Available online at www.elixirpublishers.com (Elixir International Journal)

Applied Mathematics

Elixir Appl. Math. 83 (2015) 32909-32917

ISSN: 2229-712X

Predator-Prey System with Infection in Predator Only

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ARTICLE INFO

Article history:
Received: 3 May 2015;
Received in revised form:
25 May 2015;
Accepted: 2 June 2015;

Keywords
Prey, Predator,
Stability, Equilibrium points,
Carrying capacity.
Predation rate, Recovery.

ABSTRACT

A three dimensional eco-epidemiological model consisting of prey, susceptible predator, infected predator species, is proposed and analyzed in the present work. From infected predator, the disease is transmitted to the susceptible predator species. Differential predation rate is considered due to disease in predator as the infection reduces the predation ability of infected predator. The recovery of infected predator from disease is incorporated; therefore, an SIS model is taken for predator species. The dynamics of the system is analyzed mathematically and conditions for existence and stability of disease free equilibrium point has been found out. Also conditions for disease to be endemic in predator species are obtained. Numerical simulations have been carried out to justify the results obtained.

Introduction

In natural ecosystem, regulatory mechanisms for evolution of biological species are provided by predator-prey interactions. Not only the disease in the system affects the dynamics of prey predator population, but the prey predator interactions also affect the dynamics of disease [1, 5, 8, 11, 14]. Some previous works on infectious diseases in animal populations had paid attention to the changes in the regulation of natural populations due to disease-induced mortality or disease-reduced reproduction [14, 15, 16]. The effect of parasites on biodiversity and ecosystem are the main fields to be studied in conservation biology [9, 13]. Many Authors proposed and studied different prey-predator models in presence of disease. Chattopadhyay and Arino [10] coined the name eco-epidemiology for the study of such systems. Researchers are now considering the importance of parasites not only on individual hosts but also on their population dynamics and community structure. In most of the eco-epidemiological models, studied so far, disease in only prey species has been studied [1, 2, 3, 6]. In continuity with the work on models where the infection floats in prey population only, Lafferty and Morris [12] observed that the prey killifish on getting infected tends to come closer to the sea surface which makes them easily available for the predation by birds. They observed that predation rate of infected prey is 31 times more than that of the susceptible prey. Few studies [4, 7] considered the spread of disease in predator species.

In the present paper, a predator-prey model in which only predator population is invaded by a disease is proposed and analyzed. The disease is spreading from infected predator to susceptible predator. The predator species are compartmentalized into susceptible and infected classes. The disease does not cause immunity in the predator species. Consequently, an SIS model is considered for the predator. Due to infection in infected predator, its catching ability will be reduced as compared to susceptible predator. So, differential predation rates of both susceptible and infected predator have been considered and effect of it on the dynamics of the system is investigated. The effect of recovery on the system dynamics is also observed.

Basic Assumptions and Mathematical Model

In present paper, a prey predator model is considered in which only the predator species is infected with some disease. Due to disease predator species is divided into two units say susceptible and infected ones. Let $y(t)$ and $z(t)$ are densities of susceptible and infected predator respectively at any time t. Hence the total population P of predator is given by

$$P(t) = y(t) + z(t)$$

Density of prey population at any time t is '$x(t)$' and is growing logistically with intrinsic growth rate r. Carrying capacity of system is considered to be k for prey. Law of mass action is considered for disease transmission in predator's species and disease is spreading at rate c. Mortality rates of susceptible and infected predators are taken as μ_1 and μ_2 $(\mu_2 > \mu_1)$. The susceptible and infected predators are predating the prey with differential predation rate α_1 and α_2 respectively $(\alpha_2 < \alpha_1)$. l is the conversion rate of prey into young ones of predators. The infected prey population recovers with recovery rate γ .

Based on the above assumptions, the following eco- epidemiological model is formed.

$$\begin{aligned} \frac{dx}{dt} &= rx\left(1-\frac{x}{k}\right) - \alpha_1 xy - \alpha_2 xz \\ \frac{dy}{dt} &= l\alpha_1 xy - \mu_1 y - cyz + \gamma z \qquad (1) \\ \frac{dz}{dt} &= l\alpha_2 xz - \mu_2 z + cyz - \gamma z \qquad \text{where } (x(0)>0\ y(0)>0,\ z(0)>0) \end{aligned}$$

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Mathematical analysis

The system can have following different equilibrium

(a) A trivial equilibrium point $E_0\left(0,0,0\right)$, where all the predator and prey populations extinct.

(b) A predator free equilibrium point $E_1\left(k,0,0\right)$, where prey approaches to its carrying capacity in absence of predator.

(c) A disease free equilibrium (DFE) $E_2=\left(x_2,y_2,0\right)$, where

$$x_2=\frac{\mu_1}{l\alpha_1},\quad y_2=\frac{r}{\alpha_1}\left(\frac{l\alpha_1 k-\mu_1}{l\alpha_1 k}\right),\ \text{which exist provided}\quad \alpha_1>\mu_1/lk \tag{2}$$

(d) An endemic positive equilibrium $E^*=\left(x^*,y^*,z^*\right)$, where

$$x^*=\frac{\mu_2-cy^*+\gamma}{l\alpha_2},\qquad \text{and}\quad z^*=\frac{r\left(kl\alpha_2-\mu_2-\gamma\right)+\left(cr-kl\alpha_1\alpha_2\right)y^*}{kl\alpha_2^2},$$

exists provided

$$\frac{\mu_2+\gamma}{kl}<\alpha_2<\frac{cr}{kl\alpha_1} \tag{3}$$

and y^* is the positive root of the equation $Ay^2+By+C=0$ (4)

where

$$A=rc^2>0\ ,\ B=kl\alpha_2\left(\alpha_2\mu_1+cr-\alpha_1\mu_2\right)-cr\left(2\gamma+\mu_2\right)$$

$$\text{and } C=\left(\mu_2+\gamma-kl\alpha_2\right)\gamma r$$

The equation (4) has a positive root for y^* under the following conditions

(i) If B < 0, C < 0,

$$\text{i.e}\quad \frac{\mu_2+\gamma}{kl}<\alpha_2<\frac{cr}{kl\alpha_1}\ ,$$

then the equation (4) has one positive value of y.

(ii) If B>0, C<0,

$$\text{i.e}\quad \alpha_2>Max\left\{\frac{\mu_2+\gamma}{kl},\frac{cr\left(2\gamma+\mu_2\right)}{kl\left(\alpha_2\mu_1+cr-\alpha_1\mu_2\right)}\right\} \tag{5}$$

then the equation (4) has one positive value of y.

(iii) If C>0,

$$\text{i.e}\quad \alpha_2<\frac{\mu_2+\gamma}{kl} \tag{6}$$

then

$$B=kl\alpha_2\left(\alpha_2\mu_1+cr-\alpha_1\mu_2\right)-cr\left(2\gamma+\mu_2\right)$$
$$=cr(kl\alpha_2-\mu_2-\gamma)+kl\alpha_2\left(\alpha_2\mu_1-\alpha_1\mu_2\right)-\gamma<0\qquad \text{using (6)}$$

then equation (4) has two positive roots.

From above, we conclude that if

$$\text{Max}\left\{\frac{\mu_2+\gamma}{kl},\frac{cr\left(2\gamma+\mu_2\right)}{kl\left(\alpha_2\mu_1+cr-\alpha_1\mu_2\right)}\right\}<\alpha_2<\frac{cr}{kl\alpha_1} \tag{7}$$

holds then $\left(x^*,y^*,z^*\right)$ is unique.

$$\text{further if}\ \alpha_2<\min\left\{\frac{\mu_2+\gamma}{kl},\frac{cr}{kl\alpha_1}\right\}\quad \text{and}\quad y^*>\frac{r\left(\mu_2+\gamma-kl\alpha_2\right)}{cr-kl\alpha_1\alpha_2} \tag{8}$$

then there exist two non zero equilibrium points.

Theorem 1: If $\text{Max}\left\{\frac{\mu_2+\gamma}{kl},\frac{cr\left(2\gamma+\mu_2\right)}{kl\left(\alpha_2\mu_1+cr-\alpha_1\mu_2\right)}\right\}<\alpha_2<\frac{cr}{kl\alpha_1}$ holds

then the system (1) has a non zero unique equilibrium

but if $\alpha_2<\min\left\{\frac{\mu_2+\gamma}{kl},\frac{cr}{kl\alpha_1}\right\}$ and $y^*>\frac{r\left(\mu_2+\gamma-kl\alpha_2\right)}{cr-kl\alpha_1\alpha_2}$

then (1) has two non zero equilibrium points.

Stability Analysis

The variational matrix for the system (2.1) is given by

$$\begin{pmatrix} X & -\alpha_1 x & -\alpha_2 x \\ l\alpha_1 y & l\alpha_1 x-\mu_1-cz & -cy+\gamma \\ l\alpha_2 z & cz & l\alpha_2 x-\mu_2+cy-\gamma \end{pmatrix}$$

Where

$$X=-\frac{rx}{k}+\left[r\left(1-\frac{x}{k}\right)-\alpha_1 y-\alpha_2 z\right]$$

Now we will check the stability of all the four equilibrium points

Theorem 2: The trivial equilibrium point $E_0\ (0,0,0)$ is always unstable.

Proof: - The Eigen values for $E_0\ (0,0,0)$ are given by $\xi_1=r,\quad \xi_2=-\mu_1,\quad \xi_3=-\mu_2-\gamma$

Clearly $\xi_2,\ \xi_3<0$, but $\xi_1>0$. So $E_0\ (0,0,0)$ is a saddle point.

Biological Meaning: - The predator and prey population will not extinct forever.

Theorem 3: The predator free equilibrium point $E_1(k,0,0)$, is locally asymptotically stable if

$$\alpha_1<\mu_1/lk \tag{9}$$

Proof: - The Eigen values for the equilibrium E_1 are given by

$$\xi_1=-r,\quad \xi_2=l\alpha_1 k-\mu_1,\quad \xi_3=l\alpha_2 k-\mu_2-\gamma$$

Clearly $\xi_1<0$,

Now $\xi_2<0,\quad \xi_3<0$ iff $\alpha_1<\mu_1/lk$ and $\gamma>lk\alpha_2-\mu_2$

i.e. $\alpha_1<\min\left\{\mu_1\big/lk,\frac{\gamma+\mu_2}{lk}\right\}=\mu_1/lk$

Hence the result.

Biological Meaning: - If the capture coefficient 'α_1' of the susceptible predator and conversion rate of prey into young ones of predators (*l*) are sufficiently low such that equation (9) is satisfied, then the predator population will extinct from the environment.

Theorem 4: The disease free equilibrium point (DFE) $E_2=(x_2,y_2,0)$, if exist, is locally asymptotically stable provided

$$\gamma>\frac{\alpha_2\mu_1}{\alpha_1}-\mu_2+\frac{cr}{\alpha_1}-\frac{cr\mu_1}{lk\alpha_1^2} \tag{10}$$

Proof:-The Characteristic roots corresponding to the equilibrium E_2 are given by the equation

$$(\xi-l\alpha_2 x_2+\mu_2-cy_2)\left[\xi^2+\left(\frac{rx_2}{k}-l\alpha_1 x_2+\mu_1\right)\xi+\left(\frac{r\mu_1 x_2}{k}-\left(\frac{rl\alpha_1 x_2^2}{k}\right)+l\alpha_1^2 x_2 y_2\right)\right]=0$$

$$\text{i.e.}\quad (\xi-l\alpha_2 x_2+\mu_2-cy_2)\left(\xi^2+\frac{rx_2}{k}\xi+l\alpha_1^2 x_2 y_2\right)=0$$

Here $\xi_1=l\alpha_2 x_2-\mu_2+cy_2-\gamma$ and ξ_2,ξ_3 are roots of second factor having the entire coefficients positive. So ξ_2,ξ_3 are clearly negative. For $\xi_1<0$,

$$\gamma>l\alpha_2 x_2-\mu_2+cy_2$$

$$\text{i.e}\quad \gamma>\frac{\alpha_2\mu_1}{\alpha_1}-\mu_2+\frac{cr}{\alpha_1}-\frac{cr\mu_1}{lk\alpha_1^2}.$$

Hence the proof.

Biological Meaning: - It is clear from equation (10) that reduced value of capture coefficient of infected predator species will enhance the stability of disease free equilibrium point.

Theorem 5:- The non zero equilibrium point $E^*=\left(x^*,y^*,z^*\right)$, if exist, is locally asymptotically stable provided

$$cy^*-\gamma>0. \tag{11}$$

Proof: - The characteristic equation at E^* is given as

$$\xi^3+A_1\xi^2+A_2\xi+A_3=0$$

where

$$A_1 = \frac{\gamma z^*}{y^*} + \frac{rx^*}{k}$$
$$A_2 = cz^*\left(cy^* - \gamma\right) + \frac{rx^*}{k}\left(\mu_1 + cz^* - l\alpha_1 x^*\right) + l\alpha_1^2 x^* y^* + l\alpha_2^2 x^* z^*$$
$$= cz^*\left(cy^* - \gamma\right) + \frac{rx^*}{k}\left(\frac{\gamma z^*}{y^*}\right) + l\alpha_1^2 x^* y^* + l\alpha_2^2 x^* z^* \qquad (12)$$
$$A_3 = \frac{rc}{k} x^* z^*\left(cy^* - \gamma\right) + l\alpha_1\alpha_2\gamma x^* z^* + l\alpha_2^2 x^* z^*\left(\mu_1 + cz^* - l\alpha_1 x^*\right)$$
$$= \frac{rc}{k} x^* z^*\left(cy^* - \gamma\right) + l\alpha_1\alpha_2\gamma x^* z^* + l\alpha_2^2 x^* z^*\left(\frac{\gamma z^*}{y^*}\right)$$

Again $A_1A_2 - A_3 = klx^* y^{*^2}\left(rx^* y^* + k\gamma z^*\right)\alpha_1^2 - k^2 l\gamma\alpha_1\alpha_2 x^* z^* y^{*^2}$

$$+\ z^*\left(\gamma\left(r^2 x^{*^2} y^* + ck^2 y^* z^*\left(cy^* - \gamma\right) + k\gamma r x^* z^*\right) + klr\alpha_2^2 x^{*^2} y^{*^2}\right)$$
$$= \left(k^2 l\alpha_1^2\gamma x^* y^{*^2} z^* - k^2 l\gamma\alpha_1\alpha_2 x^* z^* y^{*^2}\right) + klr x^{*^2} y^{*^3}$$
$$+\ z^*\left(\gamma\left(r^2 x^{*^2} y^* + ck^2 y^* z^*\left(cy^* - \gamma\right) + k\gamma r x^* z^*\right) + klr\alpha_2^2 x^{*^2} y^{*^2}\right)$$

As $k^2 l\alpha_1^2\gamma x^* y^{*^2} z^* - k^2 l\gamma\alpha_1\alpha_2 x^* z^* y^{*^2}$

$$= k^2 l\gamma x^* y^{*^2} z^*\left(\alpha_1 - \alpha_2\right) > 0$$

So $A_1A_2 - A_3 > 0$ if $cy^* - \gamma > 0$

Now all A_1, A_2, A_3, and $A_1A_2 - A_3$ will be positive if $cy^* - \gamma > 0$.

So by Routh-Hurwitz stability criterion, all the eigen values of the characteristic equation have negative real part if

$$cy^* - \gamma > 0$$

Hence the Proof.

Biological Meaning: - If the capture coefficient of the infected predator is so adjusted such that the equation (3) is satisfied, and the recovery rate is sufficiently low such that the equation (11) is satisfied, then all the three populations will survive in the system together.

Numerical Simulations

Numerical simulations have been carried out to investigate the dynamics of the proposed 3-D model (1). Computer simulations have been performed using MATLAB, for different set of parameters.

Consider the following set of parametric values:

$$r = 1,\ k = 100,\ \alpha_1 = 0.48,\ \alpha_2 = 0.45,\ c = 0.5,$$
$$\mu_1 = 0.49,\ \mu_2 = 0.5,\ l = 0.01,\ \gamma = 0.05 \qquad (12)$$

The system (1) has equilibrium point $E_1(100,0,0)$ for the data set (12). It is locally asymptotically stable by Theorem (3), as the computed value of α_1 is sufficiently low such that the equation (9) is satisfied. The solution trajectories in phase plane with different initial values and time series plot for the predator free equilibrium point $E_1(100,0,0)$ is shown in following Fig. 1.1.

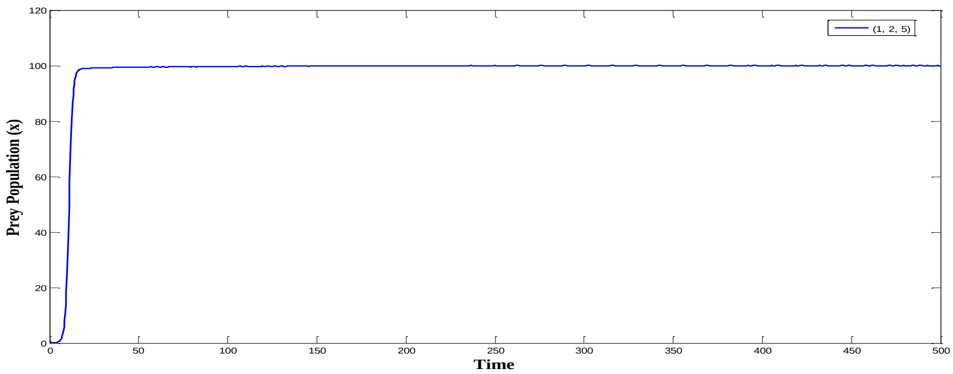

Figure 1.1 (a)

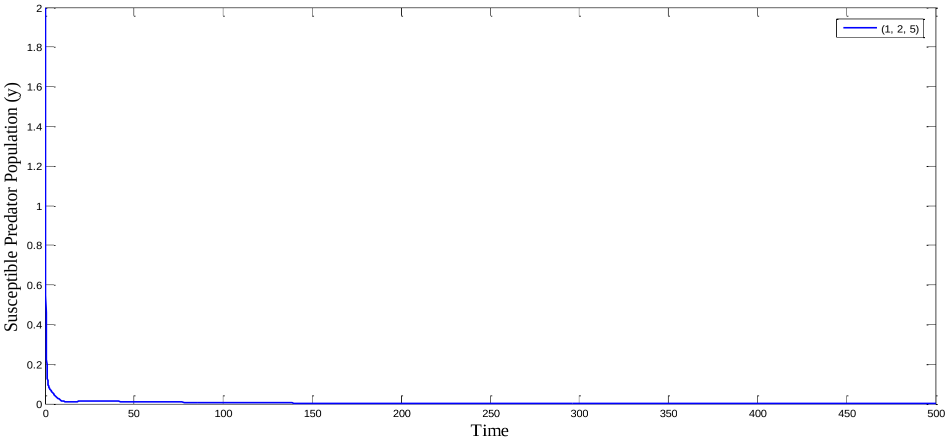

Figure 1.1 (b)

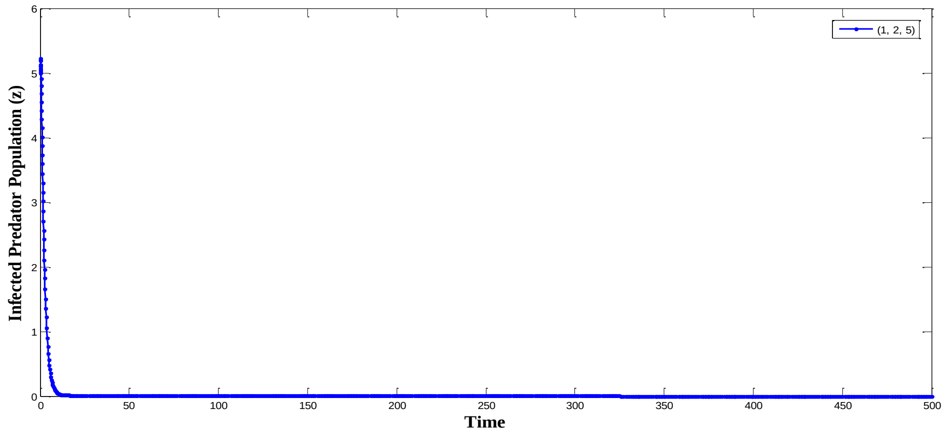

Figure 1.1 (c)

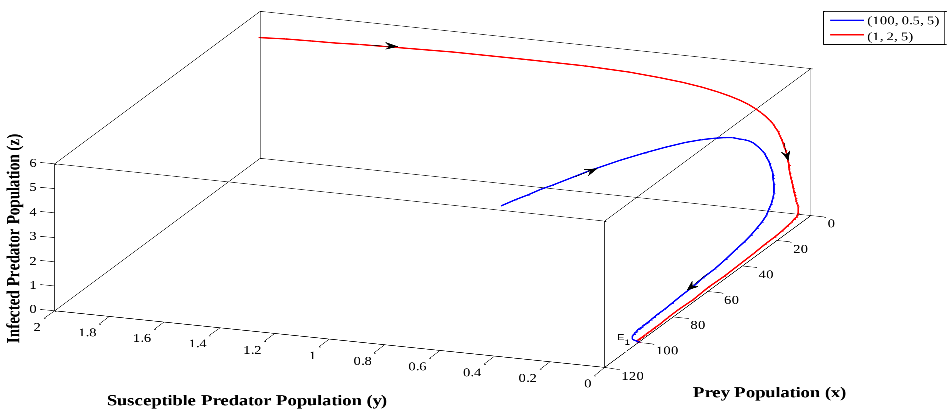

Figure 1.1 (d)

Fig. 1.1. Behavior of the model equation (1) for the stability of predator free point. Fig 1.1(a), (b) and (c) depict respectively, the time evolution of prey, susceptible predator and infected predator populations and 1.1(d) depicts the phase space

Now consider the another set of parametric values

$$r = 1,\ k = 100,\ \alpha_1 = 0.7,\ \alpha_2 = 0.45,\ c = 0.5,$$
$$\mu_1 = 0.49,\ \mu_2 = 0.5,\ l = 0.01,\ \gamma = 0.05 \qquad (13)$$

For these parametric values, the equilibrium point $E_1(100,0,0)$ for system (1) becomes unstable as the capture coefficient of susceptible predator species, α_1, is sufficiently high that condition (9) is violated and it gives rise to existence of $E_2(69.9,0.43,0)$ as condition (2) is satisfied. Also $E_2(69.9,0.43,0)$ is locally asymptotically stable as the recovery rate of the infected predator γ so increased that the condition (10) is satisfied. The solution trajectories in phase plane with different initial values and time series plot for the disease free equilibrium point is shown in Fig-1.2.

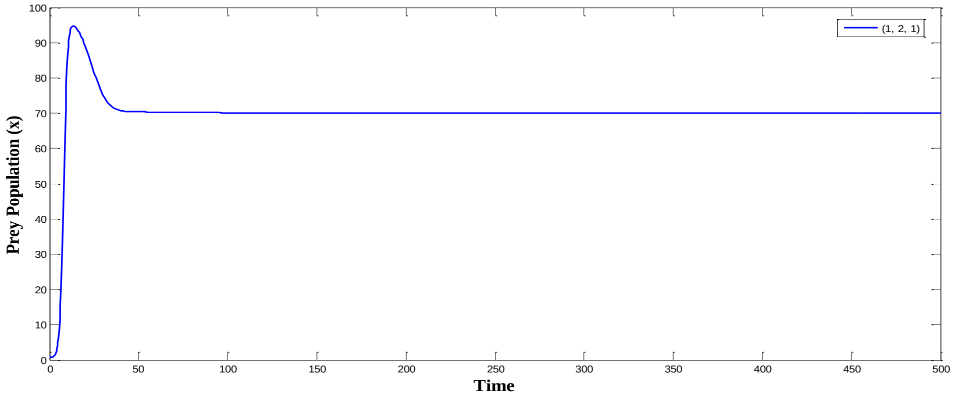

Figure 1.2 (a)

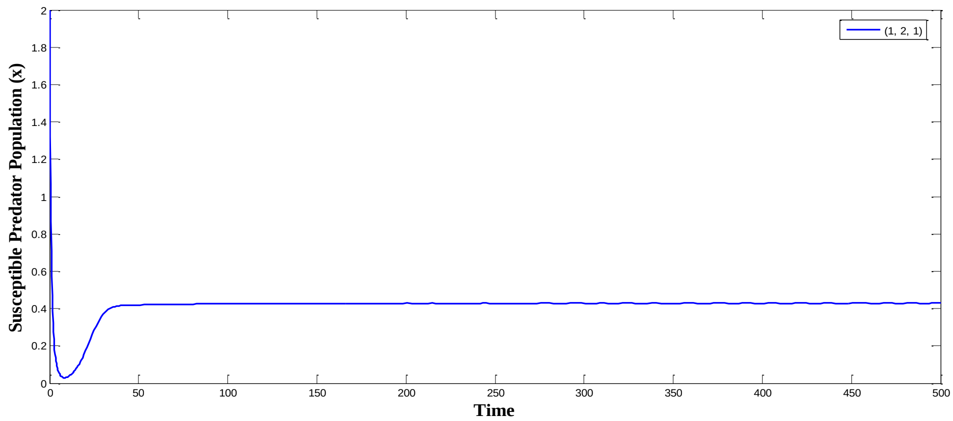

Figure 1.2 (b)

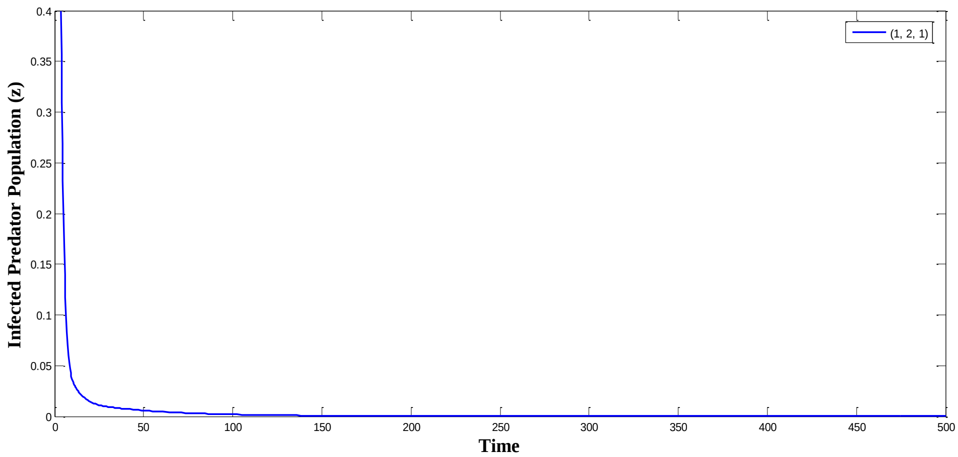

Figure 1.2 (c)

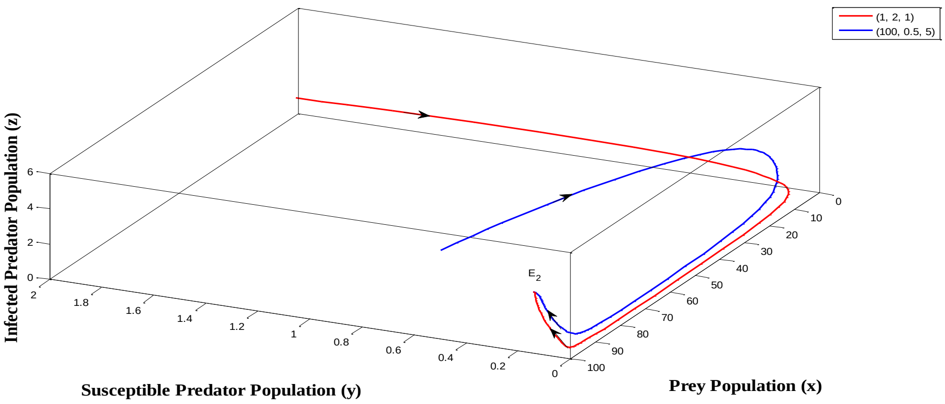

Figure 1.2 (d)

Fig 1.2. Behavior of the model equation (1) for the stability of disease free point. Fig 1.2 (a), (b) and (c) depict respectively, the time evolution of prey, susceptible predator and infected predator populations and 1.2 (d) depicts the phase space

Further, we take equal rate of predation/ catching ability coefficient for both the susceptible and infected predator. For this purpose, following set of data is considered

$$r=1,\ k=100,\ \alpha_1=0.4,\ \alpha_2=0.4,\ c=0.5,$$
$$\mu_1=0.49,\ \mu_2=0.5,\ l=0.01,\ \gamma=0.05 \tag{14}$$

Here the equilibrium point $E_2(69.9, 0.43, 0)$ becomes unstable by Theorem (4) as condition (10) is not satisfied, but the predator free equilibrium point $E_1(100,0,0)$ becomes stable as the conditions (9) is satisfied So different predation rates of susceptible and infected predators are helpful in making the system disease free and both the prey and susceptible predator population coexist where as when the predation rates are equal, then predator population extinct from the system.

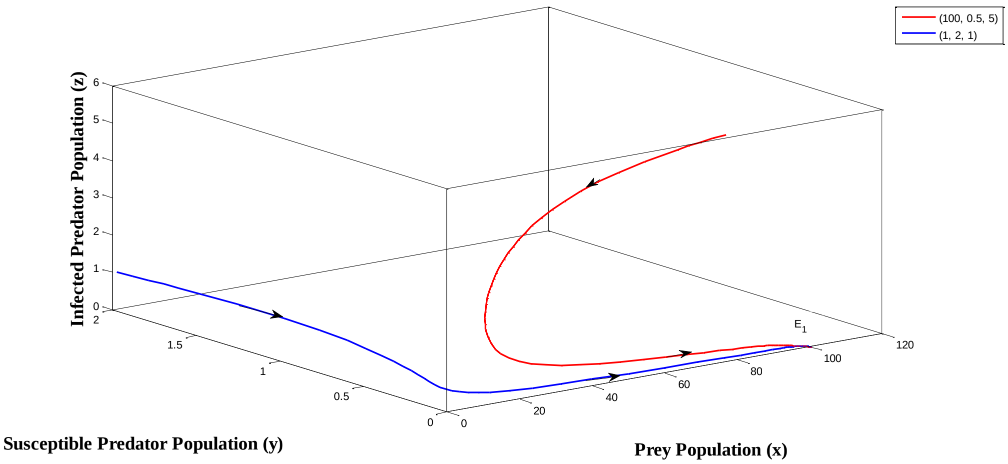

Fig. 1.3. Phase diagram showing the impact of equal predation rates.

Now consider another set of parametric values

$$r=1,\ k=100,\ \alpha_1=0.65,\ \alpha_2=0.55,\ c=0.5,$$
$$\mu_1=0.49,\ \mu_2=0.5,\ l=0.01,\ \gamma=0.05 \tag{15}$$

For these set of values, all the three populations coexist and the disease becomes endemic in the system. The system (1) has equilibrium point $E^*(79.74, 0.222, 0.1047)$ as the conditions (3), (7) and (8) are satisfied. The reduced value of recovery rate of the infected predator is helpful in making this equilibrium point locally stable. The solution trajectories in phase plane with different initial values and variation of all the three population with time is shown in Fig-1.4

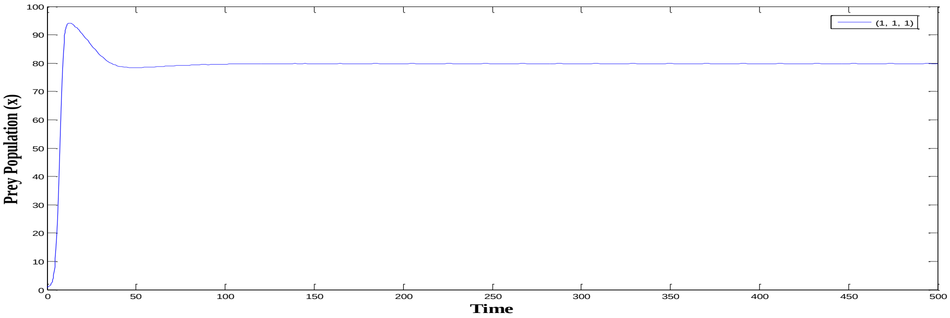

Figure 1.4 (a)

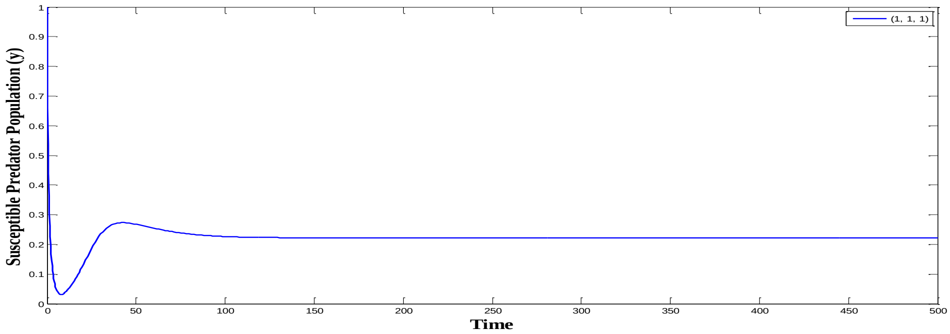

Figure 1.4 (b)

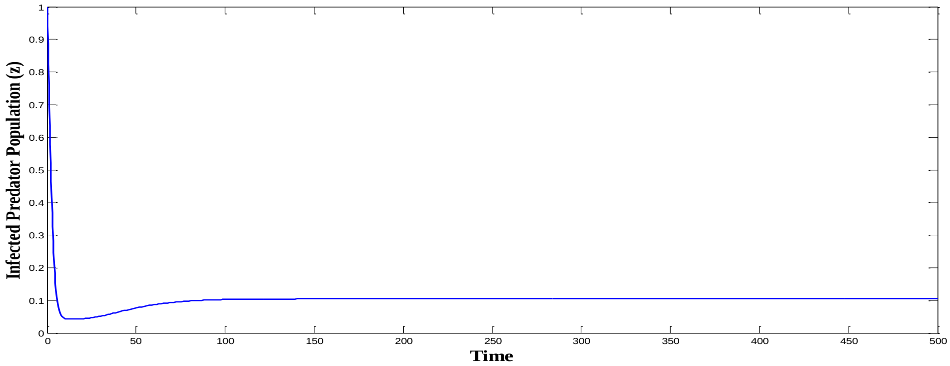

Figure 1.4 (c)

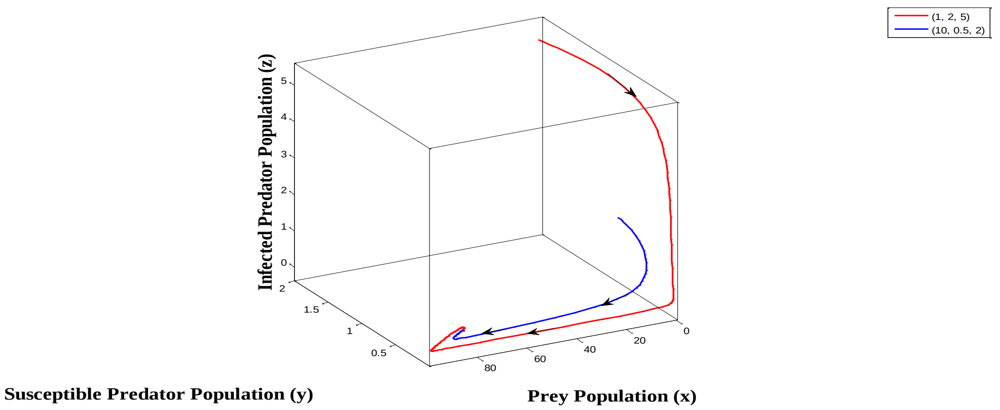

Figure 1.4 (d)

Fig 1.4. Behavior of the model equation (1) for the stability of predator free point $E^*(x^*, y^*, z^*)$. Fig 1.4 (a), (b) and (c) depict respectively, the time evolution of prey, susceptible predator and infected predator populations and 1.4(d) depicts the phase space

Discussion and Conclusion

In present paper, a prey predator model is considered in which only predator species is infected with some disease. Due to disease, predator species is divided into two compartments namely susceptible and infected ones. An SIS model is taken for the predator species. Differential catching ability of both the susceptible and infected predator is considered. Conditions for the existence and stability of disease free prey-predator system and persistence of all the three species are obtained. It is observed that the predator and prey species will not extinct forever. Further it is found that if the capture coefficient 'α_1' of the susceptible predator and conversion rate 'l' of prey into predator's young ones is sufficiently low such that equation (9) is satisfied, then the predator population will extinct from the environment. The impact of different catching ability of both the susceptible and infected predator on the stability of disease free system has been observed. It is seen from equation (10) that reduced value of capture coefficient of infected predator species will enhance the stability of disease free equilibrium point. This fact was verified with the help of data set (13) for which disease free equilibrium point becomes stable when the capture coefficient of infected predator/reduced predation rate 'α_2' of predator was taken whereas when same rate of predation was taken for both susceptible and infected predator in data set (14) then the disease free equilibrium point becomes unstable and non- zero equilibrium point becomes stable. The effect of recovery of predator species on the dynamics of the system is also observed and it is viewed that if the recovery is sufficiently high that the equation (10) is satisfied, then the disease free equilibrium point becomes locally stable. Also if the capture coefficient of the infected predator is so adjusted such that the condition (3) is satisfied, and the recovery rate is sufficiently low such that the condition (11) is satisfied, then all the three populations will survive in the system together which is favorable for ecological diversity.

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