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Separation Cordial Labeling for some Star and Bistar Related Graphs

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ABSTRACT

A separation cordial labeling of graph G is a bijection f from V to $\{1, 2, \dots, |V|\}$ such that each edge uv is assigned the label 1 if $f(u) + f(v)$ is an odd number and label 0 if $f(u) + f(v)$ is an even number. Then the number of edges labeled 0 and the number of edges labeled 1 differ by at most 1. A graph has a separation cordial labeling, then it is called separation cordial graph. Here, the bistar $B_{m,n}$ the splitting graphs of $K_{1,n}$ and $B_{m,n}$, the shadow graph of $B_{m,n}$ and square graph of $B_{m,n}$ are discussed and found to be separation cordial.

Introduction

Let $G = (V, E)$, be a finite, undirected, connected graph with no loop or multiple edges. The order and size of G are denoted by 'p' and 'q' respectively ($p = |V|$ and $q = |E|$). For basic graph theoretic terminology, we refer to **[1, 2, 3]**. Graph labeling **[4]** is a strong relation between numbers and structure of graphs **[3]** A graph labeling is a bijection 'f' from a subset of the elements of a graph to the set of positive integers. The domain of 'f' is the set of vertices, for vertex labeling and for edge labeling the domain of 'f' is the set of edges. A useful survey to know about the numerous graph labeling methods is given by J. A. Gallian **[5].** The origin of labeling can be attributed to A. Rosa **[6]** or R.L. Graham and N.J.A. Sloane **[7].** A vertex labeling **[4]** of a graph is an assignment f of labels to the vertices of G that induces for each edge uv a label depending on the vertex label $f(u)$ and $f(v)$. The two important labeling methods are called graceful and harmonious labelings. Cordial labeling is a variation of both graceful and harmonious labeling **[8].** The concept of cordial labeling was introduced by I. Cahit **[8].**

Definition 1.1: If the vertices of the graph are assigned values subject to certain condition(s), then is known as *graph labeling*

Definition 1.2: Let $G = (V, E)$ be a graph. A mapping $f: V(G) \rightarrow \{0,1\}$ is called *binary vertex labeling* of G and $f(v)$ is called the label of the vertex v of G under f.For an edge $e = uv$, the induced edge labeling $f^*: E(G) \rightarrow \{0,1\}$ is given by $f^*(e) = |f(u) - f(v)|$. Let $v_f(0)$ and $v_f(1)$ be the number of vertices of G having labels 0 and 1 respectively under f. Let $e_f(0)$ and $e_f(1)$ be the number of edges having labels 0 and 1 respectively under f^*.

Definition 1.3: A binary vertex labeling of a graph G is called a *cordiallabeling*, if $|v_f(0) - v_f(1)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$. A graph G is cordial, if it admits cordial labeling.

Main Results

Sundaram, Ponraj and Somasundaram **[9]** have introduced the notation of prime cordial labeling and proved that some graphs are prime cordial. R. Varatharajan , S. Navaneethakrishnan and K.Nagarajan **[10]** introduced divisor cordial labeling and they have proved that some graphs are divisor cordial **[11].**Motivated by the concept of prime cordial labeling and divisor cordial labeling, we introduced a new cordial labeling called separation cordial labeling **[12]**. In this paper, we prove some star and bistar related graphs such as subdivision of star, subdivided star, splitting graph of star, bistar, shadow graph, square graph and the like are separation cordial.

Definition 2.1:[12] A separation cordial labeling of a graph G with vertex set V is a bijection f from V to $\{1, 2, \dots, p\}$ such that if each edge uv assigned the label 1 if $f(u) + f(v)$ is an odd number and label 0 if $f(u) + f(v)$ is an even number then the number of edges labeled 0 and the number of edges labeled 1 differ by at most 1. A graph G is separation cordial if it admits separation cordial labeling.

Theorem 2.1: *$S(K_{1,n})$, the subdivision of the star $K_{1,n}$, is separation cordial.*

Proof: Let $V(S(K_{1,n})) = \{v, v_i, u_i, : 1 \leq i \leq n\}$ and let $E(S(K_{1,n})) = \{vv_i, v_iu_i, : 1 \leq i \leq n\}$ The order and size of $S(K_{1,n})$ are $2n + 1$ and $2n$ respectively. Define a bijection f from V to $\{1, 2, \dots, p\}$ as follows.

The labeling of the vertices are:

$f(v) = 1$

$f(v_i) = 1 + 2i$, $for\ 1 \leq i \leq n$

$f(u_i) = 2i$, $for\ 1 \leq i \leq n$

Here, $e_f(0) = e_f(1) = n$ and $|e_f(0) - e_f(1)| \leq 1$

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Thus $S\,(K_{1,n})$, is separation cordial.

Definition 2.2: The *subdivided star* $S_{2k}^{(r)}$ is the graph from 'r' paths of length '$2k$' by identifying their center vertex.

Theorem 2.2: *The graph* $G = S_{2k}^{(r)}$*, the subdivided star, is separation cordial , for* $k \geq 1, r \geq 1$

Proof: The graph G has $2k\,r+1$ vertices $v, v_1, v_2, v_3, \dots, v_{2kr}$ and $2\,kr$ edges. Define a bijection f from V to $\{1, 2, \dots, p\}$. Let v be the apex (central) vertex of G and assign the label 1 to v. Then place and label the remaining $2\,kr$ vertices into the following cases.

Case (*i*): *r = 1 and k = 1*

It is a path P_3 and it is separation cordial.

Case (*ii*): *r > 1, k > 1and k is an odd number*

The labeling of the vertices are:

$f(v) = 1$

$f(v_i) = 1 + 2i\,,\ for\ 1 \leq i \leq 2r$

$f(v_{2r+i}) = 2i\,,\ for\ 1 \leq i \leq 2r$

$f(v_{2.2r+i}) = 2.2r + 2i\,,\ for\ 1 \leq i \leq 2r$

$f(v_{3.2r+i}) = 2.2r - 1 + 2i\,,\ for\ 1 \leq i \leq 2r$

$f(v_{4.2r+i}) = 4.2r - 1 + 2i\,,\ for\ 1 \leq i \leq 2r$

$f(v_{5.2r+i}) = 4.2r + 2i\,,\ for\ 1 \leq i \leq 2r$

$f\left(v_{(k-1).2r+i}\right) = (k-1).2r + i\,,\ for\ 1 \leq i \leq 2r$

Case (*ii*): *r > 1, k > 1and k is an even number*

The labeling of the vertices are:

$f(v) = 1$

$f(v_i) = 1 + 2i\,,\ for\ 1 \leq i \leq 2r$

$f(v_{2r+i}) = 2i\,,\ for\ 1 \leq i \leq 2r$

$f(v_{2.2r+i}) = 2.2r + 2i\,,\ for\ 1 \leq i \leq 2r$

$f(v_{3.2r+i}) = 2.2r - 1 + 2i\,,\ for\ 1 \leq i \leq 2r$

$f(v_{4.2r+i}) = 4.2r - 1 + 2i\,,\ for\ 1 \leq i \leq 2r$

$f\left(v_{(k-1).2r+i}\right) = (k-2).2r + 2i\,,\ for\ 1 \leq i \leq 2r$

In each case, $e_f(0) = e_f(1) = kr$ and $|e_f(0) - e_f(1)| \leq 1$. Thus $S_{2k}^{(r)}$, the subdivided star, is separation cordial , for $k \geq 1, r \geq 1$

Definition 2.3: A *bistar* is a tree of order at least four containing exactly two non-pendent vertices. We denote the bistar with two non-pendent vertices having degrees $m+1$ and $n+1$, respectively by $B_{m,n}$.From the definition, clearly $m \geq 1$ and $n \geq 1$ and also $B_{m,n}$ is isomorphic to $B_{n,m}$. The vertex set of the bistar be $\{u_1, u_2, \dots, u_m, u, v, v_1, v_2, \dots, v_n\}$ and the edge set $E\,(B_{m,n}) = \{uu_1, uu_2, \dots, uu_m, uv, vv_1, vv_2, \dots, vv_n\}$.The order and size of $B_{m,n}$ are $m+n+2$ and $m+n+1$ respectively.

Theorem 2.3: *The bistar* $(B_{m,n})$ *is separation cordial graph.*

Proof: Consider $B_{m,n}$ with vertex set $\{u, v, u_i, v_j, 1 \leq i \leq m, 1 \leq j \leq n\}$ where u_i and v_j are pendent vertices. Let G be the graph $(B_{m,n})$, the order and size of G are $m+n+2$ and $m+n+1$ respectively. Define a bijection f from V to $\{1, 2, \dots, p\}$ as follows.

For labeling of the vertices consider the two cases:

Case (*i*): *m is even and n is odd number*

$f(v) = 1\,,\quad f(u) = 2$

$f(u_i) = 2 + i\,,\ for\ 1 \leq i \leq m$

$f(v_j) = 2 + m + j\,,\ for\ 1 \leq i \leq n$

Here, $e_f(0) = \frac{m+n+1}{2} = e_f(1)$

Case (*ii*): *Except for m is even and n is odd number*

$f(u) = 1\,,\quad f(v) = 2$

$f(u_i) = 2 + i\,,\ for\ 1 \leq i \leq m$

$f(v_j) = 2 + m + j\,,\ for\ 1 \leq j \leq n$

In the view of the above labeling pattern, we have,

$\left.\begin{array}{l} e_{f(0)} = \frac{m+n}{2}+1 \\ e_{f(1)} = \frac{m+n}{2}-1 \end{array}\right\}$, for both m and n are odd

$\left.\begin{array}{l} e_{f(0)} = \frac{m+n}{2}-1 \\ e_{f(1)} = \frac{m+n}{2}+1 \end{array}\right\}$, for both m and n are even

$e_{f(0)} = e_{f(1)} = \frac{m+n+1}{2}$, for m odd and n even

Hence, in each case, $|e_f(0) - e_f(1)| \leq 1$.

Thus, bistar $(B_{m,,n})$ is separation cordial graph.

Definition 2.4: The *splitting graph* of a graph G is obtained by adding toeach vertex v a new vertex v' such that v' is adjacent to v in G, ie , $N(v) = N(v')$. The resultant graph is denoted by $S'(G)$.

Theorem 2.4: *The splitting graph of star,* $S'(K_{1,n})$ *is separation cordial , for* $k \geq 1, r \geq 1$

Proof: Let $v_1, v_2, v_3, \ldots, v_n$ be the pendent vertices and v be the apex (central) vertex of $(K_{1,n})$ and $u_1, u_2, u_3, \ldots, u_n$ are added vertices corresponding to $v_1, v_2, v_3, \ldots, v_n$ to obtain $S'(K_{1,n})$. Let G be the graph $S'(K_{1,n})$. Then order and size of G are $2(n+1)$ and $3n$.

Define a bijection f from V to $\{1, 2, \ldots, p\}$ as follows.The labeling of the vertices are:

$f(u) = 1$, $f(v) = 2$

$f(v_i) = 2 + i$, $for\ 1 \leq i \leq n$

$f(u_i) = 2 + n + i$, $for\ 1 \leq i \leq n$

In the view of the above labeling pattern, we have,

$e_f(0) = \frac{3n+1}{2}$, $e_f(1) = \frac{3n-1}{2}$, if n is odd and $e_f(0) = e_f(1) = \frac{3n}{2}$, if n is even

Hence, in both case, $|e_f(0) - e_f(1)| \leq 1$

Thus $S'(K_{1,n})$ is separation cordial , for $k \geq 1, r \geq 1$.

Theorem 2.5: *The splitting graph of bistar,* $S'(B_{m,,n})$ *is separation cordial*

Proof: Consider $(B_{m,,n})$ with vertex set $\{u, v, u_i, v_j, 1 \leq i \leq m, 1 \leq j \leq n\}$ where u_i and v_j are pendent vertices. In order to obtain $S'(B_{m,,n})$ add vertices corresponding to u, v, u_i, v_j where $1 \leq i \leq m, 1 \leq j \leq n$. If $G = S'(B_{m,,n})$, then $|V(G)| = 2(m+n+2)$ and $|E(G)| = 3(m+n+1)$. Define a bijection f from V to $\{1, 2, \ldots, p\}$ as follows. Consider the three cases:

Case (*i*): *m is even and n is odd number*

The labeling of the vertices are:

$f(u') = 1$, $f(u) = 2$

$f(v) = 3$, $f(v') = 4$

$f(u_i) = 4 + i$, $for\ 1 \leq i \leq m$

$f(u_i') = 4 + m + i$, $for\ 1 \leq i \leq m$

$f(v_j) = 4 + 2m + j$, $for\ 1 \leq j \leq n$

$f(v_j') = 4 + 2m + n + j$, $for\ 1 \leq j \leq n$

In the view of the above labeling pattern, we have, $e_f(0) = e_f(1) = \frac{3(m+n+1)}{2}$

Case (*ii*): *m is odd and n is even number*

The labeling of the vertices are:

$f(v') = 1$, $fv = 2$

$f(u) = 3$, $f(u') = 4$

$f(v_j) = 4 + j$, $for\ 1 \leq j \leq n$

$f(v_j') = 4 + n + j$, $for\ 1 \leq j \leq n$

$f(u_i) = 4 + 2n + i$, $for\ 1 \leq i \leq m$

$f(u_i') = 4 + 2n + m + i$, $for\ 1 \leq i \leq m$

In the view of the above labeling pattern, we have, $e_f(0) = e_f(1) = \frac{3(m+n+1)}{2}$

Case (*iii*): *m and n are both odd or both even number*
The labeling of the vertices are:
$f(u') = 1$, $f(u) = 2$

$f(v) = 3$, $f(v') = 4$

$f(u_i) = 4 + i$, $for\ 1 \leq i \leq m$

$f(u_i') = 4 + m + i$, $for\ 1 \leq i \leq m$

$f(v_j) = 4 + 2m + i$, $for\ 1 \leq j \leq n$

$f(v_j') = 4 + 2m + n + j$, $for\ 1 \leq j \leq n$
In the view of the above labeling pattern, we have,

$e_f(0) = \frac{3(m+n+1)+1}{2}$ and $e_f(1) = \frac{3(m+n+1)-1}{2}$
Hence, in each case, $|e_f(0) - e_f(1)| \leq 1$
Thus the splitting graph of bistar, $S'(B_{m,n})$ is separation cordial.
Definition 2.5: The *shadow graph* $D_2(G)$ of a connected graph G is constructed by taking two copies of G say $G'$and G''.Join each vertex u' in G' to the neighbors of the corresponding vertex u'' in G''
Theorem 2.6: *The shadow graph, $D_2(B_{m,n})$ is separation cordial, for $m, n \geq 1$*
Proof: Consider two copies of $B_{m,n}$. Let $\{u, v, u_i, v_j, 1 \leq i \leq m, 1 \leq j \leq n\}$ and $\{u', v', u_i', v_j', 1 \leq i \leq m, 1 \leq j \leq n\}$ be the corresponding vertex set of each copy of $B_{m,n}$. Let G be the graph $D_2(B_{m,n})$ then the order and size of $D_2(B_{m,n})$ are $2(m + n + 2)$ and $4(m + n + 1)$ respectively. Define a bijection *f* from *V* to $\{1, 2, \dots, p\}$ as follows.
The labeling of the vertices are:
$f(u) = 1$, $f(u') = 2$

$f(v) = 3$, $f(v') = 4$

$f(u_i) = 4 + i$, $for\ 1 \leq i \leq m$

$f(u_i') = 4 + m + i$, $for\ 1 \leq i \leq m$

$f(v_j) = 4 + 2m + i$, $for\ 1 \leq j \leq n$

$f(v_j') = 4 + 2m + n + j$, $for\ 1 \leq i \leq n$
Here, $e_f(0) = e_f(1) = 2(m + n + 1)$ and $|e_f(0) - e_f(1)| \leq 1$
Hence, the shadow graph, $D_2(B_{m,n})$ is separation cordial, for $m, n \geq 1$
Definition 2.6: For a simple connected graph G, the *square of graph* G is denoted by G^2 and defined as the graph with the same vertex set as of G and two vertices are adjacent in G^2 if they are at a distance 1 or 2 apart in G.
Theorem 2.7: *The square of bistar $B^2{}_{m,n}$ is separation cordial, for $m, n \geq 1$*
Proof: Consider $B_{m,n}$ with vertex set $\{u, v, u_i, v_j, 1 \leq i \leq m, 1 \leq j \leq n\}$ where u_i and v_j are pendent vertices. Let G be the graph $B^2{}_{m,n}$ .Then the order and size of $B^2{}_{m,n}$ are $m + n + 2$ and $2(m + n) + 1$ respectively.
Define a bijection f from *V* to $\{1, 2, \dots, p\}$ as follows.
The labeling of the vertices are:
$f(u) = 1$, $f(v) = 2$

$f(u_i) = 2 + i$, $for\ 1 \leq i \leq m$
$f(v_j) = 2 + m + j$, $for\ 1 \leq j \leq n$
Here, $e_f(0) = m + n$, $e_f(1) = m + n + 1$ and $|e_f(0) - e_f(1)| \leq 1$
Thus $B^2{}_{m,n}$, is separation cordial, for all m and n

Conclusion

In this paper, we proved that the bistar, $B_{m,n}$, the splitting graphs of $K_{1,n}$ and $B_{m,n}$, the shadow graph of $B_{m,n}$ and the square graph of $B_{m,n}$, are separation cordial..

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