Available online at www.elixirpublishers.com (Elixir International Journal)

Applied Mathematics

Elixir Appl. Math. 83 (2015) 33069-33074

ISSN: 2229-712X

An Integrated Vendor-Buyer Investment Model for Reducing the Defect Rate with Delay in Payments

W.Ritha and S.Haripriya
Department of Mathematics, Holy Cross College (Autonomous), Tiruchirappalli - 620002, India.

ARTICLE INFO

Article history:
Received: 20 April 2015;
Received in revised form:
29 May 2015;
Accepted: 8 June 2015;

Keywords
Defective Items,
Imperfect Process,
Integrated Model,
Delay in Payments,
Stochastic Demand.

ABSTRACT

In modern competitive business environment, it is essential to continuously work on improving the performance of their supply chains with high customer satisfaction. Inventory management is affected by the imperfect production process. So it is contemporary for the vendor to improve quality of the product/service where customer's loyalty will be the by-product. In this paper vendor invests money to reduce the number of defective items produced. Also in many transactions regarding selling and buying, a specified delay of payment is offered or accepted by the vendor. This can be viewed as a kind of discount and has potential consequences for the order size. Differently from the existing literature this paper extends the model with the inclusion of stochastic demand, stock out cost and delay in payments. Numerical studies explore the beneficial of the vendor investment.

Introduction

Global market influences on the tighter competition and persuades the companies to improve the performance of their supply chain. Enterprises understand that efficient management of inventories across the entire supply chain can be achieved through better co-ordination and more co-operations of all parties involved as a joint benefit. Hence an integrated supply chain management is required to achieve cost reduction and increase profitability. People are forced to take advantages of any opportunity to optimize their business processes and improve the performance of the entire supply chain. To provide mathematical models that more closely conform to actual inventories and respond to the factors that contribute to inventory costs, the models must be extended or altered.

Analyzing the literature, Goyal(1976)[6] was the first to develop an integrated inventory model for a single-supplier single-buyer problem. The generalized model of integrated vendor–buyer problem is due to Banerjee (1986)[1]. Most of the models assume that demand is deterministic. Porteus(1986)[16] incorporates the effect of defective items into the basic EOQ model and introduces the option of investing in-process quality improvement by means of reducing the process quality parameter, which could move the process out of control. Lee and Rosenblatt (1987) [13] consider process inspection during the production run so that a shift to an out-of-control state could be detected and restored earlier than conventional EOQ models. Goyal (1988)[7] argued that producing a batch which is made up of equal shipments generally produced lower cost but the whole batch must be completed before the first shipment is made. Schwaller (1988)[21]extends the EOQ by adding the assumption that defective items of a known proportion are present in incoming lots and that fixed and variable inspection costs are incurred in finding and removing the items. Zhang and Gerchak (1990) [22] consider a joint lot sizing and inspection policy in an EOQ model where a random proportion of units are defective.

Salameh and Jaber's(2000)[17] developed joint lot sizing and inspection policy under an EOQ model. Goyal and Cardenas-Barron (2002) [8] presented a simple approach for determining the economic production quantity for an item with imperfect quality. Huang (2004)[9] developed an integrated vendor–buyer inventory model for items with imperfect quality and equal shipment size in a deterministic framework, Shortages or any investment was however not considered. Ouyang et al. (2006)[14] investigated an integrated model with imperfect production but did not consider any investment, re-order point or shortages. Shu and Zhou (2014) [20] proposed an integrated single-vendor single-buyer model in which the products are sold with free minimal repair warranty.

Goyal [1](1985) was the first to develop a model for a delay in payment to the supplier, making all the usual assumptions of the classic EOQ model except for when payment is due. Hwang and Shinn [10] discussed delay in payments in their model for retailer's pricing and lot sizing policy. Jamal et al. [12] presented a model for an ordering policy with allowable shortages and permissible delay in payments. Sarkar [18] considered delay in payments with stock dependent demand to investigate the retailer's optimal replenishment policy in an EOQ model.

This paper develops a model to determine an optimal integrated vendor-buyer production inventory model for flawed items. As a result of weak process control, deficient planned maintenance, inadequate work instructions and/or damage in transit, an arriving order lot often includes defective items. In an integrated model, since the production is controlled by the vendor who has to pay warranty cost for defective items, it is beneficial to him, in particular, and to the supply chain as a whole, to invest in reducing the number of defective items produced. For integrated models with imperfect production process, it is very likely that the buyer performs some sort of inspection activity before selling the products to the customers. Ignoring this inspection/screening period or assuming it to be negligible is not very practical. Therefore, we assume that the buyer performs an error- free and non-destructive screening in a non-negligible finite period. At the end of the screening period, all the defective items in each lot are returned to the vendor at the time of the next delivery. To imply the consideration of real market behaviour rework of defective items is included in this paper.

Tele:
E-mail addresses: ritha_prakash@yahoo.co.in

Here in this paper we relax the assumption of deterministic demand and consider delay in payments with stock out cost. Since it is assumed that the vendor makes an investment in improving the process quality, therefore, the defect rate is assumed to be an additional control parameter together with the number of shipments from the vendor to the buyer, and the buyer's order quantity. This paper is organized as follows; Section 2 formulates the mathematical model. Section 3 develops the solution procedure for the proposed model. Section 4 illustrates the developed model with numerical example. The paper is concluded in Section 5.

Mathematical Modeling

An integrated vendor–buyer inventory model with defective items is developed on the basis of the following assumptions and notations. In this paper we extended the model developed by O.Dey [4]

Notations

D - Expected demand rate (units/time) non-defective items
Q -Order quantity (units)
P - Production rate (P=1/p)
r - Re-order point
A - buyer's ordering cost per order
K - vendor's setup cost
F -Transportation cost per delivery
h_v - vendor's holding cost per item per year
h_{b1} - buyer's holding cost for defective items per item per year
h_{b2} - buyer's holding cost for non-defective items per item per year
s - buyer's unit screening cost
p_1 -unit cost of an item
x - buyer's screening rate
w - vendor's unit warranty cost for defective items
y - percentage of defective items produced
t_c -credit period(year)
I_d - rate of interest earned due to financial inventory
I_c - rate of interest charged due to credit balance
π - buyer's shortage cost per item per year
π_0-gross marginal profit per unit
C_R -rework cost per defective unit
η - fractional opportunity cost
δ - percentage decrease in defective items per dollar increase in investment
β -fraction of the demand backordered during the stock our period
B(r) -Expected shortage quantity at the end of cycle
S - Safety Stock
L - Lead time (in weeks)

Assumptions

- A single buyer orders items of a single product from a single vendor.
- Demand per unit time is normally distributed with mean D and standard deviation σ.
- The buyer places an order of nQ (non-defective) items to the vendor. The vendor produces these items and, on average, transfers these items to the buyer in n equal sized shipments, where n is a positive integer.
- Items in each lot will be inspected and defective items are returned to the supplier at the time of delivery of the next lot.
- The buyer follows the classical (Q,r) continuous review inventory policy.
- The lead time L is a constant. The demand during lead time is normally distributed with mean DL and standard deviation $\sigma\sqrt{L}$.
- The re-order point r =expected demand during lead-time+ safety stock (SS)
- i.e., $r = DL+k\sigma\sqrt{L}$, where k is the safety stock factor.
- Shortages are allowed.
- $y(0<y<1)$ is the percentage of defective items produced in each batch of size Q.
- The vendor's rate of production of non-defective items is greater than the demand rate
i.e., $P(1-y)>D$.
- The fraction of demand backordered during the stock out period β is considered as a constant.
- The interest rate applicable to the stock value after credit period Ic is greater than the rate of interest earned due to financial inventory I_d.
- t_c is greater than the reorder interval i.e. credit period should not be longer than the time at which next order is placed.
- The screening rate x is fixed and is greater than the demand rate i.e., $x>D$.
- The vendor incurs a warranty cost for each defective item produced.
- The vendor invests money to improve the production process quality in terms of buying new equipment, improving machine maintenance and repair, worker training, etc. We consider the following logarithmic investment function I(y) (Porteus, 1986)[16]:
$I(y)= 1/\delta \ln (y_o/y)$ where δ is the percentage decrease in y per dollar (or any other suitable currency) increase in investment and y_0 is the original percentage of defective items produced prior to investment.

Suppose that the buyer places an order of size nQ for non- defective items to the vendor. In order to reduce the production cost, the vendor produces these nQ items at one go and transfers n batches of Q items each at regular intervals of Q(1-y)/D units of time on average. The length of each ordering cycle is Q(1-y)/D and the length of the complete production cycle is nQ(1-y)/D.

Buyer's Perspective

Assume that as soon as the inventory of non-defective items reaches the level called the re-order point r, the buyer places an order of size Q for non-defective items to the vendor (Fig.1). When the order arrives, buyer inspects the items at a fixed screening rate x. It is assumed that the screening process is non-destructive and error-free. The buyer, therefore, has two types of holding cost – for defective items and non-defective items. The buyer's average inventory level for non-defective items (including those defective items which have not yet been detected before the end of the screening time Q/x) is given by

$$\frac{nQ(1-y)}{D}\left[k\sigma\sqrt{L}+\frac{Q(1-y)}{2}+\frac{DQy}{2x(1-y)}\right] \qquad (1)$$

For defective items

$$nQ^2y\left[\frac{1-y}{D}-\frac{1}{2x}\right] \qquad (2)$$

The annual expected total cost for the buyer including the ordering cost, shipment cost, holding cost, shortage cost, screening cost and total interest derived during the credit period is, therefore, given by

$$ETCB\ (Q,y,n)\ =\frac{D(A+nF)}{nQ(1-y)}+h_{b1}\left[Qy-\frac{DQy}{2x(1-y)}\right]+h_{b2}\left[k\sigma\sqrt{L}+\frac{Q(1-y)}{2}+\frac{DQy}{2x(1-y)}+(1-\beta)B(r)\right]$$
$$+\frac{\Pi D\sigma\sqrt{L}\psi(k)}{Q(1-y)}+\frac{sD}{1-y}-\frac{Dp_1t_c{}^2I_d}{2}-B(r)p_1t_cI_d \qquad (3)$$

$\psi(k)=\phi(k)-k(1-\Phi(k))>0$ where ϕ denotes the standard normal probability distribution function and Φ denotes the cumulative distribution function. Also B(r) = $\sigma\sqrt{L}\psi(k)$

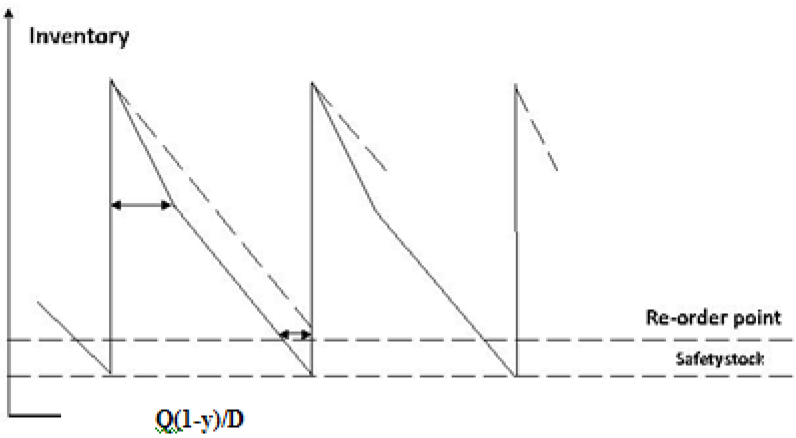

Figure 1. Inventory of the buyer

Vendor's Perspective

The vendor produces Q items in the first instance and delivers those to the buyer during the production process. After that, the vendor delivers a quantity Q to the buyer every T units of time where T =Q (1-y)/D. This continues till the vendor's production run is completed (Fig. 2). The annual expected total cost acquired by the vendor is the sum of setup cost, holding cost, warranty cost for the defective items, stock out cost, rework cost, interest charged to the portion of cycle stock.

$$\text{ETCV(Q,n)}=\frac{KD}{nQ(1-y)}+h_v\frac{Q}{2}\left[n\left(1-\frac{Dp}{1-y}\right)-1+\frac{2Dp}{1-y}\right]+\frac{wDy}{1-y}+\frac{C_R(1-y)(Q-(1-y))}{D} \qquad (4)$$
$$(\Pi+\Pi_0(1-\beta)B(r))+\frac{(Q-Dt_c)^2\ p_1I_c}{2D}$$

The total cost in (4) does not include any investment on the part of the vendor to improve the process quality. Therefore, it is quite appropriate for the vendor to make an investment to try and reduce the number of defective items produced.

Assuming a logarithmic investment function of the form I(y) = 1/δ ln (y_o/y), the expected annual total cost of the vendor can be obtained as

$$\text{ETCV(Q,n)}=\frac{KD}{nQ(1-y)}+h_v\frac{Q}{2}\left[n\left(1-\frac{Dp}{1-y}\right)-1+\frac{2Dp}{1-y}\right]+\frac{wDy}{1-y}+\frac{C_R(1-y)(Q-(1-y))}{D}$$
$$+(\Pi+\Pi_0(1-\beta)B(r))+\frac{(Q-Dt_c)^2\ p_1I_c}{2D}+\frac{\eta}{\delta}\ln\left(\frac{y_0}{y}\right) \qquad (5)$$

where η is the fractional opportunity cost. It may be noted here that this logarithmic function I(y) is a convex function with respect to y for all y ($0<y<y_0<1$).

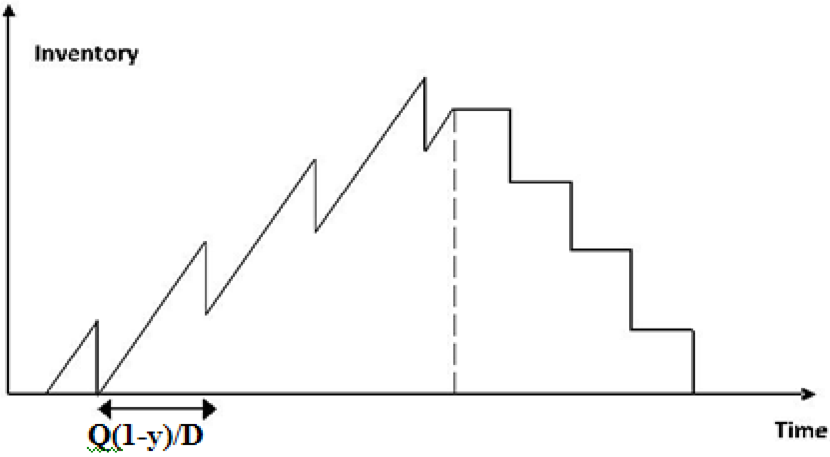

Figure 2. Inventory of the vendor

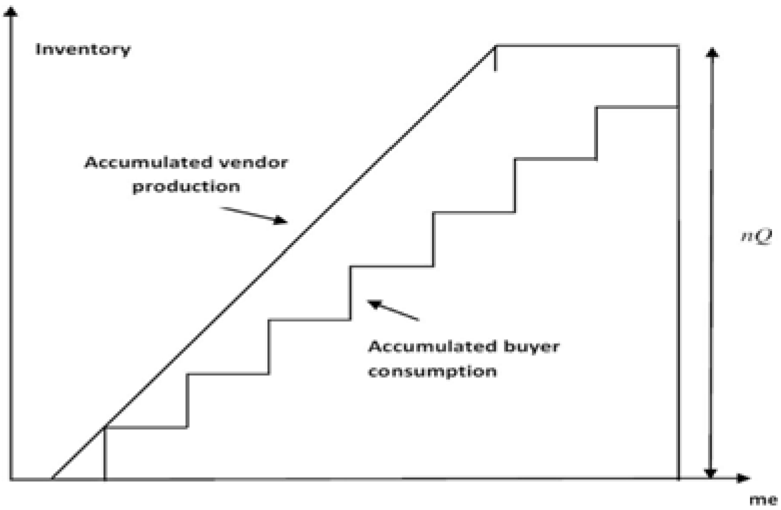

Figure 3. Vendor's inventory holding area

Integrated approach

The expected annual total cost of the integrated system is the sum of the vendor's and the buyer's expected annual total costs which is given by

$$ETC\ (Q,y,n) = \frac{D(A+K+nF)}{nQ(1-y)} + h_{b1}\left[Qy - \frac{DQy}{2x(1-y)}\right] + h_{b2}\left[k\sigma\sqrt{L} + \frac{Q(1-y)}{2} + \frac{DQy}{2x(1-y)} + (1-\beta)B(r)\right]$$
$$+\frac{\Pi D\sigma\sqrt{L}\psi(k)}{Q(1-y)} + h_v\frac{Q}{2}\left[n\left(1-\frac{Dp}{1-y}\right) - 1 + \frac{2Dp}{1-y}\right] + \frac{(s+wy)D}{1-y} - \frac{Dp_1 t_c{}^2 I_d}{2} - B(r)p_1 t_c I_d + \frac{C_R(1-y)(Q-(1-y))}{D}$$
$$+(\Pi+\Pi_0(1-\beta)B(r)) + \frac{(Q-Dt_c)^2\, p_1 I_c}{2D} + \frac{\eta}{\delta}\ln\left(\frac{y_0}{y}\right) \quad (6)$$

Solution procedure

Let G (n) = (A+ K+ nF)/n

The first derivatives of ETC with respect to Q and y is equated to zero to derive the optimal solution,

$$\frac{\partial ETC}{\partial Q} = -\frac{DG(n)}{Q^2(1-y)} + yh_{b1}\left\{1-\frac{D}{2x(1-y)}\right\} + h_{b2}\left\{\frac{1-y}{2} + \frac{Dy}{2x(1-y)}\right\} \quad (7)$$
$$+\frac{h_v}{2}\left\{-1+n\left(1-\frac{Dp}{1-y}\right)+\frac{2Dp}{1-y}\right\} - \frac{\Pi D\sigma\sqrt{L}\psi(k)}{Q^2(1-y)} + \frac{C_R(1-y)y}{D} + \frac{(Q-Dt_c)\,p_1 I_c(1-Dt_c)}{D} = 0$$

$$\frac{\partial ETC}{\partial y} = \frac{Dw}{1-y} + \frac{D(s+wy)}{(1-y)^2} - \frac{\eta}{y\delta} + \frac{DG(n)}{Q(1-y)^2} + Qh_{b1}\left\{1-\frac{D}{2x(1-y)}\right\} - \frac{DQhb_1 y}{2x(1-y)^2}$$
$$+Qh_{b2}\left\{\frac{-1}{2} + \frac{D}{2x(1-y)} + \frac{Dy}{(1-y)^2}\right\} + \frac{Qh_v}{2}\left\{\frac{2Dp}{(1-y)^2} - \frac{Dnp}{(1-y)^2}\right\} - \frac{\Pi D\sigma\sqrt{L}\psi(k)}{Q(1-y)^2} + \frac{C_R(2(1-y)-Q)}{2} = 0 \quad (8)$$

On simplification we get

$$Q^* = \sqrt{\frac{D(G(n)+\Pi\sigma\sqrt{L}\psi(k)) + \mathrm{J(I)}}{H(n,y)}} \quad (9)$$

Where $J(I)=\frac{Q_0 p_1 I_c(1-Dt_c)}{D}+t_c(Dt_c-p_1I_c)$

H(n,y) = $h_{b1}\left\{y(1-y)-\frac{Dy}{2x}\right\}+h_{b2}\left\{\frac{(1-y)^2}{2}+\frac{Dy}{2x}\right\}+\frac{h_v}{2}\left[-(n-2)Dp+(n-1)(1-y)\right]+\frac{C_R(1-y)^2 y}{D}$

It is to be noted that if the updated value of y is found to be greater than the initial value y_0, then the updated value is rejected .This follows intuitively since making an investment to improve process quality cannot end up making the production process even more imperfect than it originally was. Following the same argument, the value of y cannot be set less than zero as well. The value of Q_0 can be calculated using previous inventory record. Since the value represents the Interest of the previous credit period. Else it can be neglected.

Algorithm

Step 1 : Initially set n=1.

Step 2: Set y= y_0 and compute $Q_{0}=\sqrt{D(G(n)/H(n,y)}$ using (9)

Step 3: Compute $\psi(\mathrm{k})=\phi(\mathrm{k})-k(1-\Phi(\mathrm{k}))$

Step 4: Compute y from (9) using $\psi(\mathrm{k})$,Q_0. If y $\geq$ y_0, set y = y_0

Step 5: Compute Q* from (10)

Step 6: Compute ETC using Q*, y, n

Note: optimal value of n can also be obtained using $n^*=\frac{2D(A+K)}{Q^2h_v(1-y-DP)}$ which is obtained using first order derivative.

The total cost function ETC is convex in n, since it is easy to see that

$$\frac{\partial^2 ETC}{\partial n^2}=\frac{2D(A+K)}{n^3Q(1-y)}>0 \qquad (10)$$

$$\text{Similarly, }\frac{\partial^2 ETC}{\partial Q^2}>0,\frac{\partial^2 ETC}{\partial y^2}>0 \qquad (11)$$

Now, partially deriving ETC with respect to w, δ and y_0, we get

$$\frac{\partial ETC}{\partial w}=\frac{Dy}{1-y}>0,\frac{\partial ETC}{\partial \delta}=\frac{\eta\log\frac{y_0}{y}}{\delta^2}<0,\frac{\partial ETC}{\partial y_0}=\frac{\eta}{y\delta}>0 \qquad (12)$$

Analyzing (12) we see that ETC increases with an increase in warranty cost w and also with an increase in y_0, the original percentage of defective items. Also, if the system produces items of very poor quality then it makes sense to invest more to improve quality, there by driving up the total cost. An increase in δ implies that there is a greater reduction in the number of defective items per dollar increase in investment.

Numerical Example

Consider the following data

D=1000 units/year, P=1/p=3200 units/year, A=\$100 per order, F=\$35 per shipment, K=\$400 per setup, L=10 weeks, h_v=\$4/unit/year, h_{b1}=\$6/unit/year, h_{b2}=\$10/unit/year, s=0.25/unit/year, x=2164/order, w=\$15/unit, y=0.22, σ=2 units/week, ∏=\$50/unit, $\prod_0$=\$100/unit, η =0.2, δ =0.0002, C_R=\$2/unit, tc=0.1year,I_c=\$0.15/year, I_d=\$0.12/year, β=0.34,k=0.845,p_1=\$10/unit

Proceeding in the way of algorithm we get,

y=y_0=0.22

Let $Q_0=\sqrt{\frac{DG(n)}{H(n,y_0)}}$

Now G (n) = (A+K+nF)/n = 535

H (n,y) = 4.859

Hence Q_0 = 331.82

$\psi(k)$ = 0.2076>0

From this we find that y*= 0.22, n* =2,Q* =421.08 units, ETC* =\$9589.39.

Conclusion

Analyzing the results, we can observe that increase in defective percentage increases the warranty cost for vendor, hence investing to reduce defect rate is profitable for him. As an example we have considered the logarithmic investment function, we can also consider any other investment function such as linear power investment function. We see that if the customers are offered some credit periods to pay the cost of products instead of paying the whole amount at a time, then they are interested to buy more items. This model contributes an application in an inventory system consisting of rework of imperfect products. As a scope of future research this model can also be further extended for multiple buyers.

References

1. Banerjee., A. (1986). A joint economic-lot-size model for purchaser and vendor . *Decis. Sci.*, *17*, 292–311.

2. Ben-Daya,Hariga, M.(2004). Integrated single vendor single buyer model with stochastic demand and variable lead-time. *Int. J. Prod. Econ*, *92*, 75–80..

3. Chao-Kuei Huang. (2004). An optimal policy for a single-vendor single-buyer integrated production–inventory problem with process unreliability consideration. *Int. J. Production Economics* , *91*, 91–98.
4. 0, Dey. (2014). Optimal vendor investment for reducing defect rate in a vendor-buyer integrated system with imperfect production process. *International Journal of Production Economics* , Retrieved from http://dx.doi.org/10.1016/j.ijpe.2012.02.004
5. Glock , C. H. (2009). A comment: "integrated single vendor-single buyer model with stochastic demand and variable leadtime" . *Int. J. Prod. Econ*, *122*, 790–792.
6. Goyal, S. K. (1976). an integrated inventory model for a single supplier-single customer problem. *Int. J. Prod. Res.* , *15*, 107–111.
7. Goyal, S.K., (1988). A joint economic-lot-size model for purchaser and vendor: a comment. *Decis. Sci. 19, 236–241*.
8. Goyal,Cárdenas-Barrón, S. K. E. (2002). Note on: economic production quantity model for items with imperfect quality – a practical approach.*Int. J. Prod. Econ*, *77*, 85–87.
9. Huang, C. K. (2004). An optimal policy for a single-vendor single-buyer integrated production-inventory problem with process unreliability consideration. *Int. J Prod. Econ.* , *91*, 91–98.
10. H. Hwang, S.W. Shinn, (1997) Retailer's pricing and lot sizing policy for exponentially deteriorating products under the condition of permissible delay in payment, *Comput. Oper. Res. 24, 539–547.*
11. Jaggi CK, Goyal SK, Goel SK. (2008) Retailer's optimal replenishment decisions with credit-linked demand under permissible delay in payments. *European Journal of Operational Research 190,130–5.*
12. A.M.M. Jamal, B.R. Sarker, S. Wang,(1997)An ordering policy for deteriorating items with allowable shortage and permissible delay in payment, *J. Oper. Res.Soc. 48, 826–833*.
13. Lee, Rosenblatt, H. L. J. (1987). Simultaneous determination of production cycles and inspection schedules in a production system.*Manag. Sci*, *33*, 1125–1137.
14. L.Y, Ouyang., K.S., Wu., & C.H, Ho. (2006). Analysis of optimal vendor–buyer integrated inventory policy involving defective items. *Int. J. Adv. Manuf. Technol., 29*, 1232–1245.
15. L.Y, Ouyang., K.S., Wu., & C.H, Ho. (2007). Integrated vendor–buyer inventory model with quality improvement and lead-time reduction. (2007). *Int. J. Prod. Econ., 108*, 349–358.
16. E.L, Porteus. (1986). Optimal lot sizing, process quality improvement and setup cost reduction. *Oper. Res, 34,* 137–144.
17. M.K, Salameh., & M.Y, Jaber. (2000). Economic production quantity model with for items with imperfect quality. *Int. J. Prod. Econ., 64*, 59–64.
18. B. Sarkar, (2012) An EOQ model with delay in payments and stock dependent demand in the presence of imperfect production, *Appl. Math. Comput. 8295–8308.*
19. B.Sarkar,(2014) An integrated inventory model with variable lead time, defective units and delay in payments, *Appl. Math. Comput.237,650-658.*
20. H, Shu., & X, Zhou. (2014). An optimal policy for a single-vendor and a single-buyer integrated system with setup cost reduction and process-quality improvement. *Int. J. Syst. Sci., 45*, 1242–1252.
21. R.L, Scwaller. (1988). EOQ under inspection costs. Prod. *Invent. Manag, 29,* 22–24.
22. X, Zhang., & Y, Gerchak. (1990). Joint lot sizing and inspection policy in an EOQ model with random yield. *IIE Trans, 22*, 41–47.