\cap$ ET
percentage < 50
$50 \leq$ percentage $<80$
percentage $\geq 80$
percentage $<50$
percentage $\geq 70$
$\{0.7 / \mathrm{A} 8+0.4 / \mathrm{B} 8+0.3 / \mathrm{C} 5+0.2 / \mathrm{C} 6\}$
It gives the academically excellent student is A8.

## Discussion and conclusion

From the above evaluation of 50 student's performance 7 students have been chosen according their age group. And the performance of the student's varies only in their IQ level. So, whatever marks the student s got in their SSLC, higher secondary and graduation are not sufficient to determine their level of intelligence. The IQ level and the participation of students in extra-curricular activities are also very important to predict their intelligence and skills.

Learners with higher IQ make significantly greater academic progress in reading and writing than children with lower IQ's. Also, compared to average IQ students, high IQ students are better, more flexible, more adaptive and efficient at choosing and utilizing effective strategies. Therefore according to causal relationship between IQ and achievement it is beneficial to consider IQ when investigating predictors of achievements and relationship between scores.

## References

[1].Adlassnig.K.P,(1986) "Fuzzy Set Theory in Medical Diagnosis", IEEE Trans.Systems, Man, cybernetics, pages 260265.
[2].Angela L. Duckworth and Martin E.P. Seligman, "SelfDiscipline Outdoes IQ in Predicting Academic Performance of Adolescents", Positive Psychology Center, University of Pennsylvania.
[3]. George j. Klir/Bo Yuan, "Fuzzy Sets and Fuzzy Logic".
[4]. Geethalakshmi.M, Jose Praveena.N, Rajkumar.A(2012), "Result analysis students using fuzzy matrices", International Journal of Scientific and Research Publications, Volume 2, Issue 4, ISSN 2250-3153.
[5]. Paul Newton and Helen Bristoll, "Abstract Reasoning".
[6]. Philip Carter, "The complete book of intelligence tests".
[7]. Philip Carter and Ken Russell (2000), "Test your IQ", ISBN-10 0749448334 ISBN-13 9780749448332.
[8]. Rajeev G. Sapre and Shraddha Surve (2012), "Fuzzy Mathematical Approach for Performance Evaluation of a Student", International Journal of Fuzzy Mathematics and Systems, ISSN 2248-9940.
[9]. Shane Frederick, (2005) "Cognitive Reflection and Decision Making", Journal of economic perpectives, volume 19,number 4, pages 25-42.
[10].Saba Ghayas and Adnan Adil (2007)," Effect of Handedness on Intelligence Level of Students", Journal of the Indian Academy of Applied Psychology.
[11]. Vasantha Kandasamy .W.B (2007), "Elementary fuzzy matrix theory and fuzzy models for social scientists".
[12]. Zimmermann.H, (1991) "Fuzzy set theory and its applications", second edition, kluwer academic publishers, Germany.
[13].Zadeh.L.A and Bellman.R.E, (1970) "Decision making in a fuzzy environment", Management sci.

