



Shortest Path Problem in a Network Using Intuitionistic Fuzzy Number - A Case Study about Kanyakumari Roadways Network

R. Sophia Porchelvi¹ and G. Sudha²

¹Department of Mathematics, A.D.M College for women (Autonomous), Nagapattinam, India.

²Department of Mathematics, A.V.C. College (Autonomous), Mannampandal, India.

ARTICLE INFO

Article history:

Received: 5 April 2015;

Received in revised form:

17 May 2015;

Accepted: 22 May 2015;

Keywords

Intuitionistic Fuzzy Distance,
Shortest Path Length (SPL),
Shortest Path Problem (SPP),
Triangular Intuitionistic Fuzzy
Number (TIFN).

ABSTRACT

Finding Shortest path in a graph has been the area for many researchers. Shortest path is one of the fundamental and most widely used concepts in networks. Here, we discuss the Shortest Path Length (SPL) from a specified vertex to all other vertices in a network. For illustration, a real life example has been considered from Chennai to kanyakumari roadways transport network.

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Introduction

The SPP concentrates on finding the path with minimum distance. The lengths of arcs in the network represent travelling time, cost, distance or other variables. In real life applications, these are lengths could be uncertain and to determine the exact value of these arc lengths is very difficult for decision maker. In such a situation Intuitionistic fuzzy SPP seems to be more realistic, where the arc lengths are characterized by intuitionistic fuzzy numbers.

The SPP under the uncertain environment was first analyzed by Dubois and Prade [1]. In this paper, we will be dealing with TIFN which are expressed as the edge costs for the directed graph. The main objective of this paper is to determine the shortest path from source to destination in a network.

This paper is organized as follows: In section 2, some elementary concepts and definitions in intuitionistic fuzzy set theory are discussed. In section 3, new algorithm have been proposed for intuitionistic fuzzy SPP. In section 4, illustrative example is given to demonstrate to our proposed algorithm. This paper is concluded in section 5.

Preliminary concepts

Intuitionistic Fuzzy Set: An Intuitionistic fuzzy set \tilde{A}^1 in X is given by a set of ordered triples:

$$\tilde{A}^1 = \{(x, \mu_{\tilde{A}^1}(x), \nu_{\tilde{A}^1}(x)) / x \in X\},$$

where $\mu_{\tilde{A}^1}, \nu_{\tilde{A}^1} : X \rightarrow [0,1]$ are functions such that $0 \leq$

$\mu_{\tilde{A}^1}(x) + \nu_{\tilde{A}^1}(x) \leq 1$ for all $x \in X$. For each x the numbers $\mu_{\tilde{A}^1}(x)$ and $\nu_{\tilde{A}^1}(x)$ represent the degree of membership and degree of non-membership of the element $x \in X$ to $A \subset X$, respectively.

Intuitionistic Fuzzy Number (IFN)

An intuitionistic fuzzy subset \tilde{A}^1 = $\{(x, \mu_{\tilde{A}^1}(x), \nu_{\tilde{A}^1}(x)) / x \in X\}$, of real line R is called an Intuitionistic Fuzzy Number (IFN) if the following holds:

Tele:

E-mail addresses: soiaporchelvi@gmail.com

(i) There exists $m \in R$, $\mu_{\tilde{A}^1}(m) = 1$ and $\nu_{\tilde{A}^1}(m) = 0$

(ii) $\mu_{\tilde{A}^1}$ is a continuous mapping from R to the closed interval $[0,1]$ and for all $x \in R$, the relation $0 \leq \mu_{\tilde{A}^1}(x) + \nu_{\tilde{A}^1}(x) \leq 1$

holds. The membership and non-membership function of \tilde{A}^1 is of the following form:

$$\mu_{\tilde{A}^1}(x) = \begin{cases} 0, -\infty < x \leq m - \alpha \\ f_1(x), x \in [m - \alpha, m] \\ 1, x = m \\ h_1(x), x \in [m, m + \beta] \\ 0, m + \beta \leq x < \infty \end{cases}$$

Where $f_1(x)$ and $h_1(x)$ are strictly increasing and decreasing function

$[m - \alpha, m]$ and $[m, m + \beta]$ respectively.

$$\nu_{\tilde{A}^1}(x) = \begin{cases} 1, -\infty < x \leq m - \alpha^1 \\ f_2(x), x \in [m - \alpha^1, m] \\ 0, x = m \\ h_2(x), x \in [m, m + \beta^1] \\ 0 \leq h_1(x) + h_2(x) \leq 1 \\ 0, m + \beta^1 \leq x < \infty \end{cases}$$

Here m is the mean value of \tilde{A}^1 . α, β are called left and right spreads of membership function $\mu_{\tilde{A}^1}(x)$ respectively. α^1, β^1 represents left and right spreads of non-membership function $\nu_{\tilde{A}^1}(x)$ respectively.

Step 1

Activity	TIFN
TIFP ₁₂	$(\langle 3,4,5 \rangle, \langle 6,7,8 \rangle)$
TIFP ₁₃	$(\langle 1,2,3 \rangle, \langle 4,5,6 \rangle)$
TIFP ₁₄	$(\langle 4,5,6 \rangle, \langle 6,8,9 \rangle)$
TIFP ₂₅	$(\langle 3,4,6 \rangle, \langle 7,8,9 \rangle)$
TIFP ₂₇	$(\langle 6,7,9 \rangle, \langle 8,10,11 \rangle)$
TIFP ₃₅	$(\langle 5,6,7 \rangle, \langle 7,9,10 \rangle)$
TIFP ₃₆	$(\langle 2,3,4 \rangle, \langle 5,6,7 \rangle)$
TIFP ₄₆	$(\langle 5,8,9 \rangle, \langle 10,11,12 \rangle)$
TIFP ₅₇	$(\langle 8,9,10 \rangle, \langle 11,12,13 \rangle)$
TIFP ₅₈	$(\langle 9,10,11 \rangle, \langle 12,13,14 \rangle)$
TIFP ₆₈	$(\langle 3,4,5 \rangle, \langle 6,7,8 \rangle)$
TIFP ₇₉	$(\langle 6,7,8 \rangle, \langle 9,10,11 \rangle)$
TIFP ₈₉	$(\langle 4,5,6 \rangle, \langle 7,8,9 \rangle)$

Name of the cities with codes

Node Number	Name of City	Code	Node Number	Name of City	Code
1	CHENNAI	CHE	5	COIMBATORE	COI
2	VELLORE	VEL	6	MADURAI	MAD
3	TRICHY	TRY	7	KOCHI	KOH
4	PONDY	PON	8	TIRUNELVELI	TNV
			9	KANYAKUMARI	KAN

Triangular Intuitionistic Fuzzy Number (TIFN)

A (TIFN) \tilde{A}^1 is an intuitionistic fuzzy set in R with the following $\mu_{\tilde{A}^1}(x)$ and $\nu_{\tilde{A}^1}(x)$

$$\mu_{\tilde{A}^1}(x) = \begin{cases} \frac{x-a_1}{a_2-a_1}, & a_1 \leq x \leq a_2 \\ \frac{a_3-x}{a_3-a_2}, & a_2 \leq x \leq a_3 \\ 0, & \text{otherwise} \end{cases}$$

$$\nu_{\tilde{A}^1}(x) = \begin{cases} \frac{a_2-x}{a_2-a_1}, & a_1 \leq x \leq a_2 \\ \frac{x-a_2}{a_3-a_2}, & a_2 \leq x \leq a_3 \\ 1, & \text{otherwise} \end{cases}$$

Where $a_1^1 \leq a_1 \leq a_2 \leq a_3 \leq a_3^1$ and $\mu_{\tilde{A}^1}(x) + \nu_{\tilde{A}^1}(x) \leq 1$

Arithmetic Operations of two TIFNs

The additions of two TIFN are as follows. For two triangular intuitionistic fuzzy numbers

$A = (\langle a_1, b_1, c_1 \rangle : \mu_A, \langle e_1, f_1, g_1 \rangle : \gamma_A)$ and

$B = (\langle a_2, b_2, c_2 \rangle : \mu_B, \langle e_2, f_2, g_2 \rangle : \gamma_B)$ with $\mu_A \neq \mu_B$

and $\gamma_A \neq \gamma_B$, define

$$A+B = \left(\begin{aligned} &\langle a_1 + a_2, b_1 + b_2, c_1 + c_2 \rangle : \text{Min}(\mu_A, \mu_B), \\ &\langle e_1 + e_2, f_1 + f_2, g_1 + g_2 \rangle : \text{Max}(\gamma_A, \gamma_B) \end{aligned} \right)$$

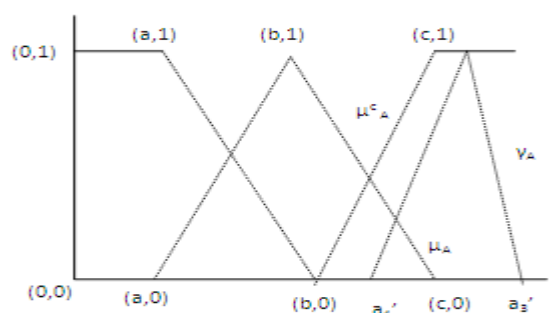


Figure 1 Triangular Intuitionistic fuzzy number

Ranking Function of TIFN with Distance method

For P and Q $\in E$,

$D(P, P_0) > D(Q, P_0)$, iff $P > Q$

$D(P, P_0) < D(Q, P_0)$, iff $P < Q$

$D(P, P_0) = D(Q, P_0)$, iff $P = Q$

Triangular Intuitionistic Fuzzy Distance

The distance of TIFN, $P = (\langle x, y, z \rangle, \langle l, m, n \rangle)$ of P_0 is defined by

$$D(P, P_0) = \sqrt{\frac{2(x+l)^2 + 3(y+m)^2}{3 + x(n-z)}}$$

Shortest Path Length Procedure in Intuitionistic Fuzzy sense

Step 1: Construct a network $G = (V, E)$ where V is the set of vertices and E is the set of edges. Here, G is an acyclic digraph and the arc length takes the TIFNs.

Step 2: Form the possible paths from source vertex to destination vertex and compute the possible triangular intuitionistic fuzzy path lengths.

Step 3: Calculate triangular intuitionistic fuzzy distances for each path length PL_{ij} using

$$D(P, P_0) = \sqrt{\frac{2(x+l)^2 + 3(y+m)^2}{3 + x(n-z)}}$$

Step 4: Identify the SPL using

$$TIFP_{ij} = \min \{ D_1(PL_{ij}), D_2(PL_{ij}), \dots, D_m(PL_{ij}) \}$$

where $i, j = 1, 2, 3, \dots, n$ for all $i \neq j$.

Minimum path is the SPL.

Numerical Example

Consider a network with the triangular intuitionistic fuzzy arc lengths as shown below:

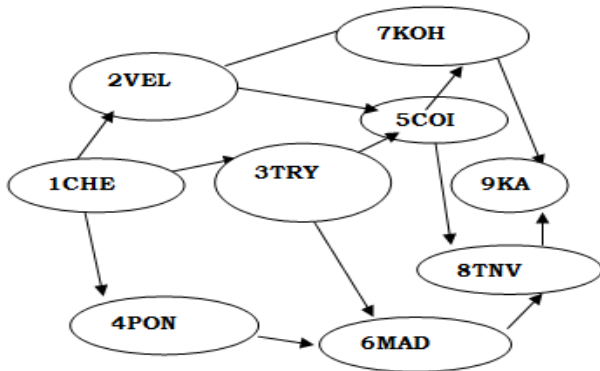


Figure 2. Triangular Intuitionistic Fuzzy Distance Network Step 2

The possible paths of an triangular intuitionistic fuzzy distance network are

P_1 : 1-2--7-9 ; P_2 : 1-2-5-7-9 ; P_3 : 1-2-5-8-9 ; P_4 : 1-3-5-8-9 ;

P_5 : 1-3-6-8-9 ; P_6 : 1-3-5-7-9 ; P_7 : 1-4-6-8-9

The possible Triangular Intuitionistic fuzzy path lengths are as follows:

$P_1 = \langle (15, 18, 22), \langle 23, 27, 30 \rangle \rangle$; $P_2 = \langle (20, 24, 29), \langle 33, 37, 41 \rangle \rangle$; $P_3 = \langle (21, 23, 28), \langle 32, 36, 40 \rangle \rangle$;
 $P_4 = \langle (19, 23, 27), \langle 30, 35, 39 \rangle \rangle$; $P_5 = \langle (10, 14, 18), \langle 22, 26, 30 \rangle \rangle$; $P_6 = \langle (20, 24, 28), \langle 31, 36, 40 \rangle \rangle$;
 $P_7 = \langle (16, 22, 26), \langle 29, 34, 38 \rangle \rangle$

Step 3:

To calculate the Triangular Intuitionistic fuzzy distances for each path P_{ij} using (2.5)

$$D(P_1) = \sqrt{\frac{2(15+23)^2 + 3(18+27)^2}{3 + 15(30-22)}} = 8.54$$

$$\text{Similarly, } D(P_2) = 8.31 ; D(P_3) = 8.03 ; D(P_4) = 8.03 ; D(P_5) = 7.46 ; D(P_6) = 8.11 ; D(P_7) = 8.31$$

Step 4

Identify the SPL using $TIFP_{ij} = \min \{ D_1(PL_{ij}), D_2(PL_{ij}), \dots, D_m(PL_{ij}) \}$. Here, the path P_5 : 1-3-6-8-9 is the SPL.

Conclusion

As a real life example Kanyakumari Roadways Transport network has been considered and with the help of ranking of TIFNs, shortest path length in this network is computed.

In this paper, new algorithm have been proposed for SPP where the shortest path is identified using the concept of ranking function with regard to the fact that the Decision Maker can

choose the best path among various alternatives from the list of ranking. We conclude that the algorithms developed in the current research are the simplest and is the alternative method for getting the shortest path intuitionistic fuzzy.

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