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The Impact of Cap and Trade Scheme on Inventory Policies

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ABSTRACT

In this paper we analyse that how the firms manage carbon footprints in inventory management under the carbon emission trading scheme. Today's modern world the carbon emission is one of the main sources for the global warming, so every mankind have the responsibility for reducing the carbon emission to mitigate global warming. It is generally accepted that carbon emission trading is one of the most effective market based mechanism to prohibit the amount of carbon emission. AmulyaGurtu, Mohammed Y.Jaber discussed the following result in their model paper "Impact of fuel price and carbon emission on inventory policies" [App.Mathematical Modelling, 39(2015) 1202-12016]. In that paper they mentioned that the order cost and lot sizes are affected by changes in fuel price, emission tax, fuel consumption, distance travelled. Now we expanded the above paper by introducing the emission minimizing policies such as a cap and trade system in the two level supply chain.Under the cap and trade system we explain about the carbon price and transfer quantity of carbon emission. We also provide the numerical example for better understanding.

Introduction

Currently companies are increasingly sensitive and responsive to the carbon emissions (emissions of carbon dioxide and other greenhouse gases) associated with their operations. Cholette and Venkat (2009),Stock et al (2010). Under the influence of their customers who increase their socially responsible consumption practices Gonzalez et al (2009) and also the government and other pressure groups, they are undertaking initiatives to reduce their carbon footprint. The report taken by intergovernmental panel on climate change (IPCC, 2007) says that the global warming is a endanger to the world ecological system and human race which is caused by the increasing level of carbon emissions and human activities such as burning of fossil fuels and deforestation .In order to reduce the global warming many countries have enacted legislation or designed mechanism to prohibited the total amount of carbon emissions. In recent days the companies are concentrating on the following activities such as redesigning products and packaging, deployment and use of less polluting sources of energy or replacing energy inefficient equipment's and facilities for reducing the carbon emissions. Cap and trade system is one of the sources for reducing the carbon emissions. It is a market based policy tool for protecting human health and environment by controlling large amounts of emission from a group of sources. Mainly the carbon emissions are caused by the logistics and warehousing.

The literature on carbon footprint management in supply chain is wide area. Some studies focus on the measurement method of carbon emissions in supply chains. Carbon trust (2006) develops a methodology to determine the carbon footprint of different products by analysing the carbon emissions generated by the energy used across the supply chain. Mtalaa at al (2009) review the current measurement and calculation models that compute carbon emission from trade transportation. Sundarakani et al (2009) present an analytical model that computes carbon emissions from both stationary and non-stationary supply chain processes. Chaabane et al (2010) induced a mixed integer linear programming based framework for sustainable supply chain design, their model demonstrated that efficient carbon management strategies will help decision makers to achieve sustainability objectives in a cost effective manner. Penkuhn et al (1997) present a nonlinear programming model for joint production planning problems by integrating emission taxes. Kim et al (2009) examine the relationship between the freight transport costs and carbon emissions in given intermodal and truck only freight network s by multi objective optimization. Benjaafar et al (2010) introduce a series of simply models to illustrate how carbon footprint considerations could be incorporated into operations decisions. Pan et al (2010) examined the environmental impact of pooling of supply chains; they found the supply network pooling is an efficient approach in reducing carbon emissions. Harris et al (2011) investigated the relationship between the total logistics costs and the environmental impact of carbon emissions from transportation and electricity usage in depots when using the tradition cost based optimization approach. Bonney and Jaber(2010) examined the importance of inventory planning to the environment and the possibility of using models to perform analysis. In this paper we examine the operations decisions in inventory management with a view to managing a firm's carbon footprint. Under the carbon emission trading mechanism where carbon footprint measures the total amount of carbon dioxide released into the atmosphere as a result of the activities of a particular individual, organisation or community. We derive the environmental inventory model under cap and trade mechanism.

The rest of the paper organised as follows. In section 2 we formulate the carbon footprint management problem and derive the optimal order quantity. In section 3 we provide some numerical example, finally we conclude the paper in section 4.

Optimal order quantity with carbon emission trading:

In this section we consider the single product replenishment for at wo level supply chain that consists of a vendor and buyer. According to the base paper we take the Hill's model and proceed with the carbon trading mechanism. Carbon trading is also known

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as cap and trade. A firm is allocated a limit or cap on carbon emissions. If its amount of carbon emissions exceeds the carbon cap it can buy the right to emit extra carbon from the carbon trading market, otherwise it can sells its surplus carbon credit. According to the base paper we concentrate on the carbon emissions caused by logistics and warehousing activities. The level of carbon emissions from logistics depends on the mode of transportation, choice of fuel used and distance travelled. We assume that the retailer continues to use his current supplier and vehicles after the implementation of the cap and trade system. The carbon price is only affected by the carbon cap of a country, region or the world and is not affected by the carbon cap allocated to the single retailer.

Assumptions and Notations

1. The product demand is known and deterministic.
2. The retail price is exogenous.
3. The retailer decided only the order size

The following notations have been used in the paper.

A_0 : Transport cost at the start of an order point.
A_1 : Annual incremental increase in transport cost.
D : demand/consumption rate per unit time.
h : unit holding per unit of time subscript b and v are added to show buyer and vendor.
K : order cost per cycle, subscript b and v are added to show buyer and vendor.
n : vendor's cycle number
P : Production rate per unit time.
t : cycle time for a lot
α: fuel price
λ : buyer's cycle in asingle vendor's cycle an integer number.
Q : order size in units
Q^0 : optimal order size in the classical EOQ model
Q^* : optimal order size with cap and trade
$Q^\wedge$: optimal order size when carbon emissions reach the minimum
μ : carbon emission quotas per unit time
$e+e_0Q$: the amount of carbon emissions in executing an order of Q units, where eis the Carbon emissions, when the truck is empty and e_0 is the variable emission factor.
g_{0+} gQ : the amount of carbon emissions in holding Q units product, where g_0 is the fixed carbon emissions and g is the variable emission factor in warehousing.
X : transfer quantity of carbon emissions
ATC (Q, X): average total cost per unit time if the order size is Q units and transfer quantity Of carbon emissions is X.

The Mathematical model

We derive the mathematical model under "cap and trade" system. First we derive the Hill's model in the base paper along with the transportation cost and emission tax. The average total cost

$$ATC(Q) = \frac{D}{Q}\left(\frac{K_v}{\lambda}+K_b+A_0+T_e\right)+Qh_v\left\{\frac{D}{P}+\lambda\left(\frac{P-D}{2P}\right)\right\}+\frac{(h_b-h_v)Q}{2}+\frac{A_1(2n\lambda-\lambda-1)}{2} \quad (1)$$

Find the first order derivative of 'Q' and get the optimal order quantity,

$$= -\frac{D}{Q^2}\left(\frac{K_v}{\lambda}+K_b+A_0+T_e\right)+h_v\left\{\frac{D}{P}+\lambda\left(\frac{P-D}{2P}\right)\right\}+\frac{(h_b-h_v)}{2} = 0 \quad (2)$$

$$Q=\sqrt{\frac{D\left(\frac{K_v}{\lambda}+K_b+A_0+T_e\right)}{h_v\left\{\frac{D}{P}+\lambda\left(\frac{P-D}{2P}\right)\right\}+\frac{(h_b-h_v)}{2}}} \quad (3)$$

Substituting the value of Q optimal in the equation (1), the total cost function reduces to

$$ATC(Q) = 2\sqrt{D(\frac{K_v}{\lambda}+K_b+A_0+T_e)[h_v\left\{\frac{D}{P}+\lambda\left(\frac{P-D}{2P}\right)\right\}+\frac{(h_b-h_v)}{2}]}+\frac{A_1(2n\lambda-\lambda-1)}{2} \quad (4)$$

Now we derive the Hill's model under the "cap and trade system" instead of emission tax.

Under the cap and trade system the cost functions are,

$$ATC(Q,X) = \frac{D}{Q}\left(\frac{K_v}{\lambda}+K_b+A_0\right)+Qh_v\left\{\frac{D}{P}+\lambda\left(\frac{P-D}{2P}\right)\right\}+\frac{(h_b-h_v)Q}{2}+\frac{A_1(2n\lambda-\lambda-1)}{2}-CX$$

We assumed that the carbon emission is caused by the logistics and warehouse. Therefore the carbon footprint from logistics with per unit time is,

$$\frac{(e+e_0Q)}{Q/D} = e_0D+eD/Q$$

Carbon footprint from warehouse with per unit time is = $g_0+gQ/2$ [average inventory is Q/2]

∴ The total carbon footprint is,

$CF(Q) = e_0D + eD/Q + g_0 + gQ/2$

$= eD/Q + gQ/2 + (e_0D + g_0)$

Which is similar to the objective function in the classical EOQ model,from that we find the,

$$\hat{Q} = \frac{eD}{Q^2} + \frac{g}{2}$$

$$\frac{g}{2} = \frac{eD}{Q^2}$$

$$Q^2 = \frac{2eD}{g}$$

$\hat{Q} = \sqrt{\frac{2eD}{g}}$ the carbon emission reaches the minimum.

The modified EOQ model under the cap and trade system is,

$$\text{Min ATC }(Q, X) = \frac{D}{Q}\left(\frac{K_v}{\lambda} + K_b + A_0\right) + Qh_v\left\{\frac{D}{P} + \lambda\left(\frac{P-D}{2P}\right)\right\} + \frac{(h_b - h_v)Q}{2} + \frac{A_1(2n\lambda - \lambda - 1)}{2} - CX \quad (1)$$

s.t $eD/Q + gQ/2 + X = \mu - (e_0D + g_0)$ (2)

Where equ(2) is the carbon balance constraint and X may be positive, negative or zero. X is positive that the retailer sells X (buy (X)) units of carbon credit to (from) the carbon market and X= 0 means that the retailer neither buys nor sells any carbon credit.

When $e_0 = g_0 = 0$, constraint (2) is, $eD/Q + gQ/2 + X = \mu$ (3)

For the convenient, without loss of generality we will focus on the following model in the rest of the paper.

$$\text{Min ATC }(Q, X) = \frac{D}{Q}\left(\frac{K_v}{\lambda} + K_b + A_0\right) + Qh_v\left\{\frac{D}{P} + \lambda\left(\frac{P-D}{2P}\right)\right\} + \frac{(h_b - h_v)Q}{2} + \frac{A_1(2n\lambda - \lambda - 1)}{2} - CX$$

s.t $eD/Q + gQ/2 + X = \mu$

From (3) $X = \mu - eD/Q + gQ/2$ (4)

Subequ (4) in (1) we have, min ATC (Q),

$$\frac{D}{Q}\left(\frac{K_v}{\lambda} + K_b + A_0\right) + Qh_v\left\{\frac{D}{P} + \lambda\left(\frac{P-D}{2P}\right)\right\} + \frac{(h_b - h_v)Q}{2} + \frac{A_1(2n\lambda - \lambda - 1)}{2} - C(\mu - (eD/Q + gQ/2)) \quad (5)$$

$$= \frac{D}{Q}\left(\frac{K_v}{\lambda} + K_b + A_0 + Ce\right) + Qh_v\left\{\frac{D}{P} + \lambda\left(\frac{P-D}{2P}\right)\right\} + \frac{(h_b - h_v + Cg)Q}{2} + \frac{A_1(2n\lambda - \lambda - 1)}{2} - C\mu \quad (6)$$

Derive the first order derivative with respect to 'Q'

$$= -\frac{D}{Q^2}\left(\frac{K_v}{\lambda} + K_b + A_0 + Ce\right) + h_v\left\{\frac{D}{P} + \lambda\left(\frac{P-D}{2P}\right)\right\} + \frac{(h_b - h_v + Cg)}{2} = 0 \quad (7)$$

From that we find the optimum order quantity, Q*

$$Q^* = \sqrt{\frac{D(\frac{K_v}{\lambda} + K_b + A_0 + \text{Ce})}{h_v\left\{\left(\frac{D}{P} + \lambda\left(\frac{P-D}{2P}\right)\right)\right\} + (h_b - h_v + \text{Cg})}} \quad (8)$$

$$\text{ATC}(Q^*) = 2\sqrt{D(\frac{K_v}{\lambda} + K_b + A_0 + Ce)[h_v\left\{\frac{D}{P} + \lambda\left(\frac{P-D}{2P}\right)\right\} + \frac{(h_b - h_v + Cg)}{2}]} + \frac{A_1(2n\lambda - \lambda - 1)}{2} - C\mu$$

From (4) & (5) , if C=0 then $Q^* = \sqrt{\frac{D(\frac{K_v}{\lambda} + K_b + A_0)}{h_v\left\{\left(\frac{D}{P} + \lambda\left(\frac{P-D}{2P}\right)\right)\right\} + (h_b - h_v)}}$

$$\text{ATC }(Q^*) = 2\sqrt{D(\frac{K_v}{\lambda} + K_b + A_0)[h_v\left\{\frac{D}{P} + \lambda\left(\frac{P-D}{2P}\right)\right\} + \frac{(h_b - h_v)}{2}]} + \frac{A_1(2n\lambda - \lambda - 1)}{2} - C\mu$$

This means that, when carbon credit is free the retailer does not care about the carbon emissions and will adopt the optimal order policy for the classical EOQ model. Suppose that the carbon credit is very expensive the retailer should minimize the carbon emission.

Numerical Example

For the clear understanding of the proposed model, the numerical example is provided. The following data is taken from the base model.

Demand rate (unit/year)	D	1000
Production rate(unit/year)	P	3200
Buyer's order cost ($/order)	K_b	25
Vendor's order cost ($/order)	K_v	400

Holding cost for the vendor ($/unit/unit of time)	h_v	4.00
Holding cost for the buyer ($/unit/unit of time)	h_b	5.00
Fuel price	α	0.5107
Transport cost start of an order point	A_0	$153.21
Transport cost start of an order point	A_1	$5.46
Carbon tax	T_e	23.20
Carbon emission	e 600	
Variable emission factor	g	1
Carbon price per unit	C	0.2
Buyer's cycles in a single vendor's cycle An integer number	λ	2
Carbon emission quotas	μ	8000
Vendor cycle's number	n	2

Using the above data, we can able to find out the optimum order quantity with carbon tax, Q = 299 and Average total cost including carbon tax = 2691. Under the cap and trade system the optimum order quantity and the total cost are Q^* = 310, ATC (Q^*) = 1441.

Suppose if the carbon credit is free (i.e) C= 0, then the order quantity $\hat{Q}$ =290.

From the numerical result we understand that the retailer should minimize the carbon emissions when the carbon credit is very expensive.

Conclusion

We conclude that the carbon tax and cap and trade scheme both add to the price of emitting carbon albeit in slightly different way. The carbon tax is known as a price instrument at the same time cap and trade is quantity instrument. We also noticed that the total cost is less in the cap and trade system compare to the total cost with the carbon tax. For that reason cap and trade system is very popular among the world.

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