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EOQ model with various Environment costs

W.Ritha[1] and P.Bharathi[2]

[1]Holy Cross College (Autonomous) ,Trichirapalli, India.

[2]Government Arts College, Thiruverumbur, Trichy, India– 620 022.

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ABSTRACT

Environment Pollution is assuming dangerous proportions all through the globe. This is the gift of modern living industrialization and urbanization. Transportation is a non-separable part of any society. It exhibits a very close relation to the style of life, the range and location of activities and the goods and services which will be available for consumption. Costs and benefits have mirror image relationship. A cost can be defined as a reduction in benefits and a benefit can be defined as a reduction in costs. Transportation benefits are often measured in terms reduced Transportation costs. Calculating costs is therefore the basis for calculating benefits. This paper deals with an Economic Order Quantity model (EOQ) which associates the vehicle cost, Damage cost, Parking cost, Environmental cost along with the costs of ordering, Purchasing and holding. This paper also investigates the attributing factors of the Environmental impacts which could be minimized to build a healthy society.

Introduction

The "Economic Order Quantity" (EOQ) is the order quantity that minimizes total inventory holding cost and ordering cost. The first mathematical treatment of inventory systems was the EOQ model developed by Harris in the 1920's (Harris 1913 / 1990). Major advances in understanding inventory problems took place in 1950's and 1960's (whitin 1953; Arrow et.al., 1962, Hadley and whitin, 1963). The increasing concern on environmental issues stresses the need to treat inventory management decisions as a whole with economic and environmental objectives (i.e. Banney and Jaber 2011; Wahab.et.al.2011) Turkay (2008) revised the standard EOQ model to incorporate sustainability considerations that include Environmental criteria. Hua.et.al (2009 and 2011) studied this problem and compared with classical EOQ model with a new one. Banjaffer et.al (2010) incorporated Carbon emission constraints on single and multi-stage lot-sizing models.

Nothing is more important to civilization than transportation and communication and apart from direct tyranny and oppression, nothing is more harmful to the well – being of a society than an irrational transportation system. Trade is essential to economic Vitality and transportation is essential to trade. Vehicles are significant sources of pollution that can damage the environment and pose public health issues. Motor Vehicles emit pollutions that contribute to global climate change. Air Pollution from vehicles is split into primary and secondary pollution. Primary pollution is emitted directly into atmosphere. Secondary pollution results from chemical reaction between pollutants in the atmosphere .Our environment has become difficult to escape from Noise Pollution. Air Pollution is one of the most obvious Environmental costs of motor vehicle use. Motor vehicles traffic imposes noise pollution. Noise costs tend to be much higher on local urban roads, where traffic tends to be closer to residences. Roads and motor vehicles use contribute to water pollution and waste disposal (such as used tires) impose a variety of costs on society

Notations and Assumptions

The following Notations and Assumptions are used through out to develop the EOQ model:-

$A \rightarrow$ Order cost (mu)

$C \rightarrow$ Unit Purchase cost (mu/unit)

$h \rightarrow$ holding cost (mu / unit / year)

$a \rightarrow$ fixed cost per trip (mu)

$b \rightarrow$ variable cost per unit transported per distance travelled (mu / unit / km)

$d \rightarrow$ distance travelled (from supplier to buyer, km)

$\alpha \rightarrow$ Proportion of demand returned $(0 < \alpha < 1)$

$D \rightarrow$ Demand rate (units / year)

$\beta \rightarrow$ Social cost from vehicle emission (mu / h)

$V \rightarrow$ Average Velocity (km / h)

$\gamma \rightarrow$ Cost to dispose waste to the environment (mu / unit)

$\theta \rightarrow$ Proportion of waste produced per lot Q

$\mu_1 \rightarrow$ Cost to purchasing fuel and oil per year

$\mu_2 \rightarrow$ Cost to fuel taxes per year

$\mu_3 \rightarrow$ Cost to tolls and tires per year

Tele:

E-mail addresses: ritha_prakash@yahoo.co.in

t_c → Number of traffic crashes
p_1 → Parking spaces cost per year
p_2 → Cost to number of Parking spaces per vehicles per year
p_3 → Equipment Maintenance cost per year
n → number of shipments
d_1 → Cost to Restore the original condition due to damage per year
d_2 → Preventive Measures cost due to damage per year
d_3 → Cost to loss of Amenity due to damage per year
β_1 → Air Pollution cost from vehicle emission
β_2 → Noise Pollution cost from vehicles
β_3 → Water Pollution cost from by the use of vehicles

Assumptions

The Mathematical Models developed in this paper are based on the following assumptions:-

1. The demand, ordering cost, proportion of the demand returned are constant per cycle.
2. The units that are transported are delivered and collection of returned items.
3. Air Pollution costs, Noise Pollution costs and Water Pollution Costs are determined for a round trip.

Model Developments:-

In classical EOQ model the cost per cycle is the sum of the ordering cost, purchasing and holding cost. Jaber [2010] has developed a new EOQ model with the cost of transportation.

EOQ cost per cycle:-

$$C(Q) = A + CQ + \frac{hQ^2}{2D} \text{........①}$$

Transportation cost per cycle:-

$$C(Q) = 2a + bdQ + bd\alpha D.T$$

$$= 2a + bdQ + bd\alpha D.\frac{Q}{D}$$

$$C_t(Q) = 2a + bdQ + bd\alpha Q\text{........②}$$

Wastage Cost associated with disposal of Vehicle wastes:-

$$C_w(Q) = \gamma_0 + \gamma Q(\theta + \alpha)\text{.........③}$$

Vehicle Cost includes purchasing of fuel, oil, fuel taxes, tolls and tires per year:-

$$C_v(Q) = (\mu_1 + \mu_2 + \mu_3).\frac{d}{T}\text{........④}$$

Damage cost includes the expenditure to restore the original condition, cost to preventive measures, cost to loss of amenity due to damage by vehicles per year:-

The total cost of Damage per cycle:-

$$C_D(Q) = (d_1 + d_2 + d_3).\frac{tc}{T}\text{.........⑤}$$

Parking cost includes the amount spends for parking spaces, Number of Parking spaces cost, Equipment maintenance cost**:-**

The total cost of Parking cost per cycle.

$$C_p(Q) = (p_1 + p_2 + p_3).\frac{n}{T}\text{.........⑥}$$

Environmental cost includes the Air Pollution, Noise pollution and Water Pollution cost:-

The Air Pollution cost from Vehicle emissions per cycle =

$$2\beta_1 \frac{d}{V} \text{..........6.1}$$

The noise Pollution cost from Vehicles per cycle = $2\beta_2 \frac{d}{V}$..........6.2

The Water Pollution cost from Vehicle per cycle = $2\beta_3 \frac{d}{V}$..........6.3

The total environment cost per cycle:-

$$C_e(Q) = 2(\beta_1 + \beta_2 + \beta_3).\frac{d}{V} \text{..........⑦}$$

The total cost per ynit time is:-

$$\psi(Q) = \frac{C(Q) + C_t(Q) + C_w(Q) + C_V(Q) + C_D(Q) + C_p(Q) + C_e(Q)}{T}$$

Where $T = \frac{Q}{D}$ is the cycle time.

$$\psi(Q) = \frac{A + CQ + \frac{hQ^2}{2D} + 2a + bdQ + bd\alpha Q + \gamma_0 + \gamma Q(\theta+\alpha) + d(\mu_1+\mu_2+\mu_3) + t_c(d_1+d_2+d_3) + n(p_1+p_2+p_3) + 2\frac{d}{V}(\beta_1+\beta_2+\beta_3)}{T}$$

$$= \frac{A + CQ + \frac{hQ^2}{2D} + 2a + bdQ + bd\alpha Q + \gamma_0 + \gamma Q(\theta+\alpha) + d(\mu_1+\mu_2+\mu_3) + t_c(d_1+d_2+d_3) + n(p_1+p_2+p_3) + 2\frac{d}{V}(\beta_1+\beta_2+\beta_3)}{Q/D}$$

$$= \frac{D(A + CQ + \frac{hQ^2}{2D}) + D(2a + bdQ + bd\alpha Q) + D(\gamma_0 + \gamma Q(\theta+\alpha)) + dD(\mu_1+\mu_2+\mu_3) + Dt_c(d_1+d_2+d_3) + nD(p_1+p_2+p_3) + 2.D\frac{d}{V}(\beta_1+\beta_2+\beta_3)}{Q}$$

$$= \frac{DA + DCQ + \frac{hQ^2}{2}) + 2aD + bdDQ + bd\alpha QD + D\gamma_0 + D\gamma Q(\theta+\alpha) + dD(\mu_1+\mu_2+\mu_3) + Dt_c(d_1+d_2+d_3) + nD(p_1+p_2+p_3) + 2D.\frac{d}{V}(\beta_1+\beta_2+\beta_3)}{Q}$$

1.

$$= \frac{DA}{Q} + DC + \frac{hQ}{2} + \frac{2aD}{Q} + bdD(1+\alpha) + \frac{D\gamma_0}{Q} + \gamma D(\theta+\alpha) + dD\frac{(\mu_1+\mu_2+\mu_3)}{Q} + \frac{Dt_c(d_1+d_2+d_3)}{Q} + n\frac{D}{Q}(p_1+p_2+p_3) + 2\frac{D}{Q}.\frac{d}{V}(\beta_1+\beta_2+\beta_3)$$

Hence,

$$\Psi(Q) = \frac{DA}{Q} + DC + \frac{hQ}{2} + \frac{2aD}{Q} + bdD(1+\alpha) + \frac{D\gamma_0}{Q} + \gamma D(\theta+\alpha) + d\frac{D}{Q}(\mu_1+\mu_2+\mu_3) + tc\frac{D}{Q}(d_1+d_2+d_3) + n\frac{D}{Q}(p_1+p_2+p_3) + 2\frac{D}{Q}.\frac{d}{v}(\beta_1+\beta_2+\beta_3)$$

The objective is to determine the optimal Quantity. The optimal value of Q is obtained by setting its first derivative w.r.to Q is equal to Zero. The necessary condition is give by $\frac{\partial\psi(Q)}{\partial Q} = 0$.

i.e. $$\frac{\partial\psi(Q)}{\partial Q} = \frac{\partial}{\alpha Q}\left\{\frac{AD}{Q} + DC + \frac{hQ}{2} + \frac{2aD}{Q} + bdD(1+\alpha) + \frac{D\gamma_0}{Q} + \gamma D(\theta+\alpha) + d\frac{D}{Q}(\mu_1+\mu_2+\mu_3) + tc\frac{D}{Q}(d_1+d_2+d_3) + n\frac{D}{Q}(p_1+p_2+p_3) + 2\frac{D}{Q}.\frac{d}{v}(\beta_1+\beta_2+\beta_3)\right\}$$

$$= \frac{\partial}{\alpha Q}\left\{ADQ^{-1} + DC + \frac{hQ}{2} + 2aDQ^{-1} + bdD(1+\alpha) + D\gamma_0Q^{-1} + \gamma D(\theta+\alpha) + dDQ^{-1}(\mu_1+\mu_2+\mu_3) + tcD(d_1+d_2+d_3)Q^{-1} + nD(p_1+p_2+p_3)Q^{-1} + 2D.\frac{d}{v}(\beta_1+\beta_2+\beta_3)Q^{-1}\right\}$$

$$= AD(-1)Q^{-2} + \frac{h}{2} + (-1)2aDQ^{-2} + (-1)D\gamma_0Q^{-2} + (-1)dD(\mu_1+\mu_2+\mu_3)Q^{-2} + (-1)t_cD(d_1+d_2+d_3)Q^{-2} + (-1)nD(p_1+p_2+p_3)Q^{-2} + (-1)2D.\frac{d}{v}(\beta_1+\beta_2+\beta_3)Q^{-2}$$

$$\frac{\partial\psi(Q)}{\partial Q} = \left\{-\frac{AD}{Q^2} + \frac{h}{2} - \frac{2aD}{Q^2} - \frac{D\gamma_0}{Q^2} - \frac{dD(\mu_1+\mu_2+\mu_3)}{Q^2} - \frac{t_cD(d_1+d_2+d_3)}{Q^2} - \frac{nD(p_1+p_2+p_3)}{Q^2} - \frac{2D.\frac{d}{v}(\beta_1+\beta_2+\beta_3)}{Q^2}\right\}$$

The necessary condition $\frac{\partial\psi(Q)}{\partial Q} = 0$

$$\Rightarrow \frac{AD}{Q^2} + \frac{2aD}{Q^2} + \frac{D\gamma_0}{Q^2} + \frac{dD}{Q^2}(\mu_1+\mu_2+\mu_3) + \frac{t_cD(d_1+d_2+d_3)}{Q^2} + \frac{nD(p_1+p_2+p_3)}{Q^2} + \frac{2D.\frac{d}{v}(\beta_1+\beta_2+\beta_3)}{Q^2} = \frac{h}{2}$$

$$\Rightarrow \frac{1}{Q^2}\left\{AD + 2aD + D\gamma_0 + dD(\mu_1+\mu_2+\mu_3) + t_cD(d_1+d_2+d_3) + nD(p_1+p_2+p_3) + 2D.\frac{d}{v}(\beta_1+\beta_2+\beta_3)\right\} = \frac{h}{2}$$

$$\Rightarrow Q^2h = 2AD + 4aD + 2D\gamma_0 + 2dD(\mu_1+\mu_2+\mu_3) + 2t_cD(d_1+d_2+d_3) + 2nD(p_1+p_2+p_3) + 4D.\frac{d}{v}(\beta_1+\beta_2+\beta_3)$$

$$\Rightarrow Q^2 = \frac{2AD + 4aD + 2D\gamma_0 + 2dD(\mu_1+\mu_2+\mu_3) + 2t_c D(d_1+d_2+d_3) + 2nD(p_1+p_2+p_3) + 4D.\frac{d}{v}(\beta_1+\beta_2+\beta_3)}{h}$$

Hence,

$$Q = \sqrt{\frac{2D\left\{A + 2a + \gamma_0 + d(\mu_1+\mu_2+\mu_3) + t_c(d_1+d_2+d_3) + n(p_1+p_2+p_3) + 2.\frac{d}{v}(\beta_1+\beta_2+\beta_3)\right\}}{h}}$$

Numerical Example:-

To illustrate the result in this paper, a numerical example is built up.

Consider an inventory system with the following characteristics:-

D = 20, A = 10, C = 5, h = 3, a = 7, b = 0.3, γ_0 = 1, d = 200, V = 180, β_1 = 200, β_2 = 150, β_3 = 175, μ_1 = 125, μ_2 = 135, μ_3 = 145, d_1 = 225, d_2 = 300, d_3 = 115, p_1 = 500, p_2 = 525, p_3 = 600, n = 3, t_c = 3.

$$Q = \sqrt{\frac{2D\left\{A + 2a + \gamma_0 + d(\mu_1+\mu_2+\mu_3) + t_c(d_1+d_2+d_3) + n(p_1+p_2+p_3) + 2.\frac{d}{v}(\beta_1+\beta_2+\beta_3)\right\}}{h}}$$

$$Q = \sqrt{\frac{2\text{x}20\left\{10 + 2x7 + 1 + 200(125+135+145) + 3(225+300+115) + 3(500+525+600)2.\frac{200}{180}(200+150+175)\right\}}{3}}$$

$$Q = 211.36$$

Social role of Transportation:-

Transportation has always played an important role in influencing the formation of urban societies. Although other facilities like availability of food and water, played a major role the contribution of transportation can be seen clearly from the formation, size and pattern and the development of societies especially urban centres.

Environmental Pollution is causing a lot of distress not only to humans but also animals, driving many animal species to endangerment and even extinction. It is widely recognized that we are hugely overspending our current budget of natural resources – at the existing rates of its exploitation. There is no way for the environment to recover in good time and continue "Performing" well in the future.

Conclusion:-

This paper presents a general outlook of transportation which comprises of its merits and demerits. An Economic order quantity model that includes the vehicle cost, damage cost, parking cost and Environmental cost. The effects of emissions of vehicles are described in brief. This paper concludes that we should adopt a holistic view of nature – it is not an entity that exists separately from us; the nature is us, we are an inalienable part of it and we should care for it in the most appropriate manner. Only then can we possible solve the problem of Environmental Pollution.

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