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Performance Analysis of Three Unit Redundant with Switch and Human Failure Using Copula Distribution

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ABSTRACT

In this paper, the author has studied performance of a three unit redundant system with the impact of switch and human failure. A system with three identical unit has been considered for assessment of performance under 2-out –of -3: G; policy. In the system, a switch is used to transfer load from one unit to another unit. All three units of system are connected in parallel configuration and working under 2-out-of-3: G; policy. The system can have two types of failure partial failure and complete failure. Partial failure degrades the efficiency of system but the complete failure breakdown the system and stop functioning of the system. Switch failure and human failure are considered as complete failure. The system has two types of failure and two types of repair. General repair is employed to the partially failed system and Gumbel-Hougaard family copula distribution complete failed system. The system is studied by supplementary variable technique and various measures of reliability, such as availability, reliability, MTTF and profit functions have been discussed. Some particular cases have been discussed by taking different failure rates.

Introduction

References to the current trend of technological advancement, industrial systems are becoming smaller and more complex due to automation and miniaturization. In safety and critical applications, it becomes necessary to improve the reliability through *k*-out-of-*n* redundancy. In contrast to improve reliability of the system, many researchers have highlighted and proposed their work, contributions by considering different types of mathematical models. Therefore, tracing and repair of fault units or components is sometimes becomes time consuming and difficult as well. Therefore, it is require having an idea of system configuration before design a new system. The architecture of system should be design so that it must consist some redundant unit in standby mode, that might be perform the intended task whenever we need them. Therefore, it is important to make a view of system configuration before designing. In series configuration the system, fail when any one unit fail and in parallel configuration the system works with less efficiency until one unit of its configuration is in good condition. Both configurations are independent in nature and used to discuss reliability characteristics of a simply designed model. In first, the chance of system failure is very high, but in second, it will work with less efficiency. Therefore, it becomes necessary to study a k-out-of-n system in which system work successfully until k of its units are in good condition. Further the k-out-of –n system configuration is categorized in k-out-of-n:G and k-out-of-n:F system. The redundant system configuration of the form: k-out-of-n system, which has wide application in industrial systems. The k-out-of-n system works, if and only if at least k of the n components works. Redundancy is a technique which, improve reliability and availability of system over the time. The k-out-of-n system plays a vital role in process industrial and design and which have received attention of researchers. Many researchers have extensively studied redundant systems, and their contributions highlighted with degree of completeness. Mangey Ram and Amit Kumar (2014) discussed the performability analysis of a system under 1-out-of-2:G scheme with perfect switching using analytical approach to compute the reliability measures of a system, which contains mixed configuration. Ibrahim and Nafiu (2012) studied a comparative analysis of three unit redundant systems with three types of failure. The result obtained shown that preventive maintenance is better than other systems without preventive maintenance. V.V.Singh, Mangeyram (2014) studied a multi- state k-out-of-n type of system. The results for elastration have been highlighted specially for 2-out-of-3: G system. V.V.Singh et. al., (2013) discussed availability, MTTF and cost analysis of a system having two units in series configuration with controller and concluded that availability of the system decreases as oppose to the increase in probability of failure. Mangeyram et. al (2013) studied performance improvement of parallel redundant system with coverage factor and analyzed under preemptive resume repair policy. Mangeyram, Amit (2014) studied performance analysis of a system under 1-out-of-2: G scheme with perfect reworking. Monika Manglik et.al. (2014) studied behavioral analysis of a hydroelectric production power plant under reworking scheme. Mangey Ram et.al. (2013) considered stochastic analysis of a standby system with waiting repair strategy. The study clearly explaine the important of waiting time to repair and human error which seem to be possible in many engineering systems.

Consecutive k-out- n systems have been studied by various researches. Ramamuthy (1997) studied reliability of a consecutive-k-out-n: F system consists of n ordered components along a line or circle such extensively applications in many field of engineering like power plants, airplane model industrial organizations. Earlier the researchers including A.K Govel (1974), K.K.Agrawal (1975), Singh et.al. (2001) & Xiaolin et.al.(2010) studied the complex system by considering the fact that the failed system may be repair by general repair and they studied the reliability characteristic of complex system under the fact that only one repair can be employed between

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two transition state, but there are many situations where more than one repair can be possible between two transition state, when such type possibility exists the system is studied using copula Nelson. R. B (2006). The authors Singh. Et. al. (2010) studied the complex systems having three units-super priority, priority and ordinary under preemptive resume repairs policy.

In continuation to the study of complex systems Singh, Singh, Mangeyram& Goel (2012, 2013) proposed the work with the system, which consists of two subsystem with controller and human and deliberately failure. The standby complex system have extensively studied by varies researchers in the reference to the study of standby complex systems Singh et .al (2013) discussed availability of standby complex system under waiting repair and human failure using Gumbel-Hougaard family copula distribution. Furthermore, a number of researchers deal with the problems of reliability field but still more attention is required.

In this paper, the authors have considered mathematical model of a system, which having three units and working under 2-out –of-3: G; policy. Initially in state S_0 the system is in perfect state where all three units, switch is in good working condition. Whenever system starts functioning and if first, second and third unit of system fail than the system will approach to $S_{1,}$ S_2 and S_3 state respectively. Further failure in any unit in the system will lead it to complete failure mode i.e. $S_{4,}$ S_5 and S_6 states. The state S_7 and S_8 indicates the switch and human failure with is assumed to complete damage the system. Now under consideration the system for 2-out- 3: G; policy it is clear that states $S_{1,}$ S_2 and S_3 are in partially failed states, which may be repair by employing general repair policy. Though the states $S_{4,}$ S_5 and S_6 are complete failed states, but general repair has already been assigned, therefore these states will be repair using general repair policy. The states S_7 and S_8 are complete damaged states due to which the functioning of entire system shutdown, therefore these states must be repair using copula distribution. The system is studied by using supplementary variable technique and Laplace transforms, and various measures of reliability has been discussed and some particular cases are also taken to highlight the result. The results are demonstrated by graphs and conclusions are drowning by graphs.

The entire paper has studied by divided in following sections; Section I of paper is introduction, which consists the related work done by previous researchers and need of study is highlighted. Section II of paper consists of state transition diagram of model and notation used for study. In section III of paper a mathematical modeling and solution of formulated model is done. The IV section of the paper is a analytical part in which the various parameters like availability, MTTF and profit analysis have been evaluated for different values of parameters. The last V section of paper is conclusion part for discussion of results and their explanation for future prospects.

State Description

State	State Description
S_0	All three units system, connecting switch, is good working condition and no human failure arises in this state. The system is in perfect state.
S_1	In state S_1 first unit of system fail by the failure rate $\lambda_{1.}$ The repair been assigned to failed unit and the system is in operational mode with partial failure.
S_2	In state S_2 first unit of system fail by the failure rate $\lambda_{2.}$ The repair been assigned to failed unit and the system is in operational mode with partial failure.
S_3	In state S_3 first unit of system fail by the failure rate $\lambda_{3.}$ The repair been assigned to failed unit and the system is in operational mode with partial failure.
S_4	In state S_4 the system is in complete failure mode, general repair is employed to previously failed unit, when the unit will repaired it will start working.
S_5	In state S_5 the system is in complete failure mode, general repair is employed to previously failed unit, when the unit will repaired it will start working.
S_6	In state S_6 the system is in complete failure mode, general repair have been employed to previously failed unit, when the unit will repaired it will start working.
S_7	System has failed due to failure of its switch.
S_8	System is completely failed due to human failure.

Assumption

The following assumptions are taken throughout the discussion of model.

(1) Initially the system is in perfect state S_0 and all units are in good working condition.

(2) The system working under 2-out-of-3: G; Policy therefore till the time two of its unit are in good condition it will work and fulfill the assignment.

(3) System fails if more than two units fail.

(4) Human failure as well as switch failure completely fails the system.

(5) Only one change is allowed at a time in the transitions.

(6) Partial failure is repaired by general time distribution.

(7) Human failure, and switch failure in the system need fast repairing and hence is repaired by using (Gumbel-Hougaard) family copula.

(8) Repaired system works like a new and repair did not damage anything**.**

State transition diagram of model

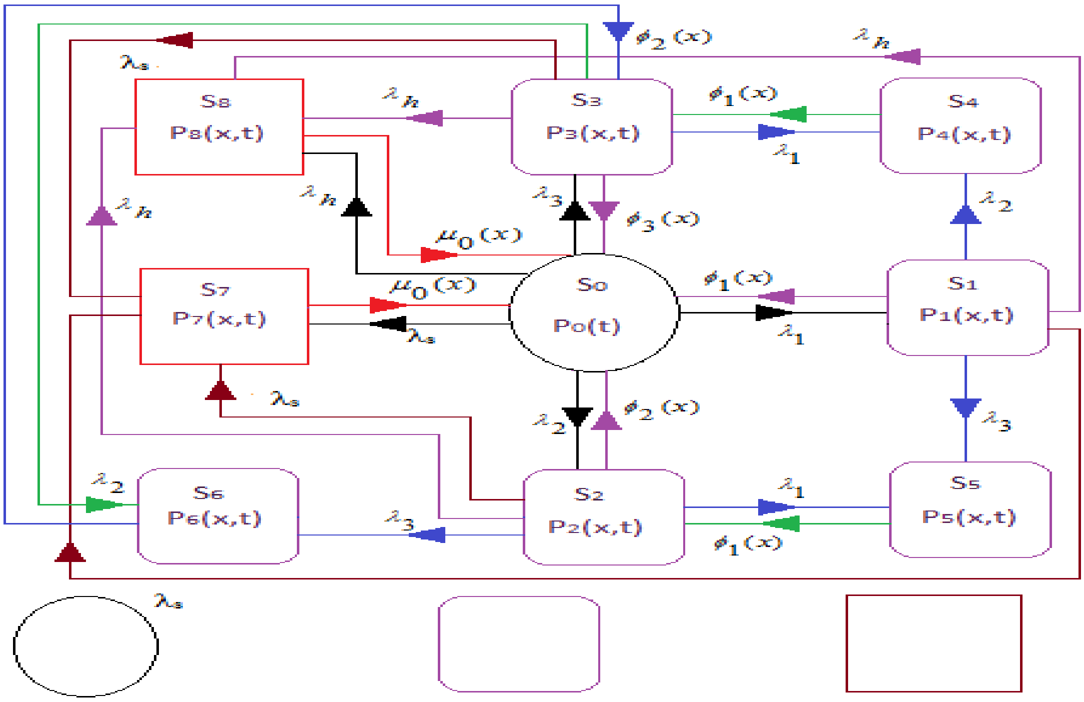

Fig 1. State Transition Diagram of model

Notations

λ_i	Failure rate of i[th] unit of plant system, i=1 , 2, 3…..n.
λ_1 / λ_2	Failure rates of system such that at most k unit /more than k units failed during operational mode.
$\lambda_D / \lambda_{CL} / \lambda_C / \lambda_S / \lambda_h$ /	Failure rate of deliberately failure/ failure due to natural calamity/ controller failure/ failure due to strike/ human failure.
$\phi(x)$	Repair rate of system for minor partial failure and repair rate due to strike in the plant.
$P_i(t)$:	State transition probabilities of system.
$\overline{P}(s)$:	Laplace transform of state transition probability
$P_i(x, t)$:	State transition probability that the system is in state $P_i(x, t)$, system is under repair with repair variable x, t.
$C_\theta (u_1, u_2(x))$:	The expression for joint probability distribution (failed state S_i to good state S_0) according to Gumbel-Hougaard family is given as: $C_\vartheta(u_1, u_2(x)) = \mu_0(x) = \exp[x^\theta + \{\log\phi(x)\}^\theta]^{1/\theta}$. Where, $u_1 = \phi(x)$, and $u_2 = e^x$
$E_p(t)$:	Expected profit in interval [0,t)

Formulation of Mathematical Model

By probability of considerations and continuity arguments, we can obtain the following set of difference differential equations governing the present mathematical model.

$$\left[\frac{\partial}{\partial t} + \lambda_1 + \lambda_2 + \lambda_3 + \lambda_h + \lambda_S\right] P_0(t) = \left[\int_0^\infty \phi_1(x) P_1(x,t)dx + \int_0^\infty \phi_2(x) P_2(x,t)dx + \int_0^\infty \phi_3(x) P_3(x,t)dx\right.$$

$$\left. + \int_0^\infty \mu_0(x) P_7(x,t)dx + \int_0^\infty \mu_0(x) P_8(x,t)dx \right] \quad ...(1)$$

$$\left[\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \lambda_2 + \lambda_3 + \lambda_h + \lambda_S + \phi_1(x)\right] P_1(x,t) = 0 \quad ...(2)$$

$$\left[\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \lambda_1 + \lambda_3 + \lambda_h + \lambda_S + \phi_2(x)\right] P_2(x,t) = 0 \quad ...(3)$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\lambda_2+\lambda_1+\lambda_h+\lambda_S+\phi_3(x)\right]P_3(x,t)=0 \quad \ldots(4)$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\phi_1(x)\right]P_4(x,t)=0 \quad \ldots(5)$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\phi_1(x)\right]P_5(x,t)=0 \quad \ldots(6)$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\phi_2(x)\right]P_6(x,t)=0 \quad \ldots(7)$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\mu_0(x)\right]P_7(x,t)=0 \quad \ldots(8)$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\mu_0(x)\right]P_8(x,t)=0 \quad \ldots(9)$$

BOUNDARY CONDITIONS

$$P_1(0,t)=\lambda_1 P_0(t) \quad \ldots(10)$$

$$P_2(0,t)=\lambda_2 P_1(0,t)+\int_0^\infty \phi_1(x)P_5(x,t)dx \quad \ldots(11)$$

$$P_3(0,t)=\lambda_3 P_1(0,t)+\int_0^\infty \phi_1(x)P_4(x,t)dx+\int_0^\infty \phi_2(x)P_6(x,t)dx \quad \ldots(12)$$

$$P_4(t)=\lambda_2 P_1(0,t)+\lambda_1 P_3(0,t) \quad \ldots(13)$$

$$P_5(t)=\lambda_3 P_1(0,t)+\lambda_1 P_2(0,t) \quad \ldots(14)$$

$$P_6(t)=\lambda_3 P_2(0,t)+\lambda_2 P_3(0,t) \quad \ldots(15)$$

$$P_7(0,t)=\lambda_s P_0(t)+\lambda_s P_2(0,t)+\lambda_s P_1(0,t)+\lambda_s P_5(0,t) \quad \ldots(16)$$

$$P_8(0,t)=\lambda_h P_0(t)+\lambda_h P_3(0,t)+\lambda_h P_1(0,t)+\lambda_h P_2(0,t) \quad \ldots(17)$$

Initials Conditions

$P_0(0)=1$ and other state probabilities are zero at t = 0

Solution of the Model

Taking Laplace transformation of equations (1)-(17) and using equation (18), we obtain.

$$\left(s+\lambda_1+\lambda_2+\lambda_3+\lambda_h+\lambda_S\right)\overline{P}_0(s)=[1+\int_0^\infty \phi_1(x)\overline{P}_1(x,s)dx+\int_0^\infty \phi_2(x)\overline{P}_2(x,s)dx++\int_0^\infty \phi_3(x)\overline{P}_3(x,s)dx$$
$$+\int_0^\infty \mu_0(x)\overline{P}_7(x,s)dx+\int_0^\infty \mu_0(x)\overline{P}_8(x,s)dx] \quad \ldots(18)$$

$$\left[s+\frac{\partial}{\partial x}+\lambda_2+\lambda_3+\lambda_h+\lambda_S+\phi_1(x)\right]\overline{P}_1(x,s)=0 \quad \ldots(19)$$

$$\left[s+\frac{\partial}{\partial x}+\lambda_1+\lambda_3+\lambda_h+\lambda_S+\phi_2(x)\right]\overline{P}_2(x,s)=0 \quad \ldots(20)$$

$$\left[s+\frac{\partial}{\partial x}+\lambda_1+\lambda_2+\lambda_h+\lambda_S+\phi_3(x)\right]\overline{P}_3(x,s)=0 \quad \ldots(21)$$

$$\left[s+\frac{\partial}{\partial x}+\phi_1(x)\right]\overline{P}_4(x,s)=0 \quad \ldots(22)$$

$$\left[s+\frac{\partial}{\partial x}+\phi_1(x)\right]\overline{P}_5(x,s)=0 \quad \ldots(23)$$

$$\left[s+\frac{\partial}{\partial x}+\phi_2(x)\right]\overline{P}_6(x,s)=0 \quad \ldots(24)$$

$$\left[s+\frac{\partial}{\partial x}+\mu_0(x)\right]\overline{P}_7(x,s)=0 \quad \ldots(25)$$

$$\left[s+\frac{\partial}{\partial x}+\mu_0(x)\right]\overline{P}_8(x,s)=0 \quad \ldots(26)$$

Laplace Transform of boundary conditions:

$$\overline{P}_1(0,s)=\lambda_1\overline{P}_0(s) \quad \ldots(27)$$

$$\overline{P}_2(0,s)=\lambda_2\overline{P}_0(s)+\int_0^\infty \phi_1\overline{P}_5(x,s)dx \quad \ldots(28)$$

$$\overline{P}_3(0,s)=\lambda_2\overline{P}_0(s)+\int_0^\infty \phi_1\overline{P}_4(x,s)dx+\int_0^\infty \phi_2\overline{P}_6(x,s)dx \quad \ldots(29)$$

$$\overline{P}_4(0,s)=\lambda_2\overline{P}_1(0,s)+\lambda_1\overline{P}_3(0,s) \quad \ldots(30)$$

$$\overline{P}_5(0,s)=\lambda_3(\overline{P}_1(0,s)+\lambda_1\overline{P}_2(0,s)) \quad \ldots(31)$$

$$\overline{P}_6(0,s)=\lambda_3(\overline{P}_2(0,s)+\lambda_2\overline{P}_3(0,s)) \quad \ldots(32)$$

$$\overline{P}_7(0,s)=\lambda_s(\overline{P}_0(s)+\overline{P}_2(0,s)+\overline{P}_1(0,s)+\overline{P}_3(0,s)) \quad \ldots(33)$$

$$\overline{P}_8(0,s)=\lambda_h(\overline{P}_0(s)+\overline{P}_3(0,s)+\overline{P}_1(0,s)+\overline{P}_2(0,s)) \quad \ldots(34)$$

Solving (19) -(26) with the help of (27) -(34), one may get

$$\overline{P}_0(s)=\frac{1}{D(s)} \quad \ldots(35)$$

$$\overline{P}_1(s)=\frac{\lambda_1}{D(s)}\frac{(1-S_{\phi_1}(s+\lambda_2+\lambda_3+\lambda_h+\lambda_S))}{(s+\lambda_2+\lambda_3+\lambda_h+\lambda_S)} \quad \ldots(36)$$

$$\overline{P}_2(s)=\frac{\overline{P}_2(0,s)}{D(s)}\frac{(1-S_{\phi_2}(s+\lambda_1+\lambda_3+\lambda_h+\lambda_S))}{(s+\lambda_1+\lambda_3+\lambda_h+\lambda_S)} \quad \ldots(37)$$

$$\overline{P}_3(s)=\frac{\overline{P}_3(0,s)}{D(s)}\frac{(1-S_{\phi_3}(s+\lambda_1+\lambda_2+\lambda_h+\lambda_S))}{(s+\lambda_1+\lambda_2+\lambda_h+\lambda_S)} \quad \ldots(38)$$

$$\overline{P}_4(s)=\frac{\overline{P}_4(0,s)}{D(s)}\frac{(1-S_{\phi_1}(s))}{s} \quad \ldots(39)$$

$$\overline{P}_5(s)=\frac{\overline{P}_5(0,s)}{D(s)}\left\{\frac{(1-S_{\phi_1}(s))}{s}\right\} \quad \ldots(40)$$

$$\overline{P}_6(s)=\frac{\overline{P}_6(0,s)}{D(s)}\left\{\frac{(1-S_{\phi_2}(s)}{s}\right\} \quad \ldots(41)$$

$$\overline{P}_7(s)=\frac{\overline{P}_7(0,s)}{D(s)}\frac{(1-S_{\mu_0}(s))}{s} \quad \ldots(42)$$

$$\overline{P}_8(s)=\frac{\overline{P}_8(0,s)}{D(s)}\frac{(1-S_{\mu_0}(s))}{s} \quad \ldots(43)$$

$$D(s)=\begin{bmatrix}(s+\lambda_1+\lambda_2+\lambda_3+\lambda_h+\lambda_s)-(\lambda_1 S_{\phi_1}(s+\lambda_2+\lambda_3+\lambda_h+\lambda_s)+\overline{P}_2(0,s)S_{\phi_2}(s+\lambda_1+\lambda_3+\lambda_h+\lambda_s)\\+\overline{P}_3(0,s)S_{\phi_3}(s+\lambda_1+\lambda_2+\lambda_h+\lambda_s)+\overline{P}_7(0,s)S_{\mu_0}(s)+\overline{P}_8(0,s)S_{\mu_0}(s))\end{bmatrix} \quad \ldots(44)$$

Where,

$$\overline{P}_2(0,s) = A_1\overline{P}_0(s),\ \ \overline{P}_3(0,s) = \left(\frac{\lambda_3 + \dfrac{\lambda_1\lambda_2\phi_1}{s+\phi_1} + \dfrac{\lambda_3 A_1\phi_2}{s+\phi_2}}{B_1} \right)\overline{P}_0(s) = B_2\overline{P}_0(s)$$

$$\overline{P}_4(0,\, s) = \lambda_1 B_3\overline{P}_0(s),\ \ \overline{P}_5(0,\, s) = \lambda_1(\lambda_3 + A_1)\overline{P}_0(s),\ \ \overline{P}_6(0,\, s) = (\lambda_3 A_1 + \lambda_2 B_2)\overline{P}_0(s)$$

$$\overline{P}_7(0,\, s) = \lambda_s(1+\lambda_1 + A_1 + B_2)\overline{P}_0(s),\ \ \overline{P}_8(0,\, s) = \lambda_h(1+\lambda_1 + A_1 + B_2)\overline{P}_0(s)$$

$$A_1 = \frac{\lambda_2 + \dfrac{\lambda_1\lambda_2\phi_1}{s+\phi_1}}{1 - \dfrac{\lambda_1\phi_1}{s+\phi_1}},\quad B_1 = \left(1 - \frac{\lambda_1\phi_1}{s+\phi_1} - \frac{\lambda_2\phi_2}{s+\phi_2}\right),\quad B_2 = \left(\frac{\lambda_3 + \dfrac{\lambda_1\lambda_2\phi_1}{s+\phi_1} + \dfrac{\lambda_3 A_1\phi_2}{s+\phi_2}}{B_1} \right)$$

$$B_3 = \lambda_2 + B_2$$

The Laplace transformations of the probabilities that the system is in up (i.e. either good or degraded state) and failed state at any time are as follows:

$$\overline{P}_{up}(s) = \overline{P}_0(s) + \overline{P}_1(s) + \overline{P}_3(s) + \overline{P}_3(s) \qquad ...(45)$$

$$\overline{P}_{failed}(s) = \overline{P}_4(s) + \overline{P}_5(s) + \overline{P}_6(s) + \overline{P}_7(s) + \overline{P}_8(s) \qquad ...(46)$$

Particular Cases

When repair follows exponential distribution.

Setting

$$\overline{S}_{\exp[x^\theta + \{\log\phi(x)\}^\theta]^{1/\theta}}(s) = \frac{\exp[x^\theta + \{\log\phi(x)\}^\theta]^{1/\theta}}{s + \exp[x^\theta + \{\log\phi(x)\}^\theta]^{1/\theta}},\ \overline{S}_{\phi_i}(s) = \frac{\phi_i}{s+\phi_i},\ \ i = 1,\, 2,\, 3$$ in equation (45) and

(1) Setting the values of different parameters as λ_1=0.03, λ_2=0.032, λ_3 = 0.025, λ_s = 0.035, λ_h = 0.025, ϕ = 1, θ = 1, x = 1, then taking inverse Laplace transform, one can obtain,

$$\begin{aligned} P_{up}(t) = &-0.01014834e^{(-1.117000t)} + 0.0043108\, e^{(-1.09000t)} + 0.0023228e^{(2.7843131t)} \\ &- 0.0215549e^{(-1.141355t)} - 0.0028393e^{(-1.0973522t)} + 0.00001238e^{(-1.004571t)} \\ &+ 0.00186353\, e^{(-0.986884t)} + 1.00512796e^{(-0.000967t)} \end{aligned} \qquad ...(47)$$

For, t= 0, 10, 20, 30, 40, 50, 60, 70, 80, 90.... One may get different values of $P_{up}(t)$ as shown inTable1.

Time(t)	Availability
0	1.000
10	0.996
20	0.986
30	0.976
40	0.967
50	0.958
60	0.949
70	0.939
80	0.930
90	0.921
100	0.913

Table 1. Time vs. Availability

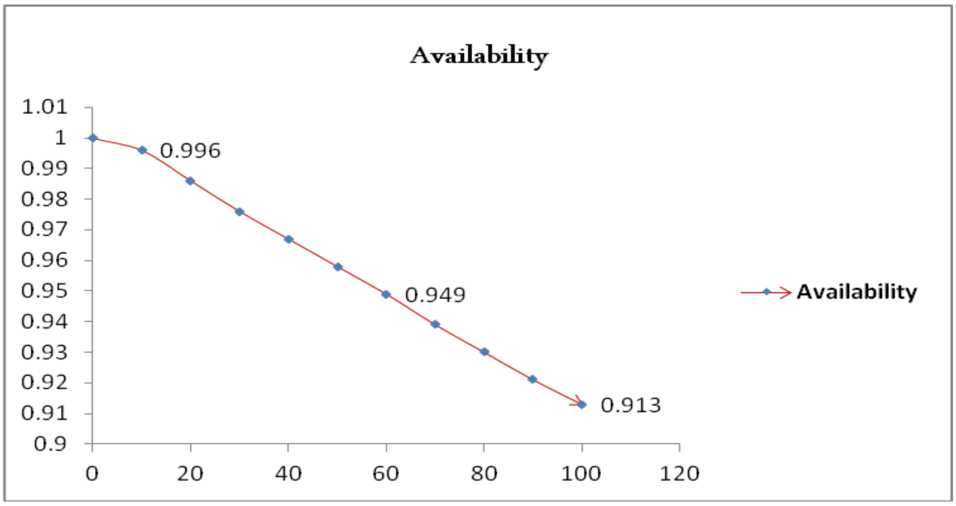

Fig 1. Time vs. Availability

Reliability Analysis

Taking all repairs to zero in equation (45) and then taking inverse Laplace transform, for given failure rates λ_1=0.03, λ_2=0.032, λ_3 = 0.025, λ_s = 0.035, λ_h = 0.025 one can get the expression for reliability;

$$R(t) = -2.0312500\, e^{(-0.1470000t)} + e^{(-0.117000t)} + 1.03125500 e^{(-0.1150000t)} + e^{(-0.122000t)} \quad .....(48)$$

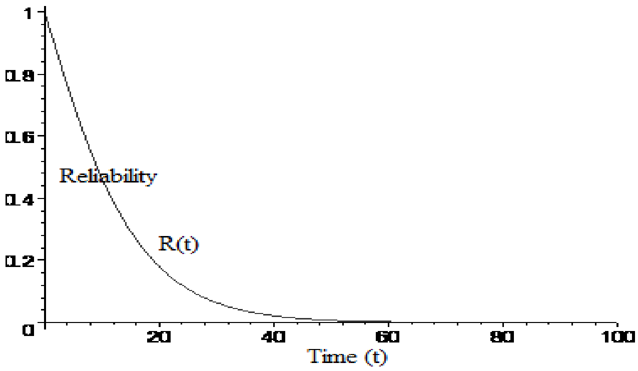

Fig 2. Time vs Reliability

Mean Time To Failure (Mttf)
Setting

$\bar{S}_{\exp[x^\theta+\{\log\phi(x)\}^\theta]^{1/\theta}}(s) = \dfrac{\exp[x^\theta + \{\log\phi(x)\}^\theta]^{1/\theta}}{s+\exp[x^\theta+\{\log\phi(x)\}^\theta]^{1/\theta}}$, $\bar{S}_{\phi_i}(s) = \dfrac{\phi_i}{s+\phi_i}$, $i = 1, 2,3$, and taking all repairs to zero in equation (45). Taking limit, as s tends to zero one can obtain the MTTF as:

$$M.T.T.F = \frac{1}{(\lambda_1+\lambda_2+\lambda_3+\lambda_s+\lambda_h)}\left[1+\frac{\lambda_1}{(\lambda_2+\lambda_3+\lambda_s+\lambda_h)}+\frac{\lambda_2}{(\lambda_1+\lambda_3+\lambda_s+\lambda_h)}+\frac{\lambda_3}{(\lambda_1+\lambda_2+\lambda_s+\lambda_h)}\right] \quad \ldots (49)$$

Setting $\lambda_2 = 0.032,\ \lambda_3 = 0.025,\ \lambda_s = 0.025,\ \lambda_h = 0.035,$ and varying λ_1 as 0.01, 0.02, 0.03, 0.04, 0.05, 0.06, 0.07, 0.08, 0.09 in (49) one may obtain Table 2 whose colamn2 demonstrates variation of MTTF with respect to λ_1.

Setting $\lambda_1 = 0.03,\ \lambda_3 = 0.025,\ \lambda_s = 0.025,\ \lambda_h = 0.035,$ and varying λ_2 as 0.01, 0.02, 0.03, 0.04, 0.05, 0.06, 0.07, 0.08, 0.09 in (49) one may obtain Table 2 whose column 3 demonstrates variation of MTTF with respect to λ_2.

Setting $\lambda_1 = 0.03,\ \lambda_2 = 0.032,\ \lambda_s = 0.025,\ \lambda_h = 0.035,$ and varying λ_3 as 0.01, 0.02, 0.03, 0.04, 0.005, 0.06, 0.07, 0.08, 0.09 in (49) one may obtain Table 2, whose column 4 shows variation of MTTF with respect to λ_3.

Setting $\lambda_1 = 0.030\, \lambda_2 = 0.032,\ \lambda_3 = 0.025,\ \lambda_h = 0.035,$ and varying λ_s as 0.01, 0.002, 0.03, 0.04, 0.05, 0.06, 0.07, 0.08, 0.09 in (49) one may obtain Table 2, which reveals variation of MTTF with respect to λ_s in column5.

Setting $\lambda_1 = 0.030\, \lambda_2 = 0.032,\ \lambda_3 = 0.025,\ \lambda_s = 0.025,$ and varying λ_h as 0.01, 0.02, 0.03, 0.04, 0.05, 0.06, 0.07, 0.08, 0.09 in (49) one may obtain Table 2, which reveals variation of MTTF with respect to λ_h in column 6.

Table 3. Variation of MTTF with respect to failure rates.

Failure rate	MTTF λ_1	MTTF λ_2	MTTF λ_3	MTTF λ_s	MTTF λ_h
0.01	1.656	1.656	1.655	2.514	3.306
0.02	1.652	1.653	1.650	2.454	3.205
0.03	1.648	1.649	1.646	2.398	3.111
0.04	1.645	1.645	1.642	2.344	3.021
0.05	1.641	1.642	1.638	2.292	2.937
0.06	1.634	1.639	1.635	2.243	2.857
0.07	1.631	1.636	1.631	2.196	2.781
0.08	1.628	1.633	1.628	2.150	2.709
0.09	1.625	1.630	1.625	2.106	2.641

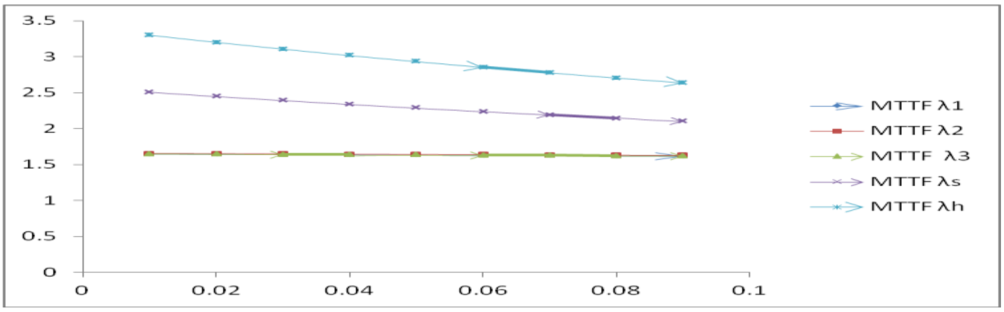

Fig 3. Failure rate v/s MTTF

Cost analysis

(a) Let the failure rates of system be λ_1=0.030, λ_2=0.032, λ_3 = 0.025, λ_s = 0.025, λ_h = 0.035, mean time to repair of be $\phi(x) = 1$, and $x = 1, \theta = 1, \phi(x) = 1$

Setting,

$$\bar{S}_{\exp[x^\theta + \{\log\phi(x)\}^\theta]^{1/\theta}}(s) = \frac{\exp[x^\theta + \{\log\phi_A(x)\}^\theta]^{1/\theta}}{s + \exp[x^\theta + \{\log\phi_A(x)\}^\theta]^{1/\theta}},\quad \bar{S}_{\phi_i}(s) = \frac{\phi_i}{s + \phi_i},\ i = 1, 2, 3$$

in equation (45) and taking inverse Laplace transform, one can obtain (50).

Let the service facility be always available, then expected profit during the interval [0, t) is

$$E_p(t) = K_1 \int_0^t P_{up}(t)dt - K_2 t$$

Where K_1 and K_2 are revenue service cost per unit time . Hence

$$\begin{aligned} E_p(t) = K_1(&0.009085355\, e^{(-1.117000t)} - 0.003954886 e^{(-1.090000t)} - 0.00834241\, e^{(-2.784313t)} \\ &+ 0.018885407 e^{(-1.1413558t)} + 0.002587397 e^{(-1.097353t)} - 0.0000123246 e^{(-1.0045700t)} \\ &- 0.00188830\, e^{(-0.9868837t)} - 1039.8293\, e^{(-0.000966627t)} + 1039.813) - K_2\, t \quad ...(50) \end{aligned}$$

Setting K_1 = 1and K_2 = 0.50, 0.40, 30, 0.20, and 0.10, respectively and varying t =0, 10, 20, 30, 40, 50, 60, 70, 80, 90,100..., one get Table 4.

Table 4. For expected profit with respect of time

Time(t)	$E_p(t)$; K_2=0.5	$E_p(t)$; K_2=0.4	$E_p(t)$; K_2=0.30	$E_p(t)$; K_2=0.20	$E_p(t)$; K_2=0.10
0	0.000	0.000	0.000	0.000	0.000
10	4.987	5.987	6.987	7.987	8.987
20	9.893	11.893	13.893	15.893	17.893
30	14.704	17.704	20.705	23.705	26.705
40	19.421	23.422	27.421	31.421	35.421
50	24.045	29.045	34.045	39.045	44.045
60	28.576	34.576	40.576	46.576	52.576
70	33.015	40.015	47.015	54.015	61.015
80	37.363	45.363	53.364	61.363	69.363
90	41.622	50.622	59.622	68.622	77.622

100	45.791	55.791	65.791	75.791	85.791

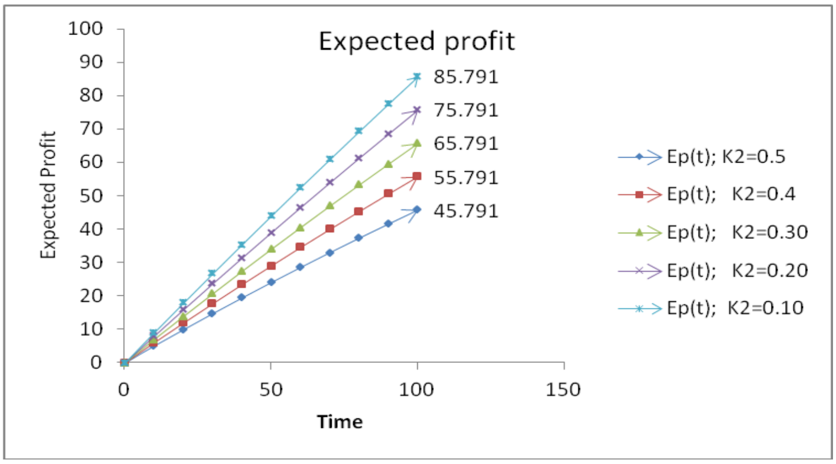

Fig 4. For expected profit with respect of time

Result discussion and conclusion

Tables 1 and Fig. 1 provide information how availability of the complex repairable system changes with respect to time when failure rates are fixed at different values. When failure rates are fixed at lower values $\lambda_1 = 0.030$, $\lambda_2 = 0.032$, $\lambda_3 = 0.025$, $\lambda_h = 0.025$, $\lambda_s = 0.035$, availability of the system decreases and probability of failure increase, with passage of time and ultimately becomes steady to the value zero after a sufficient long interval of time. Hence, one can safely predicts the future behavior of complex system at any time for any given set of parametric values, as is evident by the graphical consideration of the model.

Table 2, provide the information of reliability of system when the no repair is employed for the system. Evidently, the figure 2 clearly explains that the reliability of system is less than the availability of system. Which indicate necessity of employing repair for in repairable system?

Tables 3, yield the mean-time-to-failure (MTTF) of the system with respect to variation in λ_1, λ_2, λ_3, λ_s, and λ_h respectively when other parameters have been taken as constant and variation in values of corresponding failure rate. Fig. 3 shows the variation in MTTF corresponding to failure rates. Evidently, the MTTF decreases as failure rate increases. The MTTF corresponding to failure rates λ_1, λ_2, λ_3 very much closure but for the values λ_s and λ_h are quite different and comparatively are high. This increases that indicates that the failure λ_s and λ_h are more responsible for proper operation of the system.

When revenue cost per unit time K_1 fixed at 1, service cost $K_2 = 0.5, 0.40, 0.30, 0.20, 0.10$, profit has been calculated and results are demonstrated by graphs in figure 4. One can observed that as the service cost decreases profit increases.

Researchers further can for discussion like comparative study of copula for the particular system. This system can analyze by help of other types of copula like Archimedean copula Carleton copula, Franklin copula. Sensitivity analysis of the system is left for future research.

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