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Pricing and Advertisement in a Manufacturer- Consumer Supply Chain with Credit Period System

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ABSTRACT

This paper introduces manufacturer consumer supply chain model to investigate pricing and advertising decision when price discounts and credit facilities for certain period are offered by the manufacturer. In this supply chain, the manufacturer is the supreme power to decide the level of price discount, advertising, brand name investment and has the power to introduce the credit period facility. For the first case when the manufacturer proposes price discount, we found that as there is no contribution of retailer in the supply chain, the manufacturer only utilises the entire profits and through the intensive marketing strategies, the manufacturer can equilibrium their consumers. For the second case, when the manufacturer introduces the credit period facilities to attract more number of consumers in wide range, we found that both the manufacturer and consumer have substantial benefits which lead to augment of profit margin and derive fulfilment respectively. Numerical examples were given to illustrate the optimal profit margins in the manufacturer consumer supply chain.

Introduction

The sequence of processes involved in the production and distribution of a commodity is very difficult to understand to set it right according to our objectives. To reach our plan, it is important to analyse the manufacturer-consumer supply chain which has been developed in the recent years as the manufacturer retailer supply chain is complex and critical to adjust the terms and policies between them. In this supply chain, the manufacturer acts independently to fix the price of the commodity, advertising expenses and so on . He takes all the efforts according to the determination of him only. Everything is in the hands of the manufacturer. The advantages or impact of different operation strategies are based on the manufacturer's strategies.

Price discounts can be provided by the manufacturer here to augment the profit and sales through attraction of the consumers. The price discount by the manufacturer is to be compensated the increasing cost of ordering higher than the economic ordering quantity (Monahan, 1984; Lee and Rosenblatt, 1986; Chiang et al., 1994). The other studies assumed that if the discount is provided to the consumers , they can pay price lower than the Manufacturer's Suggested Retail Price to purchase the brand name product(Abad, 1994; Li et al., 1996 and Ye.et.al.2006). Therefore, the discount directly to the consumers may motivate the demand of the brand name product market and boost up the profit for the manufacturer. It may by situation to the consumer to buy at bulk quantity of product at the retail price from the manufacturer.

Advertising an evitable general marketing activity should be considered different between local advertising and brand name investment. The brand name investment is a long term process conducted traditionally by the manufacturer in the national level. It is mainly concentrated on the building the brand image among the consumers. But the local advertising, is to cover the local consumers. It is different from time to time. It is done by the retailer or by the manufacturer in case of direct sales. In the recent years, the both of the advertisement is made by the manufacturer to cover directly the consumer in the retail marketing. In the direct sales method, the consumer has more benefits and the manufacturer has undertaking all the strategies what the retailer's perform. It is to eliminate the profit shared with the retailer and to provide bulk sales at retail price.

In the manufacturer-consumer supply chain, the consumer has more advantages on payment of amount of purchase. The main of this sales method is to cover the poor level consumers. So the manufacturer introduces credit period facility to the consumers. For this facility, the manufacturer charges certain percentage of amount on the manufacturer's retail price. The price discounts, brand name investment, advertising expenses and credit period facilities are illustrated in the following chapters to understand how the manufacturer yield more profit margin.

Review of Literature

The research literature related to this paper can be divided into those on manufacturer's pricing decisions, advertising policy, manufacturer consumer supply chain, and credit period facilities for marketing strategies. In order to investigate the pricing and advertising in manufacturer-consumer supply chain with credit period system, early research from Cachon and Netessine(2004, chap.2) and Wang and Parlar(1989) . Leng and Parlar(2005) identified four different classes of research; two classes on inventory games, a third one related to production and pricing competition, and a fourth one was Games with other attributes, where one can find game-theoretic analyses of capacity, service, product quality, and advertising decisions. Vertical co-operative advertising, the most common comprehension of cooperative advertising where a manufacturer offers to share a certain percentage of his retailer's advertising expenditures(Bergen & John (1997)). In a leader-follower two stage supply chain, the manufacturer usually anticipates

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the reactions of the retailer and decides its first move, and then prescribes the behaviour of the retailer(Gaski, 1984). Power shifting from manufacturers to retailers to fix the price discounts and other expenses has been a new trend. According to Kadiyali et al.(2002), the common thinking is that retailers hold more channel power than manufacturers. However, Messinger and Narasimhan(1996) found no evidence that the profitability of retailers was better than manufacturers from 1961-1991, indicating the manufacturers hold more power than retailers.

If both the manufacturer and the retailer have powers, they may consider either competing or coorperating. Taylor(2001) found from his study of products sold in declining price environments that co-ordination in the frorm of price returns and price protection provides a win-win situation. Lee and Rosenblatt(1986) studied price protection in the personal computer industry and found that cooperation in the channel benefits the total chain and the retailer. Dant and Berger(1996) found from his game theory to obtain stackelberg equilibrium in advertising investment sharing where allowance from a manufacturer promotes a retailer's advertising expense and increases the profit for the whole chain. Huang, Li, and Mahajan(2002) observed that manufacturers pay not only brand name investments but also part of local advertising expenses incurred by retailers. Hutchins(1953) argues that manufacturers adopt cooperative advertising, because it generates immediate sales.

While manufacturer's global advertising creates a brand image and is more general and nationwide than retailer's local advertising, the latter treats more of promotions and prices. Hence, global advertising makes for publicity and reputation of the product(cf.Herrington & Dempsey(2005) and Young & Greyser(1983). Due to these complementary goals and effects, manufacturers are somehow reliant on a certain degree of local advertising. However, it may occur that the retailer's advertising level is not sufficient from the manufacturers's point of stimulate the retailer's advertising expenditures to a sufficient level.

Many manufacturers or retailers in the hardware and auto industries may prefer rebates because using rebates helps to advertise products and it may cost less than direct discount. With a different level of promotional benefit target, Hardest and Bearden suggested that price discounts and bonus packs were valued for both low and moderate promotional benefits levels. Krishnan et al. focused on the use of retailer rebates in the presence of retailer efforts. Eilon and Mallaya were the first authors to analyze the pricing policy for perishable items. Goh and Sharafali incorporated discounts at random time in a decision model with temporary price discounts.

In the previous research, Yue et al, 2006 proposed a manufacturer-retailer coordinate advertisement model with both the manufacturer leading a Stackelberg game and the optimal policy for integrated channel when only the manufacturer provides a price discount. In the present scenarios, instead of both the retailer and the manufacturer can provide the price discount to the consumers, the manufacturer provides price discount, brand name investment, advertising expenses and credit period facilities. It is necessary to discuss the business decisions in a manufacturer-consumer supply chain.

- When the manufacturer provides price discounts,
- Why the manufacturer tries to provides price discounts, brand name investment, advertising expenses and credit period facilities.
- Whether the contract of manufacturer with the consumer will assure to achieve their desired profit sharing.
- How to share the profit gain to price discount, brand name investment, advertising expenses and credit period facilities.

Solutions will be provided for the above topics in following sections of this paper. The rest of the paper is organised as follows, Section 2 provides literature review, Section 3 provides Demand function and profit function Determination with price discounts in manufacturer – consumer supply chain. Section 4 deals with demand function and profit function determination with credit period facility without price discounts. Section 5 concludes the study.

Demand function and profit function Determination with price discounts in manufacturer consumer supply chain

In this model, we determine the demand function with local advertising expense, brand name invest and price discount effects and further determine the profits of the manufacturer. Assume a products MRP(Manufacturer's Retail Price) is $\boldsymbol{P_0}$, the variable cost in the whole supply chain is $\boldsymbol{VC_w}$, the manufacturer profit margins of each product unit sold at $\boldsymbol{P_0}$ are $\boldsymbol{\rho_m}$.

$$\boldsymbol{P_0 = VC_w + \rho_m} \qquad \text{……………………. (1)}$$

A one period market demand (Sale volume) function with the effects of local advertising, brand name investment and price discount by Yue.et.al(2006) is

$$S(a,q,\boldsymbol{P}) = (\boldsymbol{\alpha} - \boldsymbol{\beta} a^{-\gamma} q^{-\delta})\left(\frac{\boldsymbol{P}}{\boldsymbol{P_0}}\right)^{-\boldsymbol{e}} \qquad \text{-------------------------(2)}$$

Where, a = local advertising expense;
q = National brand name invesment;
$\boldsymbol{\gamma}$ = quasi – advertising elasticity;
$\boldsymbol{\delta}$ = quasi – investment elasticity;
$\boldsymbol{\beta}$ = scalling parameter;
e = price elasticity
P = discount retailing price charged to customers .
$\boldsymbol{\alpha}, \boldsymbol{\beta}, \boldsymbol{\alpha}, \boldsymbol{\gamma}$ & $\boldsymbol{\delta}$ are other parameters.

Assuming that the manufacturer offers $\boldsymbol{\epsilon_m}$ percentage of full price $\boldsymbol{P_0}$ as the price discounts to consumers, the manufacturers profits margins will reduce to $(\boldsymbol{\rho_m - \epsilon_m p_0})$ and the discount retailing price P can be expressed as $\boldsymbol{P = (1 - \in_m)P_0}$.
Therefore, the market demand function (2) can be written as,

$$S(\boldsymbol{a, q, P}) = \boldsymbol{S(a, q, \in_m)} = \left(\boldsymbol{\alpha - \beta a^{-\gamma} q^{-\delta}}\right)(\boldsymbol{1 - \in_m})^{-\boldsymbol{e}} \qquad \text{……….(3)}$$

In this model, we assume that the manufacturer provides t percentage of local advertising expense. The manufacturer's gross profit for one period becomes

$$\boldsymbol{\pi_{m\,(\in_m)}} = (\boldsymbol{\rho_m - \epsilon_m p_0})\left(\boldsymbol{\alpha - \beta a^{-\gamma} q^{-\delta}}\right)(\boldsymbol{1 - \in_m})^{-\boldsymbol{e}} \boldsymbol{- ta - q} \qquad \text{…………(4)}$$

$$= (\boldsymbol{\rho_m - \epsilon_m p_0})\left(\boldsymbol{\alpha - \beta a^{-\gamma} q^{-\delta}}\right)(\boldsymbol{1 - \in_m})^{-\boldsymbol{e}} \boldsymbol{- a - q} \qquad \text{………..(5)}$$

Optimal Policy for integrated Channel

We obtained the local advertising a^*, brand name investment q^*, and total price discount $\in_m{}^*$. It is the maximisation problem of

$$\max_{q,a,\epsilon} \pi_m = (\rho_m - \in_m P_0)(\alpha - \beta a^{-\gamma} q^{-\delta})(1-\in_m)^{-e} - a - q \quad \ldots\ldots\ldots\ldots\ldots(7)$$

Subject to $(\alpha - \beta a^{-\gamma} q^{-\delta}) \geq 0$ & $\frac{\rho_m}{P_0} \geq e > 0$

Theorem :1

Assuming that the manufacturer offers price discount to consumer, the manufacturer's margins will reduce their profit. Find the demand function and profit function with price discount also find the unique optimal policy for integrated channel.

(see the proof in the Appendix 1).

Before to prove that the optimal policy for the integrated channel will maximise total profit of the whole chain, calculate the first partial derivatives of $\in_m$, a and q to zero and the second partial derivatives of $\in_m{}^2$, $\in_m$ and a , $\in_m$ and q, a^2, a and q and q^2 at solutions of local advertising a^*, brand name investment q^*, and total price discount $\in_m{}^*$.

$$\in_m{}^* = \begin{cases} [\frac{e(\rho_m - P_0)}{(e-1)P_0} & (e-1) > 0 \\ 0 & otherwise \end{cases} \quad \ldots\ldots\ldots\ldots\ldots\ldots(8)$$

The optimal local advertising expense is:

$$a^* = \left[\beta\delta^{-\delta}\gamma^{(\delta+1)} e^{-e}(e-1)^{(e-1)} P_0{}^e (P_0 - \rho_m)^{-(e-1)}\right]^{\frac{1}{(\delta+\gamma+1)}} \quad \ldots\ldots\ldots..(9)$$

The optimal national brand name investment is :

$$q^* = \left[\beta\delta^{(\gamma+1)}\gamma^{-\gamma} e^{-e}(e-1)^{(e-1)} P_0^e (P_0 - \rho_m)^{-(e-1)}\right]^{\frac{1}{(\delta+\gamma+1)}} \quad \ldots\ldots\ldots(10)$$

and the total profit in the supply chain is

$$\pi^*{}_{m\,(\in_m{}^*)} = \alpha \left(\frac{P_0}{e}\right)^e \left(\frac{e-1}{P_0-\rho_m}\right)^{(e-1)} - (\delta+\gamma+1)\left[\beta\delta^{-\delta}\gamma^{-\gamma}\right]^{1/(\delta+\gamma+1)} \times \left[\left(\frac{P_0}{e}\right)^e \left(\frac{e-1}{P_0-\rho_m}\right)^{(e-1)}\right]^{1/(\delta+\gamma+1)} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(11)$$

Numerical Example: 1

Assume that the MRP (P_0) is \$500. Manufacturer profit margin ρ_m is \$125 at MRP. Other parameters are $= 50,000,000$, e = 16.3, $\beta = 25,000,000, \gamma = 0.4$ and $\delta = 0.3$.

Solution:

The manufacturer's price discount $\in_m$ is 20.10% . The manufacturer's local advertisement expenses is a^* = \$ 778,495.3461 , the national brand investment is q^* = \$583,871.5096, the market demand function $S(a^*, q^*, \in_m{}^*)$ = 1, 938,383,497 units and the total profit is $\pi^*{}_{m\,(\in_m{}^*)}$= \$ 47,489,036,890.

Demand Function and Profit Function Determination With credit period facility without Price Discount

Theorem : 2. When the manufacturer offers credit period for purchasing product and the manufacturer does not offer any price discount ($\in_m = 0$) , assume that the charge of credit period Q can be expressed as Q= (1+n C_m) P_0.

(see the proof in the Appendix 2).

We obtained the optimal policy for this model and to obtain the local advertising a^*, brand name investment q^*, and total price discount $C_m{}^*$. It is the maximisation problem of

$$\max_{q,a,\epsilon} \pi_m = (\rho_m - C_m P_0)(\alpha - \beta a^{-\gamma} q^{-\delta})(1-C_m)^{-e} - a - q \quad \ldots.(7)$$

Subject to $(\alpha - \beta a^{-\gamma} q^{-\delta}) \geq 0$ & $\frac{\rho_m}{P_0} \geq e > 0$

Before to prove that the optimal policy for the integrated channel will maximise total profit of the whole chain, calculate the first partial derivatives of C_m , a and q to zero and the second partial derivatives of $C_m{}^2$, $\in_m$ and a , C_m and q, a^2, a and q and q^2 at solutions of local advertising a^*, brand name investment q^*, and total price discount $C_m{}^*$.

$$C_m{}^* = \begin{cases} [\frac{e\rho_m - P_0}{n(1-e)P_0} & (1-e) > 0 \\ 0 & otherwise \end{cases} \quad \ldots\ldots\ldots\ldots\ldots\ldots(8)$$

The optimal local advertising expense is:

$$a^* = \left[\beta\delta^{-\delta}\gamma^{(\delta+1)} e^{-e}(e-1)^{(e-1)} P_0{}^e (P_0 - \rho_m)^{(1-e)}\right]^{\frac{1}{(\delta+\gamma+1)}} \quad \ldots\ldots\ldots..(9)$$

The optimal national brand name investment is :

$$q^* = \left[\beta\delta^{(\gamma+1)}\gamma^{-\gamma} e^{-e}(e-1)^{(e-1)} P_0^e (P_0 - \rho_m)^{(1-e)}\right]^{\frac{1}{(\delta+\gamma+1)}} \quad \ldots\ldots\ldots\ldots(10)$$

and the total profit in the supply chain is

$$\pi^*{}_{m\,(C_m^*)} = \alpha \left(\frac{P_0}{e}\right)^e \left(\frac{e-1}{P_0-\rho_m}\right)^{(e-1)} - (\delta+\gamma+1)\left[\beta\delta^{-\delta}\gamma^{-\gamma}\right]^{1/(\delta+\gamma+1)} \times \left[\left(\frac{P_0}{e}\right)^e \left(\frac{e-1}{P_0-\rho_m}\right)^{(e-1)}\right]^{1/(\delta+\gamma+1)} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(11)$$

Numerical Example : 2

Assume that the MRP (P_0) is \$500. Manufacturer profit margin ρ_m is \$100 at MRP. Other parameters are $= 50,000,000$, e = 4.2, $\beta = 25,000,000, \gamma = 0.4$ and $\delta = 0.3$.

Solution

The manufacturer's price discount $\boldsymbol{C_m}$ is 1.67% . The manufacturer's local advertisement expenses is a^* = $ 209,325.2008, the national brand investment is q^* = $156,993.9006, the market demand function $\boldsymbol{S}(\boldsymbol{a}^*, \boldsymbol{q}^*, \boldsymbol{C_m}^*)$ = 40,715,202.86 units and the total profit is $\boldsymbol{\pi}^*_{\boldsymbol{m}}\,(C_m{}^*)$= $ 5,091,070,108.

Concluding Remarks

In this research, we study the pricing and advertisement in a manufacturer-consumer supply chain with credit period system offered by the manufacturer. We considered that the manufacturer will decide the manufacturer's price discount, local allowance and national brand name investment and local advertising expense with credit period facilities. We first proved that the optimal policy for integrated channel is the unique global optimal solution. In manufacturer-consumer supply chain, when the manufacturer is the leader of the chain, Our research also showed that the manufacturer alone stimulate the supply chain to coordinate the optimal level by providing a price discount and local allowance.

When manufacturer has only power in the supply chain, we studied the competition game and manufacturer's equilibrium. Finally, we proposed two different ways to determine the profit by using price discount and credit period facilities. If the manufacturer provides the price discount, brand name investment, local advertising and credit period facilities, the profit margin is increased because no retailer is available in this supply chain to share. However, the manufacturer should have good relationship with the consumer to stimulate as the retailers. This model is the recent scenario in the retailing market.

Appendix : 1

Proof: To prove that the optimal policy for integrated channel will maximize total profit of the whole chain, calculate the second particl derivatives of in (7) with respect of at solution of , respectively, we have

$$\frac{\partial \pi_m(\epsilon_m)}{\partial \epsilon_m} = -P_0\left(\alpha - \beta a^{-\gamma} q^{-\delta}\right)(1-\epsilon_m)^{-e} + \mathrm{e}(\rho_m - \epsilon_m P_0)\left(\alpha - \beta a^{-\gamma} q^{-\delta}\right)(1-\epsilon_m)^{-e-1} = 0$$

$$\frac{\partial^2 \pi_m(\epsilon_m)}{\partial \epsilon_{m^2}} = -e^{(e-1)}(e-1)^{(e+2)}P_0^{(e+2)}\left(\alpha - \beta a^{-\gamma} q^{-\delta}\right)(P_0 - \rho_m)$$

$$\frac{\partial \pi_m(\epsilon_m)}{\partial a} = (\rho_m - \epsilon_m P_0)\left(\gamma\, \beta a^{-\gamma-1} q^{-\delta}\right)(1-\epsilon_m)^{-e} - 1 = 0$$

$$\frac{\partial \pi_m(\epsilon_m)}{\partial q} = (\rho_m - \epsilon_m p_0)\left(\delta \beta a^{-\gamma} q^{-\delta-1}\right)(1-\epsilon_m)^{-e} - 1 = 0$$

$$\frac{\partial^2 \pi_m(\epsilon_m)}{\partial \epsilon_m \partial a} = 0, \quad \frac{\partial^2 \pi_m(\epsilon_m)}{\partial \epsilon_m \partial q} = 0,$$

$$\frac{\partial^2 \pi_m(\epsilon_m)}{\partial a^2} = -\beta\gamma(\gamma+1)\, a^{-(\gamma+2)} q^{-\delta}\, e^{-e}(e-1)^{(e-1)} P_0{}^e (P_0 - \rho_m)^{-(e-1)}$$

$$\frac{\partial^2 \pi_m(\epsilon_m)}{\partial a \partial q} = -\beta\gamma\delta\;\, a^{-(\gamma+1)} q^{-(\delta+1)}\, e^{-e}(e-1)^{(e-1)} P_0{}^e (P_0 - \rho_m)^{-(e-1)}$$

$$\frac{\partial^2 \pi_m(\epsilon_m)}{\partial q^2} = -\beta\delta(\delta+1)\, a^{-\gamma} q^{-(\delta+2)}\, e^{-e}(e-1)^{(e-1)} P_0{}^e (P_0 - \rho_m)^{-(e-1)}$$

Since

$$\frac{\partial^2 \pi_m(\epsilon_m)}{\partial \epsilon_{m^2}} < 0, \quad \begin{vmatrix} \frac{\partial^2 \pi_m(\epsilon_m)}{\partial \epsilon_{m^2}} & \frac{\partial^2 \pi_m(\epsilon_m)}{\partial \epsilon_m \partial a} \\ \frac{\partial^2 \pi_m(\epsilon_m)}{\partial a \partial \epsilon_m} & \frac{\partial^2 \pi_m(\epsilon_m)}{\partial a^2} \end{vmatrix} > 0 \; and \; \begin{vmatrix} \frac{\partial^2 \pi_m(\epsilon_m)}{\partial \epsilon_{m^2}} & \frac{\partial^2 \pi_m(\epsilon_m)}{\partial \epsilon_m \partial a} & \frac{\partial^2 \pi_m(\epsilon_m)}{\partial \epsilon_m \partial q} \\ \frac{\partial^2 \pi_m(\epsilon_m)}{\partial a \partial \epsilon_m} & \frac{\partial^2 \pi_m(\epsilon_m)}{\partial a^2} & \frac{\partial^2 \pi_m(\epsilon_m)}{\partial a \partial q} \\ \frac{\partial^2 \pi_m(\epsilon_m)}{\partial q \partial \epsilon_m} & \frac{\partial^2 \pi_m(\epsilon_m)}{\partial q \partial a} & \frac{\partial^2 \pi_m(\epsilon_m)}{\partial q^2} \end{vmatrix} < 0$$

From the above calculus, there is a maximum solution at $(a^*, q^*, \epsilon_m{}^*)$.

Appendix : 2

Proof . To prove that the optimal policy for integrated channel will maximize total profit of the whole chain, calculate the second particl derivatives of in (7) with respect of at solution of , respectively, we have

$$\frac{\partial \pi_m(C_m)}{\partial C_m} = nP_0\left(\alpha - \beta a^{-\gamma} q^{-\delta}\right)(1+nC_m)^{-e} - \mathrm{ne}(\rho_m + nC_m P_0)\left(\alpha - \beta a^{-\gamma} q^{-\delta}\right)(1+nC_m)^{-e-1} = 0$$

$$\frac{\partial^2 \pi_m(C_m)}{\partial C_{m^2}} = -e^{(e-1)}(e-1)^{(e+2)}P_0^{(e+2)}\left(\alpha - \beta a^{-\gamma} q^{-\delta}\right)(P_0 - \rho_m)$$

$$\frac{\partial \pi_m(C_m)}{\partial a} = (\rho_m + nC_m P_0)\left(\gamma\, \beta a^{-\gamma-1} q^{-\delta}\right)(1+nC_m)^{-e} - 1 = 0$$

$$\frac{\partial \pi_m(C_m)}{\partial q} = (\rho_m + nC_m p_0)\left(\delta \beta a^{-\gamma} q^{-\delta-1}\right)(1+nC_m)^{-e} - 1 = 0$$

$$\frac{\partial^2 \pi_m(C_m)}{\partial C_m \partial a} = 0, \frac{\partial^2 \pi_m(C_m)}{\partial C_m \partial q} = 0,$$

$$\frac{\partial^2 \pi_m(C_m)}{\partial a^2} = -\beta\gamma(\gamma+1)\, a^{-(\gamma+2)} q^{-\delta}\, e^{-e}(e-1)^{(e-1)} {P_0}^e (P_0 - \rho_m)^{(1-e)}$$

$$\frac{\partial^2 \pi_m(C_m)}{\partial a \partial q} = -\beta\gamma\delta\; a^{-(\gamma+1)} q^{-(\delta+1)}\, e^{-e}(e-1)^{(e-1)} {P_0}^e (P_0 - \rho_m)^{(1-e)}$$

$$\frac{\partial^2 \pi_m(C_m)}{\partial q^2} = -\beta\delta(\delta+1)\, a^{-\gamma} q^{-(\delta+2)}\, e^{-e}(e-1)^{(e-1)} {P_0}^e (P_0 - \rho_m)^{(1-e)}$$

Since

$$\frac{\partial^2 \pi_m(C_m)}{\partial C_m^2} < 0, \begin{vmatrix} \frac{\partial^2 \pi_m(C_m)}{\partial {\epsilon_m}^2} & \frac{\partial^2 \pi_m(C_m)}{\partial \epsilon_m \partial a} \\ \frac{\partial^2 \pi_m(C_m)}{\partial a \partial c_m} & \frac{\partial^2 \pi_m(C_m)}{\partial a^2} \end{vmatrix} > 0 \; and \begin{vmatrix} \frac{\partial^2 \pi_m(C_m)}{\partial {\epsilon_m}^2} & \frac{\partial^2 \pi_m(C_m)}{\partial \epsilon_m \partial a} & \frac{\partial^2 \pi_m(C_m)}{\partial \epsilon_m \partial q} \\ \frac{\partial^2 \pi_m(C_m)}{\partial a \partial \epsilon_m} & \frac{\partial^2 \pi_m(C_m)}{\partial a^2} & \frac{\partial^2 \pi_m(C_m)}{\partial a \partial q} \\ \frac{\partial^2 \pi_m(C_m)}{\partial q \partial \epsilon_m} & \frac{\partial^2 \pi_m(C_m)}{\partial q \partial a} & \frac{\partial^2 \pi_m(C_m)}{\partial q^2} \end{vmatrix} < 0$$

From the above calculus, there is a maximum solution at $(a^*, q^*, {C_m}^*)$.

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