

## Presenting New Solution for Optical Waves in Nonlinear Systems

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There are lots of nonlinear formulas in mathematics and physics and there are, many types of solitons to talk over. This revision is dedicated to the straight technical particulars of how to produce soliton solution of Maxwell's equations and intensive confirmation is placed on mathematical report analysis. The quasi-stationary state waveguides has been founded and with a small deviation from equilibrium, we measured and advanced a solution for a formula. Classical parting of parameters is presented with a special B.C. is introduced on a slab waveguide to obtain a general form of solution of the coordinates x, y z.

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**Introduction**

In optical beam propagation, the nonlinear parameter of refraction is responsible for self-focusing effect. R.Y. Chiao et al. reported that [1] this effect appears when an electromagnetic field stimulate a refractive index in the medium through which the field propagates. From the third order nonlinear polarization, the self-focusing appears. The interaction of strong pulses and beams of electromagnetic radiation with the interface between two semi-infinite media is investigated analytically. Start with a strong plane of constant amplitude and s-polarized wave is incident at an angle  $\theta$  at an interface between linear and nonlinear non-dispersive medium. The nonlinear medium of dielectric permittivity [2],  $\epsilon_o = \epsilon_{NL} + \alpha|E|^2$ ,  $\alpha > 0$ , occupies the half space for  $y > 0$ , and because of the

square term of the electric field in the dielectric permittivity has slow molecular motion which is responsible for the nonlinearity response. When there are two dimensional networks of waveguides, self-localized states can travel along paths and routed to ant destination port[3]. Here we take x,z axis in the space of the interface and y-axis of the Cartesian coordinates perpendicular to the interface. For a vector wave equation in the nonlinear media, the propagation of electromagnetic field is adjusted by Maxwell's equations [4].

$$\nabla^2 \vec{E} + \frac{\omega^2 n_o^2}{c^2} \vec{E} + \frac{1}{n_o^2 \epsilon_o} \vec{\nabla} \left( \vec{\nabla} \cdot \vec{P}_{NL} \right) + \frac{\omega^2}{c^2 \epsilon_o} \vec{P}_{NL} = 0 \quad (1.1)$$

Here  $\vec{E}$  is the electric field vector function of r and t,  $\vec{P}_{NL}$  is nonlinear polarization [5] vector function of r and t in the form

$\vec{P}_{NL} = \epsilon_o \alpha |E|^2 \vec{E}$ , where  $\alpha$  is nonlinear coefficient,  $\omega$  is the angular frequency of the wave,  $c$  is the speed of the light in vacuum,  $\epsilon_o$  is the permittivity of the free space and  $n_o$  is linear refractive index. Time derivatives will be removed by achieving the fields in the frequency domain. The solution for this equation will be by assuming lowest order simple soliton form and by successive approximations, spatial mathematical technique, Eq. (1.1) reduces to a familiar plane waves equation [6].

**Materials And Methods**

The nonlinearity phenomena [7] can be found in nonlinear optics and is characterized by an exponential propagation optical perturbation from the steady state. When there is a balance between linear and nonlinear processes is gained over, electromagnetic solitons arise and special solution to Maxwell's equation has been achieved. Solitons is the strong structure of solutions that hold on collisions and such formula may be founded for the waves in the deep water. In the simplest form, the solution is the stationary state which propagates with unchanged profile [8].

$$\vec{E} = \hat{r} \left( \eta f(y) \vec{B}(x) \exp(i\Gamma z) \right) \quad (2.1)$$

Where  $B(x)$  is a vector function of x and is allowed to vary slowly with propagation direction along z-axis,  $(\Gamma z)$  is the center of the beam,  $\eta$  is an inverse of the beam width and  $f(y)$  is a model profile of the field. So

$$\eta f \vec{B}_{xx} + \eta f_{yy} \vec{B} + \eta f \vec{B} \left( -\Gamma^2 \right) + \frac{\omega^2 n_o^2}{c^2} \eta f \vec{B} + \frac{1}{n_o^2 \epsilon_o} \vec{\nabla} \left( \vec{\nabla} \cdot \vec{P}_{NL} \right) + \frac{\omega^2}{c^2 \epsilon_o} \vec{P}_{NL} = 0 \quad (2.2)$$

Consider a linearly polarized beam of electromagnetic field, where the electric field E into x-direction and the magnetic field H into y-direction:

$$\vec{E} = x(\eta f(y) B(x) e^{i\Gamma z}) \quad (2.3)$$

When  $K_o^2 = \frac{\omega^2 n_o^2}{c^2}$ , Eq. (2.2) becomes

$$\frac{B_{xx}}{B} + \frac{f_{yy}}{f} + (k_o^2 - \Gamma^2) + \frac{k_o^2}{n_o^2} \alpha \eta^2 f^2 |B|^2 + \frac{1}{n_o^2 \epsilon_o} \vec{\nabla}_x \left( \frac{\vec{\nabla} \cdot \vec{P}_{NL}}{\vec{E}} \right)_x = 0 \quad (2.4)$$

The unchanged profile function  $f(y)$  decreases as  $y \rightarrow \infty$  and satisfies the boundary condition that the tangential component of the electric and magnetic fields are continuous. Using the separation of variables method for the last equation:

$$\frac{B_{xx}}{B} + (k_o^2 - \Gamma^2) + \frac{k_o^2}{n_o^2} \alpha \eta^2 \bar{f}_a^2 |B|^2 = \mu^2 \quad (2.5)$$

$$\frac{f_{yy}}{f} + \frac{1}{n_o^2 \epsilon_o} \vec{\nabla}_x \left( \frac{\vec{\nabla} \cdot \vec{P}_{NL}}{\vec{E}} \right)_x + \mu^2 = 0 \quad (2.6)$$

For  $f(y)$  is the only function of  $y$  in equation Eq. (2.5), the square of the linear modal profile  $f^2(y)$  goes to the average value of the function as  $(\bar{f}_a^2)$ . The eigenmodes may assort as transverse electric TE modes which has a have a unique component of electric field along x-axis. With  $(\beta^2 = \mu^2 + \Gamma^2 - k_o^2)$ , and  $(\Pi = \frac{\alpha}{n_o^2} \eta^2 \bar{f}_a^2)$ , Eq. (2.5) becomes [9]

$$B_{xx} - [\beta^2 - k_o^2 \Pi |B|^2] B = 0 \quad (2.7)$$

This equation is a nonlinear differential equation that can be solved exactly. We may get the first integral of the last equation in the form [10]

$$\left( \frac{\partial B}{\partial x} \right)^2 - \left[ \beta^2 - \frac{1}{2} k_o^2 \Pi |B|^2 \right] B^2 = C \quad (2.8)$$

where C is constant of integration. Her under the consumption, the constant C vanishes, because as  $x \rightarrow \infty$ , one may write  $\frac{\partial B}{\partial x} \rightarrow 0$  and  $B \rightarrow 0$ . The solution [11] for Eq. (2.8) is

$$B(x) = \frac{\beta}{k_o} \sqrt{\frac{2}{\Pi}} \operatorname{sech}[\beta(x - x_o)] \quad (2.9)$$

Where  $x_o$  is the constant of integration which satisfy the position of the self-focused peak in B. This solution can be shown as the exact solution of Eq. (2.8) by direct substitution. This solution is called stationary state because it does not vary as the wave propagates along y-axis and also satisfy Eq. (2.7) with  $(\beta(x - x_o) = n\pi/2)$ , where n is an odd numbers. The expression  $\frac{1}{k_o} \sqrt{\frac{2}{\Pi}}$  is the number of modes [12, 13] which assign the soliton order. When this term goes to unity, the beam of

$|B|^2$  view keep its shape as propagation and as the same term exceeds the unity, the shape is changed [14].

For Eq. (2.6) define the displacement as  $\vec{D} = \epsilon_o \vec{E} + \vec{P}$ , and for the propagation of EM waves the differential equation of Gauss's law  $\vec{\nabla} \cdot \vec{D} = \rho$  where  $\rho$  is the space charge density. For an isotropic dielectric medium, the parameter  $\rho$  vanishes which means that the assumption  $\vec{\nabla} \cdot \vec{D} = 0$  and  $\vec{\nabla} \cdot \epsilon_o \vec{E} = -\vec{\nabla} \cdot \vec{P}$ . Now for non-magnetic material where  $\vec{\nabla} \cdot \vec{P} = 0$ , equation (2.6) becomes

$$f_{yy} + \mu^2 f = 0 \quad (2.10)$$

The solution of this equation is given by

$$f(y) = C \exp(i\mu y) \quad (2.11)$$

### Conclusion

The reflection of the beams of the strong s-polarized electromagnetic radiation from nonlinear self-focusing medium is investigated mathematically here. More convenient approach here is associated with problems in nonlinear wave guide and the founding of new nonlinear material which has self-focusing properties. Now for isotropic dielectric medium, the stationary nonlinear transverse electric TE wave has an electric field in the form of the lowest order solutions of equation [15]

$$E(x, y, z) = \eta \frac{\beta}{k_o} \sqrt{\frac{2}{\Pi}} \operatorname{sech}(\beta(x - x_o)) \exp(i(\Gamma z + \mu y)) \quad (3.1)$$

This model has a considerable improvement over nonlinear Helmholtz equation. By direct substitution in equation (1.1),  $(1 - 2 \tanh^2 \beta x)(1 - 2 \operatorname{sech}^2 \beta x) = 0$  and Eq. (3.1) is satisfied. For the normalization of Eq. (3.1), the constant of the nonlinearity vector  $\alpha$  will be given as  $(4\beta(\frac{n_o}{k_o f_a})^2)$ . The centre of the beam is  $x = x_o + 2z\beta$ , set up  $\tan^{-1}(2\beta)$  as an angle which has been

made with between the propagation direction of the beam with the z- axis. Now if  $\theta_o$  is an initial angle, the phase angle will be  $\theta = \theta_o + (\frac{k_o^2 \Pi}{2} + \beta^2)z$ . Note that in all cases the wavelength  $\lambda$  is constant and we may note that the beam profile is damping and

irregular for a spatial value of  $k_o$ , which means as  $k_o$  becomes smaller the amplitude grows so fast. The nonlinearity will be weak to be treated as a perturbation for such layer. Time derivatives will be removed by achieving the fields in the frequency domain. The nonlinear polarization vector was suggested [16]

$$\vec{P}_{NL} = \epsilon_o \alpha |E|^2 \vec{E} \quad P_{NLX} = \epsilon_o \alpha \frac{2\eta^3 \beta^3}{K_o^3 \pi} \sqrt{\frac{2}{\Pi}} \operatorname{sech}^3(\beta x) \exp(i(\Gamma z + \mu y)) \quad (3.2)$$

...

We define  $\beta$  as the inverse of the width and  $\eta = k_o \sqrt{\frac{\pi}{2}}$  as the peak amplitude of the beam. For more information define

$$E(x, 0, 0) = \eta \frac{\beta}{k_o} \sqrt{\frac{2}{\Pi}} \operatorname{sech}(\beta x) \quad (3.3)$$

Where the integer N represents the soliton order and A is the peak amplitude of the wave. With N = 1, the first order soliton case and  $|E|^2$  does not change with z as the wave propagates. But when N = 2 for the second order soliton order, the wave  $|E|^2$  change the wave propagates along z-axis. For an electromagnetic waves, and since E is the complex amplitude of the electric field, then the time-averaged energy density of the wave [17]

$$U = \frac{1}{2} \epsilon_o n^2 |E(x, y, z)|^2 \quad (3.4)$$

and consequently, the intensity

$$I = \frac{1}{2} \epsilon_o n c |E|^2 \quad (3.5)$$

where c is the speed of light in vacuum, n is the refractive index and  $\epsilon_o$  is vacuum permittivity. Then the power intensity along x- direction

$$I = \frac{1}{2} \epsilon_o n c \eta^2 \frac{\beta^2}{K_o^2} \frac{2}{\Pi} \operatorname{sech}^2(\beta x) \quad (3.6)$$

$$\text{or} \quad I = I_o \operatorname{sech}^2(\beta x) \quad (3.7)$$

Where  $I_o$  is the peak amplitude intensity. Since the nonlinearity of the given field is weakly considered, the intensity of that field is unaffected to the first order. Also the total power [18,19]

$$P = \int I dx \quad \text{and} \quad P = 2\epsilon_o n c n_o^2 \frac{2\beta}{f_a^2 \alpha K_o^2} \quad (3.8)$$

Which can be estimated with an experimental capacity.

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