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Development of a calculator software for determining the dominant eigenvalue and eigenvector of n x n matrices

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ABSTRACT

The desire to economize time and mental effort in an arithmetical computations and to eliminate human liability to error is probably as old as the science of arithmetic itself. This desire led us to the design of a calculator software to estimate the dominant eigenvalue and the corresponding eigenvectors of an n x n matrix up to the tenth order. The method used in designing the software includes developing a Graphical User Interface (GUI) application with MATLAB (Matrix Laboratory) using the Rapid Application Development (RAD). RAD is a software development methodology that involves methods like iteration development and software prototyping. This RAD is used to refactor our codes and amend the design should there be complaints from the users. The application software was tested to estimate the largest eigenvalue and the corresponding eigenvector of an n x n matrix using the power method with accuracy and reduced time.

Introduction

A calculator is a device that can be designed to process many different types of mathematical calculations and there are many different types of calculators [18]. The design and size of calculators vary based on what they are to be used for. Some calculators are designed for specific purposed such as different trades or businesses while others can be used for basic mathematical calculations like addition and multiplication [17].

A standard calculator is rectangular in shape and has a keypad through which numbers and operations are entered, as well as a display on which the entered numbers and the results of calculations are shown [12]. Modern calculators can perform many types of mathematical computations, as well as permit the user to store and access data from memory. Common handheld calculators have the ability to use complicated geometric, algebraic, trigonometric, statistical, and calculus functions. Many can also be programmed for specialized tasks. Calculators operate on electrical power supplied by either batteries, solar cells, or standard electrical current. Modern calculators have digital displays, usually using some form of Liquid Crystal Display (LCD) [6].

Calculator programs are common accessories included with most personal computer operating systems. For example, both the Macintosh and Windows operating systems have a simple desktop calculator program.

Modern calculators generally have many more features than a standard four or five function calculator and the feature set differs between manufacturers and models. However, the defining features of a scientific calculator includes: scientific notation, floating point arithmetic, trigonometric functions, logarithmic function using base 10 and base e, exponential functions and roots beyond the square root, quick access to constants such as pi (π) and e [3, 7, 9].

In this paper, we have developed a scientific calculator software that can evaluate the largest (dominant) eigenvalues and eigenvectors of n x n matrices using the power method.

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Eigenvalues and Eigenvectors

Eigenvalues and eigenvectors play an important part in the application of linear algebra. The words are derived from the German word eigen which means “owned by” or “peculiar to”. Eigenvalues and eigenvectors are sometimes called the following names either characteristics values and characteristics vectors or proper values and proper vectors or latent values and latent vectors [2].

For an n x n matrix A, scalars λ and vector $x \neq 0$ satisfying $Ax = \lambda x$ are called the eigenvalues and eigenvectors of A respectively and any such pair (λ, x) is called an eigenpair for A.

To find the eigenvalues and eigenvectors of a matrix say A, we try to solve the equation $\det(A - \lambda I) = 0$ which is called the characteristic polynomial for A. Consequently, the eigenvalues for A are the solutions of the characteristic equation $p(\lambda) = 0$ i.e. the roots of the characteristic polynomial. The eigenvectors associated with the eigenvalues say $\lambda 1$, $\lambda 2$, ---, λn are simply the non-zero vectors in the eigen-spaces. Although, there are an infinite number of eigenvectors associated with each eigenvalue and each eigen-space is one dimensional [5]. In industrial sized matrices, however, this method is not feasible, and the eigenvalues must be obtained by other means. Fortunately, there exist several other techniques for finding eigenvalues and eigenvectors of a matrix, some of which fall under the realm of iterative methods for example the power method..

Let $\lambda 1$, $\lambda 2$, ---, λn be the eigenvalues of an n x n matrix A. $\lambda 1$ is called the dominant eigenvalue of A if $| \lambda 1| > | \lambda i|$ (i = 2, 3, - - -, n). The eigenvectors corresponding to $\lambda 1$ are called the dominant eigenvectors of A [8].

Power Method

Like the Jacobi and Gauss – Seidel methods, the power method for approximating eigenvalue is iterative [8]. First, we assume that the matrix A has a dominant eigenvalue with the corresponding dominant eigenvectors. Then, we choose an initial approximation x0 of one of the dominant eigenvectors of

A. This initial approximation must be a non-zero vector in Rn. Finally, we form the sequence given by;

$x_1 = Ax_0$ (1)

$x_2 = Ax_1 = A(Ax_0) = A^2x_0$ (2)

$x_3 = Ax_2 = A(Ax_1) = A^3x_0$ (3)

$x_4 = Ax_3 = A(Ax_2) = A^4x_0$ (4)

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$x_{n+1} = Ax_n = A(A^nx_0) = A^{n+1}x_0$ (5)

For large powers of n and by properly scaling this sequence, we will see that we obtain a good approximation of the dominant eigenvector of A [5].

For large values of n, polynomial equations may be difficult and time – consuming to solve. Moreover, numerical techniques for approximating roots of polynomial equations of high degree are sensitive to rounding errors. As shown above, the power method can be used only to find the eigenvalue of A that is largest in absolute value – we call this eigenvalue the dominant eigenvalue of A. Although, this restriction may seem severe, dominant eigenvalues are of primary interest in many physical applications [3].

Related Works on Calculating Devices

Early devices used to aid in calculation include the abacus (still common in East Asia) and the counting rods, or "bones," of the Scottish mathematician John Napier. The slide rule, invented in 1622 by William Oughtred, an English mathematician, was widely used to make λ (lambda) approximate calculations, but it has been replaced by the electronic calculator. In 1642, Blaise Pascal devised what was probably the first simple adding machine using geared wheels. In 1671 an improved mechanism for performing multiplication by the process of repeated addition was designed by Gottfried W. von Leibniz. A machine using the Leibniz mechanism was the first to be produced successfully on a commercial scale; devised in 1820 by the Frenchman Charles X. Thomas, it could be used for adding, subtracting, multiplying, or dividing. A mechanism permitting the construction of a more compact machine than the Leibniz mechanism was incorporated into a machine devised late in the 19th century by the American inventor Frank S. Baldwin [16].

Later the machine was redesigned by Baldwin and another American inventor, Jay R. Monroe. At about the same time, W. T. Odhner of Russia constructed a machine using the same device as Baldwin's. Charles Babbage, an English mathematician, and William S. Burroughs, an American inventor, also made important contributions to the development of the calculating machine [15].

Early mechanical adding machines were equipped with a keyboard on which numbers to be added were entered, a lever to actuate the addition process, and an accumulator to display the results. A full keyboard consisted of 10 columns of keys with 9 keys in each column, numbered 1 through 9. Each column could be used to enter a figure in a particular decimal place so that a number up to 10 digits long could be entered; if no key was pressed in a given column, a zero was entered in that decimal place. The lever was pulled in one direction when a number was to be added and in the opposite direction when it was to be subtracted. The accumulator was a set of geared wheels, each corresponding to a decimal place and having the digits 0 through 9 printed on its circumference. When a given wheel made a complete rotation, the next wheel was advanced by one digit. The mechanical adding machine remained essentially the same until the mid-1960s, with improvements consisting of motors to actuate additions and subtractions and mechanisms to print out results on a paper tape [1, 10].

Electronic calculators, which became available in the early 1960s, at first were merely faster and quieter adding machines. The invention of the microprocessor and advances in integrated-circuit technology made small, but highly sophisticated, calculators possible, and by the mid-1970s they were in wide use [14].

Simple calculators perform only the basic four functions of addition, subtraction, multiplication, and division. More sophisticated calculators can perform trigonometric, statistical, logarithmic, and other advanced calculations. Some electronic calculators are actually small computers with limited memory and programming capabilities. Some of these programmable calculators can accept plug-in semiconductor memory cards or programming modules for special applications, such as financial calculations, unit, currency, or number- system conversions, or engineering calculations. Others are also available that include nonmathematical functions such as data storage and schedule organizing. The personal digital assistant, a hand-held device optimized as an organizer with communications capability and accepting handwritten input, is a bridge from calculators to full computer function. Early electronic calculators had numeric displays made from Light-Emitting Diodes (LEDs). They have been supplanted by Liquid-Crystal Displays (LCDs), whose lower power consumption helps to reduce battery drain. Some calculators use an LCD readout to provide a graphic, as well as numeric, display CMOS or complementary metal-oxide-semiconductor (see integrated circuit), technology is also preferred for battery-operated models because of its low-power requirements. Some calculators are powered by solar cells in ordinary room light [11].

Design Methodology

For the implementation of this calculator software for power method, the software required are Microsoft Windows Operating System for the program to be usable on any system which supports the windows operating system and Matlab (Matrix Laboratory), for both the front end and the back end of the application [4].

The MATLAB Graphical User Interface Development Environment provides a set of tools for creating graphical user interfaces (GUIs). These tools greatly simplify the process of laying out and programming GUIs. The GUIDE Layout Editor enables you to populate a GUI by clicking and dragging GUI components into the layout area. There you can resize, group and align buttons, text fields, sliders, axes, and other components you add [4].

The hardware requirement is computer and its peripheral devices. There are no existing external and central databases or files that will be needed by this program. The peripheral devices includes CD-ROM or an external flash drive as an auxiliary storage to store the installer of the application software.

The development of the calculator software was done using Rapid Application Development (RAD) Approach. RAD is a software development methodology that involves methods like iteration development and software prototyping. RAD is used because it allows us to refactor our code and also to amend the designs should there be complaints from the users during the process of debugging. RAD usually embrace Object Oriented Programming methodology which inherently fosters software re-use. It develops in modules where each module is secluded from every other module and error in one does not restrict other modules from functioning properly.

The following subheadings discuss each of the referenced namespaces (modules) that were used in the development of the calculator software for power method:

Polynomial Equation Solver

A polynomial is an expression of finite length constructed from variables (also called indeterminate) and constants, using only the operation of addition, subtraction, multiplication, and non-negative integer exponents. Solving polynomial in matlab involves entering the coefficient of the polynomial and storing it as a variable, say c, and then r = roots(c) returns a column vector whose elements are the roots of the polynomial c.

Row vector c contains the coefficients of a polynomial, ordered in descending powers. If c has n+1 components, the polynomial it represents is

$$C_1 s^n + \cdots + C_n s + C_{n+1}$$

Experimental Design

This section explains the graphical user interface (GUI) design used in this work. The work is opened into five iterations of the window application.

The first tab in the iteration is arithmetic solver which contains arithmetic calculator in the figure 1. The arithmetic calculator is used for solving simple calculation; it contains the function that we would be found in a normal calculator.

Figure 1. Arithmetic calculator Interface

Note: To use the arithmetic calculator, the user must first input a number and (not expressions like *,-, +, /, %) into the parser, if not the user will get an invalid entry error which is shown in the figure 2. The second tab contains the scientific calculator which is for advanced calculation for trigonometry and other mathematical function such as factorial, logarithm, exponential, square, cube, pi, open and close brackets which is shown in figure 3

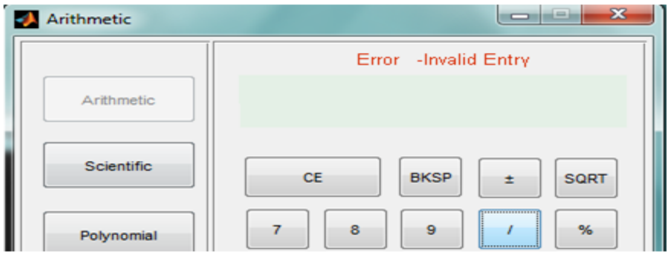

Figure 2. Arithmetic calculator showing invalid entry error

Figure 3. Scientific Calculator Interface

Note: To use the scientific calculator, the user must first input a number and (not expressions like *,-,+,/,%,sin,cos,tan,abs,pi,n!,ln etc.) into the parser, if not the user will get an invalid entry error which is shown in figure 4.

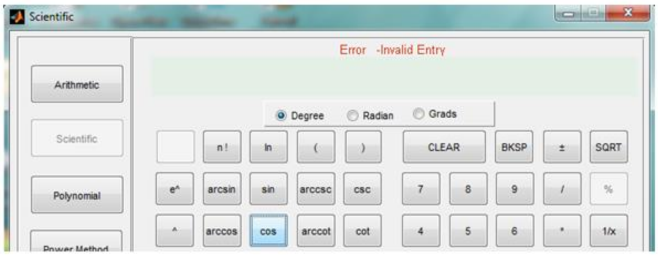

Figure 4. Scientific Calculator showing invalid entry error

The third tab is the polynomial solver shown in figure 5, which is capable of solving polynomial equations up to the tenth degree. To use the polynomial calculator, the user needs to enter the coefficient of the polynomial equation into the square brace. Although the brace is already indicated so the user just has to change the value in the brace.

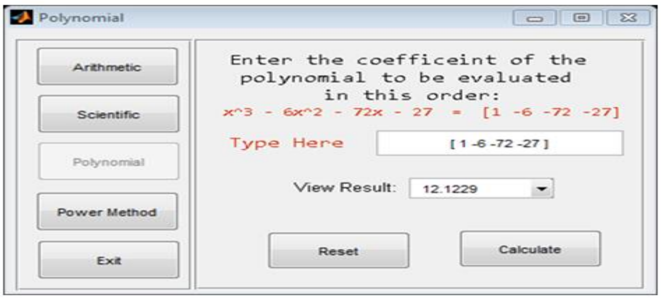

Figure 5. Polynomial Calculator interface

The fourth tab is for Power Method shown in figure 6, which estimates the largest (dominant) eigenvalue and the corresponding eigenvector. Firstly the user must input the following to be able to do an Eigen calculation;

- Matrix into the matrix textbox,
- Initial value for xo
- Max number of iteration.

After the calculate button is clicked the calculation is displayed in the command-prompt and the results and graph are displayed respectively.

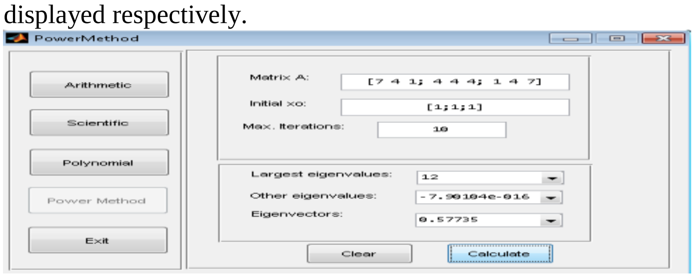

Figure 6: Power Method Calculator Interface

Quitting the Program

To quit the program, the user has to click on the exit tab or button and cancel the session that was created

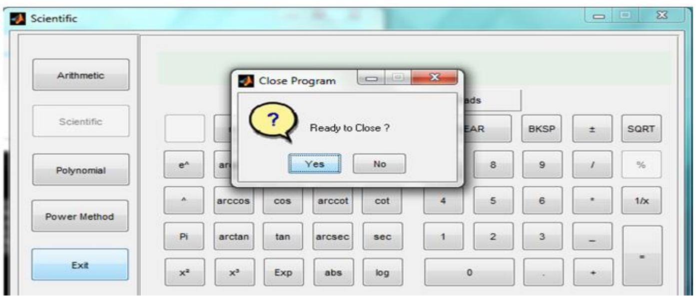

Figure 7. How to quit a program

Conclusion

The calculator software for evaluating the dominant eigenvalues and eigenvectors using power method has been developed, tested and found to meet the intended requirements. This software can be installed on personal computers since many people make use of laptops or desktops nowadays. It will therefore be very important to have this calculator software for power method installed on the laptops because it offers an additional modules like the polynomial solver, which is not present on the Microsoft calculator that comes with the Windows installation. Ultimately, the power method solver is to encourage convenience and efficiency when solving for dominant eigenvalues and eigenvectors which are used in eigenfaces calculation and other calculations that have to do with eigenvalue and eigenvector computations.

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