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Characterization of 3 – Branched Starlike Spanning Tree of a Given Two Dimensional Mesh m(m, n)

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ABSTRACT

A tree T is called starlike **[2],** if it contains a vertex v for which deg (v) ≥ 3 and all other vertices of T have degree 1 or 2. If deg (v) = k, the starlike tree T is k – branched and T – v has k components, each of which are trees. In this paper, we characterize the 3 – branched starlike spanning trees of a given two dimensional mesh M(m, n) m, n ≥ 3 and then find its number with junction vertex **[2]** of degree 3.

Introduction

By a graph $G = (V, E)$, we mean a finite, undirected, connected graph with no loop or multiple edges. For basic graph theoretic terminology, we refer to **[1]**. The *two dimensional mesh* $M(a, b)$ **[2]** of positive integers $a \geq 2$ and $b \geq 2$ is defined as follows: The vertices of $M(a, b)$ are the points (x, y) in R^2 such that x and y are positive integers satisfying $x \leq a$ and $y \leq b$. Vertices (x, y) and (x', y') are adjacent if either $x = x'$ and $|y - y'| = 1$ or $y = y'$ and $|x - x'| = 1$. If $a = 2$ or $b = 2$, then the mesh $M(a, b)$ is called a *ladder* **[2]**. Also, the two – dimensional mesh $M(a, b)$ is defined as the *Cartesian product* of the paths P_a and P_b, that is, $M(a, b) = P_a \times P_b$ **[1]**. The number of edges in a path is called the length of the path. A path P of length n is called n – path or a path having n + 1 vertices and is denoted by P_{n+1}. A tree T is called *starlike* **[2]** if it contains a vertex v for which deg $(v) \geq 3$ and all other vertices of T have degree 1 or 2. The vertex v is called the *junction* **[2]**. If deg $(v) = k$, we say that the starlike tree T is k – branched **[2]**. $T - v$ has k components, each of which is either K_1, K_2 or P_n, $n \geq 3$.

Let $S(a_1, a_2, a_3, \ldots, a_k)$ denote a k – branched starlike tree T with junction**[2]** v such that the components of $T - v$ are paths of orders $a_1, a_2, a_3, \ldots a_k$. If all the k components (paths) $k \geq 3$, are of equal order, ie $a_1, a_2, a_3, \ldots, a_k$. are all equal, we define it as *k – branched starlike spanning equi tree* and it is denoted as $S^{(k)}(e)$. In particular, if all the three paths are of order four, it is expressed as $S^{(3)}(4)$. If the spanning tree have paths of least order and greatest order, which the starlike spanning tree spans the mesh, it is called *k – branched starlike spanning l.g tree* and is denoted by $S^{(k)}(l, g)$. In a starlike spanning tree if all the paths are of order an odd number, or an even number we denote this as *od* $S^{(k)}$ *and ev* $S^{(k)}$ respectively. A bipartite graph **[2]** has the property that each of its vertices can be colored black or white, so that adjacent vertices are oppositely colored. A bipartite graph is *equitable* **[2]**, if the two color sets have the same cardinality. It is *nearly equitable* **[2]**, if the cardinalities of the two color sets differ by unity. Otherwise, a bipartite graph is *skewed* **[2]**. Figure 1 shows 3 – branched starlike tree of order 12 (*left*) and 15 (*middle and right*). The junction vertex is labeled *v*.

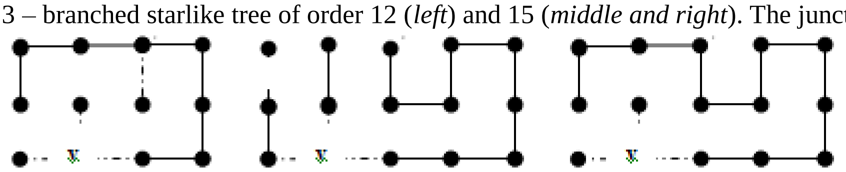

Figure 1

S(1, 1, 9) can't span M(3, 4) (left). S(2, 2, 10) can't span M(3, 5) (middle)

$S^{(k)}(l, g)$, where, k = 3, l = 1 and g = 12 spans M(3, 5) (right)

S(1, 1, 9), S(1, 3, 7), S(1, 5, 5) and S(3, 3, 5) are od $S^{(3)}$ and they can't span M(3,4). S(2, 2, 10), S(2, 4, 8), S(2, 6, 6) and S(4, 4, 6) are ev $S^{(3)}$ and they can't span M(3,5). $S^{(k)}(l, g)$ is S(1, 2, 8) in M(3, 4) and is denoted by $S^{(3)}(1, 8)$.

Previous Results

Theorem 1: An equitable, 3 – branched starlike tree on 2n vertices, where n ≥ 3, spans the ladder M (2, n).**[1]**

Theorem 2: Given positive integers a, b, c, m and n, where m, n > 2, such that mn = 1 + a + b + c, m | a and 2 | 1 + b + c, then S (a, b, c)spansM (m, n).**[1].**

Main Result

Theorem 3. 1: Let S(a, b, c) be a 3 – branched starlike spanning tree. Then S(a, b, c) spans the two dimensional mesh M (m, n), m, n ≥ 3 if it is neitherod $S^{(3)}$ nor ev $S^{(3)}$

Proof: Consider a 3 – branched starlike spanning tree $S(a, b, c)$, where a, b and c are all paths of order odd, ie, $S(a, b, c)$ is *od* $S^{(3)}$. Then $1 + a + b + c \equiv 0 \pmod 6$. Since mn and $1 + a + b + c$ are equal, and a and b are paths of order odd, the rest of the vertices in the mesh is an odd number, say c. We can't embed using the rest of the c number of vertices in the mesh into a path of order c. We can

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construct only a path of order $c-1, c-3, c-5, \ldots$ ie, a path of order an even number. Hence $S(a, b, c)$ is never *od* $S^{(3)}$.
Similarly if $S(a, b, c)$, where a, b and c are all paths of order even, ie, $S(a, b, c)$ is *ev* $S^{(3)}$. Then $1 + a + b + c \equiv 3 \ (mod\ 6)$. Since mn and $1 + a + b + c$ are equal, and a and b are paths of order even, the rest of the vertices in the mesh is an even number, say c. We can't embed using the rest of the c number of vertices in the mesh into a path of order c. We can construct only a path of order $c-1, c-3, c-5, \ldots$ ie, a path of order an odd number. Hence $S(a, b, c)$ can't be *ev* $S^{(3)}$.

Theorem 3. 2: The 3 – branched starlike spanning equi tree, $S^{(3)}(e)$ can't span the two dimensional mesh M (m, n), m, n ≥ 3, where, $e = \frac{mn-1}{3}$.

Proof: $S^{(3)}(e)$ is either *od* $S^{(3)}$ or *ev* $S^{(3)}$. By lemma 1, the result follows.

Theorem 3: The 3 – branched starlike spanning tree spans the two dimensional mesh M (m, n), m, n ≥ 3 if and only if the three paths of order are:
(i) two even number and one odd number.
(ii) two odd number and one even number.

Proof: Let $S(a, b, c)$ be a 3 – branched starlike spanning tree which spans the the two dimensional mesh M (m, n). Then $1 + a + b + c$ is either even number or odd number. If $1 + a + b + c$ is even, $a + b + c$ is odd, then at least one is odd and the others two are both even or both odd. But by lemma 1, the later possibility doesn't occur. Hence the three paths of order are two even numbers and one odd number.
If $1 + a + b + c$ is odd, $a + b + c$ is even, then at least one is odd and the others two are both even or both odd. The first possibility doesn't happen by lemma 1.Therefore, the three paths of order are two odd numbers and one even number.
Conversely, the 3 – branched starlike spanning tree $S(a, b, c)$ have paths of order two even numbers and one odd number. Then $1 + a + b + c \equiv 0 \ (mod\ 6), 2 \ (mod\ 6)$ or $4 \ (mod\ 6)$. Since $1 + a + b + c = mn$, $S(a, b, c)$ spans $M(m, n)$. If the paths of order two odd numbers and one even number, $1 + a + b + c \equiv 1 \ (mod\ 6), 3 \ (mod\ 6)$ or $5 \ (mod\ 6)$. Since $1 + a + b + c = mn$, $S(a, b, c)$ spans $M(m, n)$. This proves the theorem.

Theorem 3. 4: The number of 3 – branched starlike spanning tree with junction vertex of degree 3 of two dimensional mesh M (m, n), m,n ≥ 3 are

$$\sum_{k=1}^{\frac{mn-2}{2}} k - \sum_{k=0}^{\left[\frac{mn-2}{6}\right]} (3k+1) - \sum_{k=0}^{\left[\frac{mn-2}{6}\right]} \left\{(3k+2) - \left[\frac{3k+2}{2}\right]\right\}, if\ mn \equiv 0\ (mod\ 6)$$

$$\sum_{k=1}^{\frac{mn-3}{2}} k - \sum_{k=1}^{\left[\frac{mn-3}{6}\right]} 3k - \sum_{k=0}^{\left[\frac{mn-3}{6}\right]} \left\{(3k+1) - \left[\frac{3k+1}{2}\right]\right\}, if\ mn \equiv 1\ (mod\ 6)$$

$$\sum_{k=1}^{\frac{mn-2}{2}} k - \sum_{k=1}^{\left[\frac{mn-2}{6}\right]} (3k-1) - \sum_{k=1}^{\left[\frac{mn-2}{6}\right]} \left\{3k - \left[\frac{3k}{2}\right]\right\}, if\ mn \equiv 2\ (mod\ 6)$$

$$\sum_{k=1}^{\frac{mn-3}{2}} k - \sum_{k=1}^{\left[\frac{mn-3}{6}\right]} (3k-2) - \sum_{k=1}^{\left[\frac{mn-3}{6}\right]} \left\{(3k-1) - \left[\frac{(3k-1)}{2}\right]\right\}, if\ mn \equiv 3\ (mod\ 6)$$

$$\sum_{k=1}^{\frac{mn-2}{2}} k - \sum_{k=1}^{\left[\frac{mn-2}{6}\right]} 3k - \sum_{k=1}^{\left[\frac{mn-2}{6}\right]} \left\{(3k+1) - \left[\frac{(3k+1)}{2}\right]\right\}, if\ mn \equiv 4\ (mod\ 6)$$

$$\sum_{k=1}^{\frac{mn-3}{2}} k - \sum_{k=1}^{\left[\frac{mn-3}{6}\right]} (3k-1) - \sum_{k=1}^{\left[\frac{mn-3}{6}\right]} \left\{3k - \left[\frac{3k}{2}\right]\right\}, if\ mn \equiv 5\ (mod\ 6)$$

where $[x]$ *denotes the greatest integer less than or equal to x.*

Proof: Without loss of generality, we separate the two dimensional mesh $M\ (m, n)$ into residue mod 6. If the 3 – branched starlike spanning tree $S(x, y, z)$ spans $M\ (m, n)$, mn and $1 + x + y + z$ are equal. The rest of the proof follows from theorems *1*, *2* and *3*

Conclusion

In this paper, we have discussed characterization of the of 3 – branched starlike spanning tree of a given two dimensional mesh M (m, n), m, n ≥ 3 and then find its number with junction vertex of degree 3.

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