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Mixed Convection of Couple Stress Permeable Fluid in a Vertical Channel in The Presence of Heat Generation or Heat Absorption

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ABSTRACT

Fully developed laminar mixed convection of permeable couple stress fluid in a vertical channel has been investigated analytically in the presence of heat generation or absorption. Uniform wall temperatures with asymmetric and symmetric heating have been considered. An analytical solution has been developed by using perturbation technique. Results are depicted graphically on the flow for different values of couple stress parameter, porous parameter, mixed convection parameter and heat generation or absorption coefficient. The results show that the flow reversal occurs near the walls of the channel for sufficiently large value of the ratio of Grashof number and Reynolds number.

Introduction

With the development of modern industry the importance of using non-Newtonian fluids as lubricants has been emphasized. Common lubricants exhibiting non-Newtonian behavior are the polymer-thickened oils, greases and natural lubricant fluids that appear in animal joints. These lubricants violate Newtonian postulate, which assumes a linear relationship between shear stress and rate of shear. Various theories have been postulated to describe the flow behavior of non-Newtonian fluids. The micro continuum theories have developed to explain the peculiar behavior of fluids containing sub-structure such as polymeric fluids (Ariman et al. [1,2]). Some theoretical studies of the couple stress model to biomechanics problems has been proposed in the study of peristaltic transport by Srivastava [3], Shehawey and Mekheimer [4] and blood flow in the microcirculation by Dulal Pal et al. [5]. In lubrication problems many authors have investigated the couple stress effects on different lubrication problems (Chiang et al. [6], Naduvinamani et al. [7], Jian et al. [8] and Lu et al. [9]). Stokes [10] reported a review of couple stress (polar) fluid dynamics. The flow and heat transfer characteristics of Oberbeck convection of a couple stress fluid in a vertical porous stratum was analyzed by Umavathi and Malashetty [11]. Umavathi et al. [12] also analyzed the flow and heat transfer of a couple stress fluid sandwiched between viscous fluid layers. Finally, Srinivasacharya and Kaladhar [13] in the very recent paper have investigated the Hall and Ion-slip effects on fully developed electrically conducting couple stress fluid flow between vertical parallel plates in the presence of a temperature dependent heat source.

The current research interest on the fluid flow and heat transfer in porous media has been documented in several comprehensive works published recently (see, e.g., Pop and Ingham, [14], Bejan et al. [15], Vafai, [16] and Nield and Bejan, [17]) and is motivated by numerous applications of this class of phenomenon in the modern technologies. Recent technological implications have given rise to increased interest in combined free and forced convection flow in vertical channels in which the objective is to secure a quantitative understanding of a configuration having current engineering applications (Al-Hadharami et al. [18]). Parang and Keyhani [19] studied fully developed buoyancy assisted mixed convection in a vertical annulus by using the Brinkman-extended Darcy model. The mixed convection in narrow vertical ducts without the effect of viscous dissipation has been investigated by Pop et al. [20]. The effect of viscous dissipation has been included in the study of the combined free and forced convection in a porous medium between two vertical walls by Ingham et al. [21]. Recent contributions to the effect of viscous dissipation in addition to the buoyancy effects have been published by Nield [22,23] and Magyari et al. [24]. Umavathi et al. [25] analyzed steady and unsteady mixed convection flow through a channel. Prathap Kumar et al. [26] studied mixed convection in a vertical channel containing a porous and fluid layer with isothermal or isoflux boundaries.

The fluid flow and heat transfer in the wall-bounded forced flow through a porous medium has been extensively studied in the past, because it relates to various applications which are solid matrix heat exchanger, thermal insulation, nuclear waste disposal, geothermal energy extraction and other practical interesting designs. In the laminar forced convection in a porous channel, Vafai et al. [27] presented on exact solution for the velocity and temperature fields by using Darcy-Brinkman-Forchheimar model. They showed that for a high permeability porous medium the thickness of the momentum boundary layer depends on both the Darcy number and the inertia parameter, and neglecting the inertia effect could lead to serious errors for Nusselt number calculations. Nield et al. [28] also presented a theoretical analysis of fully developed forced convection in a porous channel. Hadim and Chen [29] investigated the non-Darcy mixed convection in a vertical porous channel with asymmetric wall heating. Their results showed that as the Darcy number is decreased, distortions in the velocity profile lead to increased heat transfer. The fully developed mixed convection in a vertical porous channel with imposed uniform heat flux was performed using DBF model by Chen et al. [30]. It was shown that the buoyancy force could significantly affect Nusselt number for higher Rayleigh numbers, higher modified Darcy number and or lower

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Forchheimer number. A comprehensive review in the laminar wall bounded forced or mixed convection is given by Nield and Bejan [17].

Quite a number of physical phenomena involve combined free and forced convection driven by heat source/sink. The study of heat source in moving fluids is important in view of several physical problems such as those dealing with chemical reactions and those concerned with dissociating fluids. Possible heat generation/absorption effects may change the temperature distribution and, therefore, the particle deposition rate. This may occur in such applications related to nuclear reactor cores, fire and combustion modeling, electronic chips and semiconductor wafers. In fact, the literature is replete with examples dealing with heat transfer in laminar flow of viscous fluids. Other investigations dealing with internal heat generation or absorption can be seen in the works of Sparrow and Cess [31] and Chamkha [32]. Mahanthi and Gaur [33] investigated the effect of the viscosity and thermal conduction on the steady free-convective flow of a viscous incompressible fluid along an isothermal vertical plate in the presence of heat sink. Recently, Umavathi et al. [34] studied the effects of viscous dissipation and heat source/sink on fully developed mixed convection for the laminar flow in a parallel-plate vertical channel. Umavathi and Jaweriya Sultana [35] investigated the fully developed mixed convection for a laminar flow of a micropolar fluid mixture in a vertical channel with a heat source/sink.

Mixed convection fluid flows play an important role in engineering and technology, geothermal and bio-fluids. Therefore in this chapter, mixed convection of permeable couple stress fluid in a vertical channel in the presence of heat generation or heat absorption is investigated for three different thermal boundary conditions. The basic equations governing the flow model are coupled and non-linear which can be solved by using the regular perturbation method.

Mathematical Formulation

It is assumed that the flow is steady, fully developed and that the fluid properties are constants. Using the conditions of the equilibrium state of the fluid and further assuming that the density of the fluid is a function of temperature alone, the equation of motion, and energy are

$$\rho = \rho_0\left(1-\beta\left(T-T_0\right)\right) \tag{1}$$

$$g\beta\left(T-T_0\right)-\frac{1}{\rho_0}\frac{\partial P}{\partial X}+\frac{\mu}{\rho_0}\frac{d^2U}{dY^2}-\frac{\eta}{\rho_0}\frac{d^4U}{dY^4}-\frac{\mu}{\rho_0 k}U=0 \tag{2}$$

where $P = p+\rho_0 g X$ is the difference between the pressure and the hydrostatic pressure. The temperature is T_1, at the left wall $Y=-L/2$ and the temperature is T_2, at the right wall $Y=-L/2$, with $T_2 \ge T_1$. These conditions are compatible with equation (2) only when dP/dX is independent of X. Hence, there exists a constant A such that,

$$\frac{dP}{dX}=A \tag{3}$$

The energy balance equation in the presence of heat generation or absorption is

$$\alpha\frac{d^2T}{dY^2}+\frac{\mu}{\rho_0 C_p}\left(\frac{dU}{dY}\right)^2\pm\frac{Q\left(T-T_0\right)}{\rho_0 C_p}=0 \tag{4}$$

Solving the momentum and energy balance equations from (2) and (4) to obtain a differential equation for U, namely

$$\frac{d^6U}{dY^6}=\left(\frac{\mu}{\eta}\mp\frac{Q}{K}\right)\frac{d^4U}{dY^4}\pm\left(\frac{\mu Q}{K\eta}-\frac{\mu}{\eta k}\right)\frac{d^2U}{dY^2}-\frac{\mu\rho_0\beta g}{K\eta}\left(\frac{dU}{dY}\right)^2-\frac{Q\mu}{K\rho_0 k}U\mp\frac{QA}{K\mu} \tag{5}$$

The corresponding boundary conditions on U becomes,

$$U=\frac{d^2U}{dY^2}=0 \quad \text{at} \quad Y=\pm\frac{L}{2}$$

$$\frac{d^4U}{dY^4}=-\frac{A}{\mu}+\frac{\beta g\rho_0\left(T_1-T_0\right)}{\eta} \quad \text{at} \quad Y=-\frac{L}{2}$$

$$\frac{d^4U}{dY^4}=-\frac{A}{\mu}+\frac{\beta g\rho_0\left(T_2-T_0\right)}{\eta} \quad \text{at} \quad Y=\frac{L}{2} \tag{6}$$

Introducing the non-dimensional parameters

$$u=\frac{U}{U_0};\quad \theta=\frac{T-T_0}{\Delta T};\quad y=\frac{Y}{D};\quad Gr=\frac{g\beta\Delta T D^3}{\nu^2};\ k=\frac{\eta}{\mu D^2};\ a^2=\frac{1}{k};\ \lambda=\frac{Gr}{\mathrm{Re}};\ R_T=\frac{T_2-T_1}{\Delta T};\ \sigma^2=\frac{D^2}{k}\ \mathrm{Re}=\frac{U_0 D}{\nu};\ \mathrm{Pr}=\frac{\nu}{\alpha};$$

$$Br=\frac{\mu U_0^{\,2}}{K\Delta T};\ \phi=\frac{QD^2}{K};\ A=-\frac{48\mu U_0}{D^2};\ U_0=-\frac{AD^2}{48\mu};\quad T_0=\frac{T_1+T_2}{2} \tag{7}$$

The temperature difference ΔT is

$$\Delta T = T_2-T_1 \quad \text{if} \quad T_1<T_2 \quad \text{or by} \tag{8}$$

$$\Delta \mathrm{T}=\frac{\nu^2}{C_p D^2} \quad \text{if} \quad T_1=T_2 \tag{9}$$

By use of the above non-dimensional quantities, equations (4) to (6) becomes

$$\frac{d^2\theta}{dy^2}=-Br\left(\frac{du}{dy}\right)^2\mp\phi\theta \tag{10}$$

$$\frac{d^6u}{dy^6} = \left(a^2 \mp \phi\right)\frac{d^4u}{dy^4} - a^2(\sigma^2 \mp \phi)\frac{d^2u}{dy^2} - \lambda Bra^2\left(\frac{du}{dy}\right)^2 \mp a^2\sigma^2\phi u \pm 48\phi a^2 \tag{11}$$

$$u = \frac{d^2u}{dy^2} = 0 \quad \text{at} \quad y = \pm\frac{1}{4}$$

$$\frac{d^4u}{dy^4} = 48a^2 - \frac{R_T\lambda a^2}{2} \quad \text{at} \quad y = -\frac{1}{4}$$

$$\frac{d^4u}{dy^4} = 48a^2 + \frac{R_T\lambda a^2}{2} \quad \text{at} \quad y = \pm\frac{1}{4} \tag{12}$$

Temperature field can also be obtained while substituting the dimensional parameters from equation (7) in momentum equation (2) and one obtains,

$$\theta = -\frac{1}{\lambda}\left(48 - \sigma^2 u + \frac{d^2u}{dy^2} - \frac{1}{a^2}\frac{d^4u}{dy^4}\right) \tag{13}$$

Equation (11) is highly nonlinear through viscous dissipation term. If the viscous dissipation is negligible so that $Br = 0$, the dimensionless temperature θ and dimensionless velocity u are uncoupled. In this case, the solution of equation (11) by applying the boundary conditions (12) becomes

$$u_0 = C_1 CoshP_1 y + C_2 SinhP_1 y + C_3 CoshP_2 y + C_4 SinhP_2 y + C_5 CosP_3 y + C_6 SinP_3 y + \frac{48}{\sigma^2} \tag{14}$$

where,

$$C_1 = -\frac{C_3 RCoshP_{14}}{PCoshP_{13}};\ C_2 = \frac{1}{PSinhP_{13}}\left(C_6\phi SinP_{15} - C_4 RSinhP_{14}\right),\ C_3 = -\frac{48a^2}{R(P-R)CoshP_{14}};\ C_4 = -\frac{R_T\lambda a^2}{2(P-R)(R+\phi)SinhP_{14}},$$

$$C_5 = 0;\ C_6 = \frac{R_T\lambda a^2}{2(P+\phi)(R+\phi)SinP_{15}}$$

for the case of heat generation and

$$u_0 = C_1 CoshP_1 y + C_2 SinhP_1 y + C_3 CoshP_2 y + C_4 SinhP_2 y + C_5 CoshP_3 y + C_6 SinhP_3 y + \frac{48}{\sigma^2} \tag{15}$$

where, $C_1 = -\frac{C_3 RCoshP_{14}}{PCoshP_{13}}$; $C_2 = \frac{-1}{PSinhP_{13}}\left(C_6\phi SinhP_{15} + C_4 RSinhP_{14}\right)$;

$$C_3 = -\frac{48a^2}{R(P-R)CoshP_{14}};\ C_4 = -\frac{R_T\lambda a^2}{2(P-R)(R-\phi)SinhP_{14}};$$

$$C_5 = 0;\ \ C_6 = \frac{R_T\lambda a^2}{2(P-\phi)(R-\phi)SinhP_{15}},$$

for the case of heat absorption. If porous parameter is negligible then the velocity and temperature fields for both cases becomes,

$$u = \frac{3}{2} - \frac{2R_T\lambda}{\phi}y - 24y^2 - \frac{48}{a^2}\left(1 - \frac{Coshay}{Cosha/4}\right) + \frac{\lambda\ R_T}{2(a^2+\phi)}\left(\frac{a^2 Sin\sqrt{\phi}y}{\phi\, Sin\sqrt{\phi}/4} + \frac{Sinhay}{Sinha/4}\right) \tag{16}$$

$$u = \frac{3}{2} + \frac{2R_T\lambda}{\phi}y - 24y^2 - \frac{48}{a^2}\left(1 - \frac{Coshay}{Cosha/4}\right) + \frac{\lambda\ R_T}{2(a^2-\phi)}\left(\frac{a^2 Sinh\sqrt{\phi}y}{\phi\, Sinh\sqrt{\phi}/4} + \frac{Sinhay}{Sinha/4}\right) \tag{17}$$

Substituting these velocity fields in equation (13), we obtain

$$\theta = \frac{R_T}{2}\frac{Sin\sqrt{\phi}y}{Sin\sqrt{\phi}/4} \tag{18}$$

$$\theta = \frac{R_T}{2}\frac{Sinh\sqrt{\phi}y}{Sinh\sqrt{\phi}/4} \tag{19}$$

Considering the porous parameter and neglecting the couple stress parameter, the velocity expressions of both cases reduces to,

$$u = \frac{48}{\sigma^2}\left(1 - \frac{Cosh\sigma y}{Cosh\sigma/4}\right) + \frac{\lambda\ R_T}{2(\sigma^2+\phi)}\left(\frac{Sin\sqrt{\phi}y}{Sin\sqrt{\phi}/4} - \frac{Sinh\sigma y}{Sinh\sigma/4}\right) \tag{20}$$

$$u = \frac{48}{\sigma^2}\left(1 - \frac{Cosh\sigma y}{Cosh\sigma/4}\right) + \frac{\lambda\ R_T}{2(\sigma^2-\phi)}\left(\frac{Sinh\sqrt{\phi}y}{Sinh\sqrt{\phi}/4} - \frac{Sinh\sigma y}{Sinh\sigma/4}\right) \tag{21}$$

and the temperature expressions are same as equations (18) and (19). In the absence of the couple stress and porous parameters, the velocity field for both the cases becomes,

$$u=\frac{3}{2}-24y^2-\frac{2\lambda\ R_T}{\phi}\left(y-\frac{Sin\sqrt{\phi}y}{4Sin\sqrt{\phi}/4}\right) \tag{22}$$

$$u=\frac{3}{2}-24y^2+\frac{2\lambda\ R_T}{\phi}\left(y-\frac{Sinh\sqrt{\phi}y}{4Sinh\sqrt{\phi}/4}\right) \tag{23}$$

and the temperature field remains same as given in equations (18) and (19).

In the absence of couple stress, porous and heat generation or heat absorption, the velocity and temperature fields reduces to

$$u=\left(\frac{R_T\ \lambda}{3}y+24\right)\left(\frac{1}{16}-y^2\right) \tag{24}$$

$$\theta=2R_T\ y \tag{25}$$

which corresponds to the velocity and temperature fields determined by Aung and Worku [36]. In the case of asymmetric heating, when buoyancy forces are dominated i.e.,

when $\lambda\to\pm\infty$, equations (20) and (21) for heat generation and absorption gives

$$\frac{u}{\lambda}=\frac{1}{2(\sigma^2+\phi)}\left(\frac{Sin\sqrt{\phi}y}{Sin\sqrt{\phi}/4}-\frac{Sinh\sigma y}{Sinh\sigma/4}\right) \tag{26}$$

$$\frac{u}{\lambda}=\frac{1}{2(\sigma^2-\phi)}\left(\frac{Sinh\sqrt{\phi}y}{Sinh\sqrt{\phi}/4}-\frac{Sinh\sigma y}{Sinh\sigma/4}\right) \tag{27}$$

In the absence of sources and sink, the above equations for clear viscous fluid reduces to

$$\frac{u}{\lambda}=\frac{y}{3}\left(\frac{1}{16}-y^2\right) \tag{28}$$

which is Batchelor's velocity profile for free convection (Batchelor, [37]).

When buoyancy forces are negligible and viscous dissipation is relevant, i.e. $\lambda=0$, so that a purely forced convection occurs, the velocity and temperature field for heat generation or heat absorption becomes,

$$u=\frac{48a^2}{(P-R)}\left(\frac{CoshP_1y}{PCoshP_{13}}-\frac{CoshP_2y}{RCoshP_{14}}\right)+\frac{48}{\sigma^2} \tag{29}$$

$$\theta=C_1CosP_3y+C_2SinP_3y+l_1Cosh2P_1y+l_2CoshP_2y+l_3CoshP_4y+l_4CoshP_5y+l_5 \tag{30}$$

where,

$$l_1=-\frac{1152\,Bra^4}{P(P-R)^2(4P+\phi)Cosh^2P_{13}};\quad l_2=-\frac{1152\,Bra^4}{R(P-R)^2(4R+\phi)Cosh^2P_{14}};\quad l_3=\frac{2304\,Bra^4}{P_1P_2(P-R)^2(P+R+2P_1P_2+\phi)CoshP_{13}CoshP_{14}};$$

$$l_4=-\frac{2304\,Bra^4}{P_1P_2(P-R)^2(P+R-2P_1P_2+\phi)CoshP_{13}CoshP_{14}};\quad l_5=\frac{2304\,Bra^4}{2\phi(P-R)^2}\left(\frac{1}{PCosh^2P_{13}}+\frac{1}{RCosh^2P_{14}}\right);$$

$$C_1=-\frac{1}{CosP_{15}}\begin{pmatrix}l_1CoshP_{10}+l_2CoshP_{11}+l_3CoshP_{16}\\ l_4CoshP_{17}+l_5\end{pmatrix};\ C_2=\frac{R_T}{2SinP_{15}},$$

for the case of heat generation.

$$\theta=C_1CoshP_3y+C_2SinhP_3y+l_1Cosh2P_1y+l_2CoshP_2y+l_3CoshP_4y+l_4CoshP_5y+l_5 \tag{31}$$

where $$l_1=-\frac{1152\,Bra^4}{P(P-R)^2(4P-\phi)Cosh^2P_{13}};\quad l_2=-\frac{1152\,Bra^4}{R(P-R)^2(4R-\phi)Cosh^2P_{14}};$$

$$l_3=\frac{2304\,Bra^4}{P_1P_2(P-R)^2(P+R+2P_1P_2-\phi)CoshP_{13}CoshP_{14}};\quad l_4=-\frac{2304\,Bra^4}{P_1P_2(P-R)^2(P+R-2P_1P_2-\phi)CoshP_{13}CoshP_{14}};$$

$$l_5=-\frac{2304\,Bra^4}{2\phi(P-R)^2}\left(\frac{1}{PCosh^2P_{13}}+\frac{1}{RCosh^2P_{14}}\right);\ C_1=-\frac{1}{CoshP_{15}}\begin{pmatrix}l_1CoshP_{10}+l_2CoshP_{11}+l_3CoshP_{16}\\ l_4CoshP_{17}+l_5\end{pmatrix};\ C_2=\frac{R_T}{2SinhP_{15}},$$

for the case of heat absorption.

Solution of equations (10) and (11) for clear viscous fluid in the absence of buoyancy force, source and sink leads to the Hagen-Poiseuille velocity profile

$$u=24\left(\frac{1}{16}-y^2\right) \tag{32}$$

and temperature profile is given by

$$\theta=-192\ Br\ y^4+2R_Ty+\frac{3Br}{4} \tag{33}$$

which agree with the results obtained by Cheng and Wu [38] in the case of forced convection with asymmetric heating.

Solutions

Equation (11) is nonlinear differential equation and hence it is difficult to find the closed form solution. We employ perturbation series method by using dimensionless parameter

$$\varepsilon = Br\ \lambda = \text{Re Pr}\ \frac{\beta g D}{C_p} \tag{34}$$

The temperature field is obtained from equation (13). The solution of equation (11) can be expressed by the perturbation expansion

$$u(y) = u_0(y) + \varepsilon\, u_1(y) + \varepsilon^2 u_2(y) + \cdots\cdots = \sum_{n=0}^{\infty} \varepsilon^n u_n(y) \tag{35}$$

The second and higher order terms of ε gives a correction to u_0 accounting for the viscous dissipation effect.

Isothermal-isothermal $(T_1 - T_2)$ walls

Substituting equation(35) in equation (11) and equating coefficients like powers of ε to zero, one obtains the boundary value problem for n = 0 and n = 1 as

$$\frac{d^6 u_0}{dy^6} = \left(a^2 \mp \phi\right)\frac{d^4 u_0}{dy^4} \mp a^2(\sigma^2 - \phi)\frac{d^2 u_0}{dy^2} \mp a^2\sigma^2\phi\, u_0 \pm 48\phi a^2 \tag{36}$$

$$\frac{d^6 u_1}{dy^6} = \left(a^2 \mp \phi\right)\frac{d^4 u_1}{dy^4} \mp a^2(\sigma^2 - \phi)\frac{d^2 u_1}{dy^2} - a^2\left(\frac{du_0}{dy}\right)^2 \mp a^2\sigma^2\phi u_1 \tag{37}$$

for the case of heat generation and absorption respectively, and the corresponding boundary conditions of u_0 and u_1 are

$$u_0 = \frac{d^2 u_0}{dy^2} = 0 \quad \text{at} \quad y = \pm\frac{1}{4}$$

$$\frac{d^4 u_0}{dy^4} = 48a^2 - \frac{R_T \lambda a^2}{2} \quad \text{at} \quad y = -\frac{1}{4}$$

$$\frac{d^4 u_0}{dy^4} = 48a^2 + \frac{R_T \lambda a^2}{2} \quad \text{at} \quad y = \frac{1}{4} \quad \text{and}$$

$$u_1 = \frac{d^2 u_1}{dy^2} = \frac{d^4 u_1}{dy^4} = 0 \quad \text{at} \quad y = \pm\frac{1}{4} \tag{38}$$

Equations (36) and (37) are ordinary linear differential equations and their exact solutions can be found. These solutions obviously coincide with the solution of equation (11) in the case of $Br = 0$. Evaluation of exact solution for $n = 2$ becomes very complicated and hence neglecting the terms for $n = 2$, zeroth and first order solutions are

$$u_0 = C_1 CoshP_1 y + C_2 SinhP_1 y + C_3 CoshP_2 y + C_4 SinhP_2 y + C_5 CosP_3 y + C_6 SinP_3 y + \frac{48}{\sigma^2} \tag{39}$$

for the case of heat generation and

$$u_0 = C_1 CoshP_1 y + C_2 SinhP_1 y + C_3 CoshP_2 y + C_4 SinhP_2 y + C_5 CoshP_3 y + C_6 SinhP_3 y + \frac{48}{\sigma^2} \tag{40}$$

for the case of heat absorption. The solution of equation (37) by using (38) is

$$\begin{aligned} u_1 = {} & C_7 CoshP_1 y + C_8 SinhP_1 y + C_9 CoshP_2 y + C_{10} SinhP_2 y + C_{11} CosP_3 y + C_{12} SinP_3 y + l_1 CoshP_1 y + l_2 CoshP_2 y + l_3 Cos2P_3 y \\ & + l_4 Sinh2P_1 y + l_5 SinhP_2 y + l_6 Sin2P_3 y + l_7 CoshP_4 y + l_8 CoshP_5 y + l_9 SinhP_4 y + l_{10} SinhP_5 y + l_{11} CoshP_1 y CosP_3 y \\ & + l_{12} CoshP_2 y CosP_3 y + l_{13} SinhP_1 y SinP_3 y + l_{14} SinhP_2 y SinP_3 y + l_{15} CoshP_1 y SinP_3 y + l_{16} CoshP_2 y SinP_3 y \\ & + l_{17} SinhP_1 y CosP_3 y + l_{18} SinhP_2 y CosP_3 y + l_{19} \end{aligned} \tag{41}$$

for the case of heat generation and

$$\begin{aligned} u_1 = {} & C_7 CoshP_1 y + C_8 SinhP_1 y + C_9 CoshP_2 y + C_{10} SinhP_2 y + C_{11} CoshP_3 y + C_{12} SinhP_3 y + l_1 Cosh2P_1 y + l_2 Cosh2P_2 y \\ & + l_3 Cosh2P_3 y + l_4 Sinh2P_1 y + l_5 Sinh2P_2 y + l_6 Sin2P_3 y + l_7 CoshP_4 y + l_8 CoshP_5 y + l_9 CoshP_6 y + l_{10} CoshP_7 y \\ & + l_{11} CoshP_8 y + l_{12} CoshP_9 y + l_{13} SinhP_4 y + l_{14} SinhP_5 y + l_{15} SinhP_6 y + l_{16} SinhP_7 y + l_{17} SinhP_8 y + l_{18} SinhP_9 y + l_{19} \end{aligned} \tag{42}$$

for the case of heat absorption.

The dimensionless temperature field is obtained from equation (13) considering velocity field defined as in equations (39) to (42) which is given by

$$\theta = \frac{1}{\lambda}\left[\begin{aligned} & l_{36}\left((C_1 + \varepsilon C_7) CoshP_1 y + (C_2 + \varepsilon C_5) SinhP_1 y)\right) + l_{37}\left((C_3 + \varepsilon C_9) CoshP_2 y + (C_4 + \varepsilon C_{10}) SinhP_2 y\right) + \\ & l_{38}\left((C_5 + \varepsilon C_{11}) CosP_3 y + (C_6 + \varepsilon C_{12}) SinP_3 y\right) + \varepsilon\left(l_{39}(l_1 Cosh2P_1 y + l_4 Sinh2y) + l_{40}(l_2 Cosh2P_2 y + l_5 Sinh2P_2 y) + \right. \\ & l_{41}(l_3 Cos2P_3 y + l_6 Sin2P_3 y) + l_{42}\left(l_7 CoshP_4 y + l_9 SinhP_4 y\right) + l_{43}\left(l_8 CoshP_5 y + l_{10} SinhP_5 y\right) + l_{44} CoshP_1 y CosP_3 y \\ & + l_{45} CoshP_2 y CosP_3 y + l_{46} SinhP_1 y SinP_3 y + l_{47} SinhP_2 y SinP_3 y + l_{48} CoshP_1 y SinP_3 y + l_{49} CoshP_2 y SinP_3 y \\ & \left. + l_{50} SinhP_1 y CosP_3 y + l_{51} SinhP_2 y CosP_3 y + l_{19}\sigma^2\right) \end{aligned}\right] \tag{43}$$

for the case of heat generation and

$$\theta=\frac{1}{\lambda}\begin{bmatrix} l_{20}\left((C_1+\varepsilon C_7)CoshP_1y+(C_2+\varepsilon C_5)SinhP_1y\right)+l_{21}\left((C_3+\varepsilon C_9)CoshP_2y+(C_4+\varepsilon C_{10})SinhP_2y\right)+ \\ l_{22}\left((C_5+\varepsilon C_{11})CoshP_3y+(C_6+\varepsilon C_{12})SinhP_3y\right)+\varepsilon\left(l_{23}(l_1Cosh2P_1y+l_4Sinh2P_1y)+l_{24}(l_2Cosh2P_2y+l_5Sinh2P_2y)\right. \\ +l_{25}(l_3Cosh2P_3y+l_6Sinh2P_3y)+l_{26}\left(l_7CoshP_4y+l_{13}SinhP_4y\right)+l_{27}\left(l_8CoshP_5y+l_{14}SinhP_5y\right)+ \\ l_{28}\left(l_9CoshP_6y+l_{15}SinhP_6y\right)+l_{29}\left(l_{10}CoshP_7y+l_{16}SinhP_7y\right)+l_{30}\left(l_{11}CoshP_8y+l_{17}SinhP_8y\right)+ \\ \left. l_{31}\left(l_{12}CoshP_9y+l_{18}SinhP_9y\right)+l_{19}\sigma^2\right) \end{bmatrix} \tag{44}$$

for the case of heat absorption.

Isoflux-isothermal (q_1-T_2) walls

In this case, the thermal boundary condition for the channel walls can be written in the dimensional form as

$$q_1=-K\frac{dT}{dY} \quad \text{at} \quad Y=-\frac{L}{2}$$
$$T=T_2 \quad \text{at} \quad Y=\frac{L}{2} \tag{45}$$

$$\frac{d^3U}{dY^3}-\frac{\eta}{\mu}\frac{d^5U}{dY^5}+\frac{\beta g}{\nu}\frac{dT}{dY}=0 \quad \text{at} \quad Y=-\frac{L}{2} \tag{46}$$

The dimensionless form of above equation (45) and (46) can be obtained by using equation (7) with $\Delta T=q_1D/K$ to give

$$\frac{d\theta}{dy}=-1 \quad \text{at} \quad y=-\frac{1}{4}$$
$$\theta=R_{qt} \quad \text{at} \quad y=\frac{1}{4} \tag{47}$$

$$\frac{d^3u}{dy^3}-\frac{1}{a^2}\frac{d^5u}{dy^5}=\lambda \quad \text{at} \quad y=-\frac{1}{4} \tag{48}$$

where $R_{qt}=(T_2-T_0)/\Delta T$ is the thermal ratio parameter. The other boundary condition at the right wall can be shown to be the same as that given for the isothermal-isothermal case with R_T replaced by R_{qt} such that

$$\frac{d^4u}{dy^4}=48a^2+\frac{R_T\lambda a^2}{2} \quad at\ y=\frac{1}{4} \tag{49}$$

The integrating constants appeared in equations (39) to(44) are evaluated using boundary conditions (38s) along with (48) and (49).

Isothermal-isoflux (T_1-q_2) walls

In this case, the thermal boundary conditions are

$$q_2=-K\frac{dT}{dY} \quad \text{at} \quad Y=\frac{Y}{2}$$
$$T=T_1 \quad \text{at} \quad Y=-\frac{Y}{2} \tag{50}$$

The dimensionless form of equation (50) can be obtained by using the equation (7) with $\Delta T=q_2D/K$ to give

$$\frac{d\theta}{dy}=-1 \quad \text{at} \quad y=\frac{1}{4}$$
$$\theta=R_{tq} \quad \text{at} \quad y=-\frac{1}{4} \tag{51}$$

where $R_{tq}=(T_1-T_0)/\Delta T$ is the thermal ratio parameter for the isothermal-isoflux case.

Similar to the procedure done in the previous section on isoflux-isothermal walls, the dimensionless form of the boundary conditions obtained from using equation(2) and applying equation (51) can be written as

$$\frac{d^3u}{dy^3}-\frac{1}{a^2}\frac{d^5u}{dy^5}=\lambda \quad \text{at} \quad Y=\frac{1}{4} \tag{52}$$

The other boundary condition at the right wall can be shown to be the same as that given for the isothermal-isothermal case with R_T replaced by R_{tq} such that

$$\frac{d^4u}{dy^4}=48a^2+\frac{R_{tq}\lambda a^2}{2} \quad at\ y=\frac{1}{4} \tag{53}$$

The integrating constants appeared in equations (39) to (44) are evaluated using boundary conditions (38) along with (52) and (53).

Result and discussion

The laminar and fully developed mixed convection in a vertical channel for couple stress permeable fluid has been analyzed by taking into account the effect of viscous dissipation. Effect of λ and ε is shown in Figures 1 and 2 on the flow. When the flow is upward, λ and ε are positive and the flow is downward for negative values of λ and ε. Velocity and temperature at each position are increasing function of ε for $\lambda = 500$. For $\lambda = -500$, as ε increases velocity increases in the downward direction where as temperature increases in upward direction. It is very interesting to note that the velocity profiles are not reversed either at cool or hot walls which is a result different from considering only couple stress fluid or viscous fluid.

The effect of heat generation coefficient ϕ on velocity and temperature is shown in Figures 3 and 4 respectively. As ϕ increases velocity and temperature decreases for the upward flow and increases on the reversal side for downward flow. These results are similar to that for couple stress fluid with no flow reversal at the boundaries. Figures 5 and 6 shows the effect of couple stress parameter $'a'$ on the flow. As $'a'$ increases velocity is promoted in the upward flow and also in the downward flow at the reversal side. The maximum velocity occurs in the middle of channel, which is not true for small values of $'a'$ for only couple stress fluid and also there is no flow reversal at the boundaries. The effect of couple stress parameter $'a'$ is to promote the temperature both for small and large values of ε which is a similar result obtained for couple stress fluid. Figures 7 and 8 shows the effect of porous parameter σ on the flow. As σ increases flow is suppressed in the upward flow for $\varepsilon = 0.1$ and $\varepsilon = 5.0$ whereas, it increases in the flow at the reversal side for $\varepsilon = -0.1$ and $\varepsilon = -5.0$. The temperature decreases as σ increases both for positive and negative values of λ. This effect is due to dampening effect of Darcy resistance, which is true for permeable viscous fluid also.

When the boundary temperatures are equal velocity and temperature is a symmetric function of y which depends only on dimensionless parameter ε also θ is a symmetric function of y and depends only on ε.

Figures 9 and 10 are for heat generation case and the results remains same for heat absorption except the on ϕ and $'a'$. Figures 11 and 12 shows that as ϕ increases velocity decreases for upward flow and increases for downward flow but variation is insignificant. The effect of ϕ on temperature is almost invariable and is constant in the middle of the channel. In figure 13 the effect of couple stress parameter $'a'$ on velocity is to promote the flow both for upward and downward flow which is a similar result for heat generation except the velocity profiles are in the upward direction for heat absorption. Figure 14 shows that the effect of temperature remains invariant for ε and couple stress parameter $'a'$, which is a different result, obtained for heat absorption.

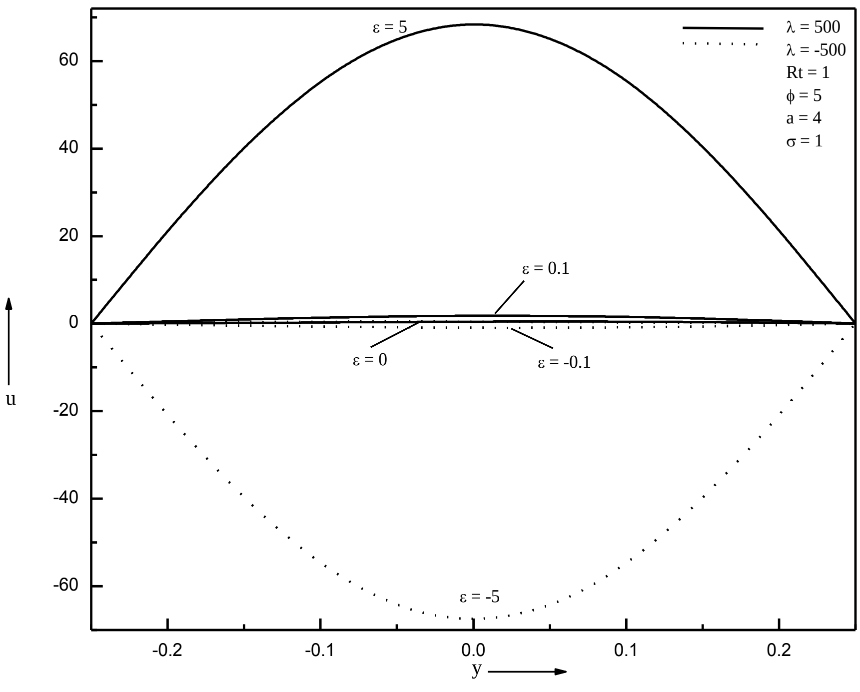

Fig.1 Plots of u versus y in the case of asymmetric heating for different values of λ and ε

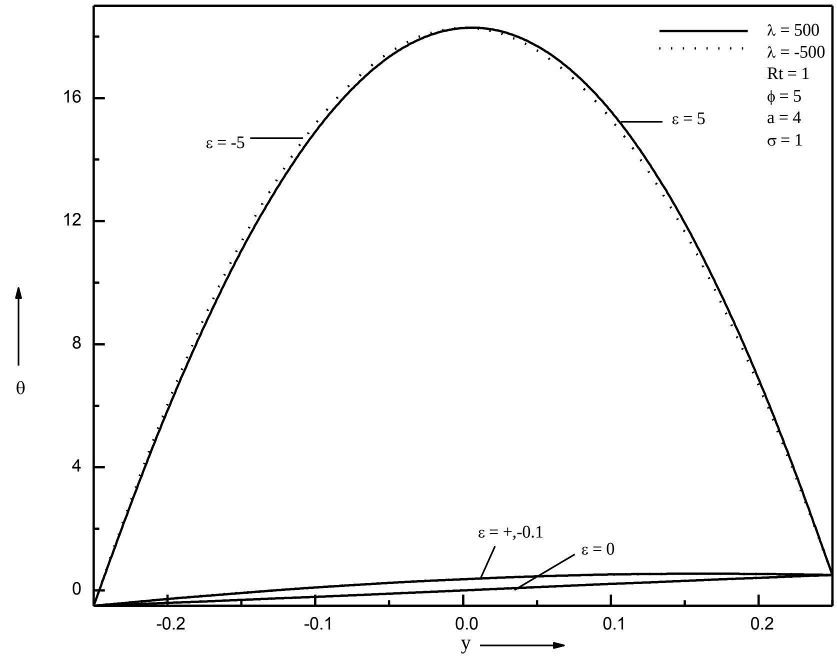

Fig.2 Plots of θ versus y in the case of asymmetric heating for different values of λ and ε

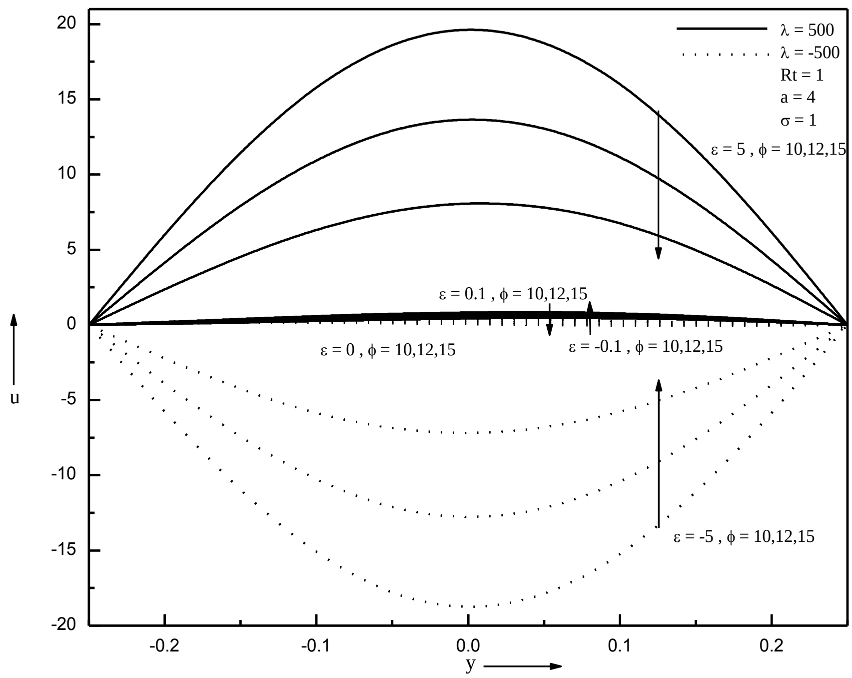

Fig. 3 Plots of u versus y in the case of asymmetric heating
for different values of heat generation coefficient ϕ and ε

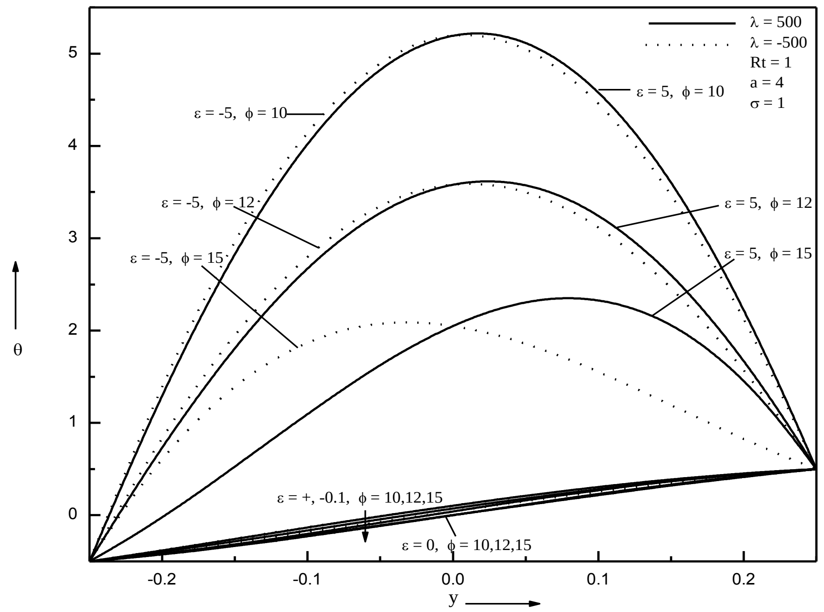

Fig. 4 Plots of θ versus y in the case of asymmetric heating for different values of heat generation coefficient ϕ and ε

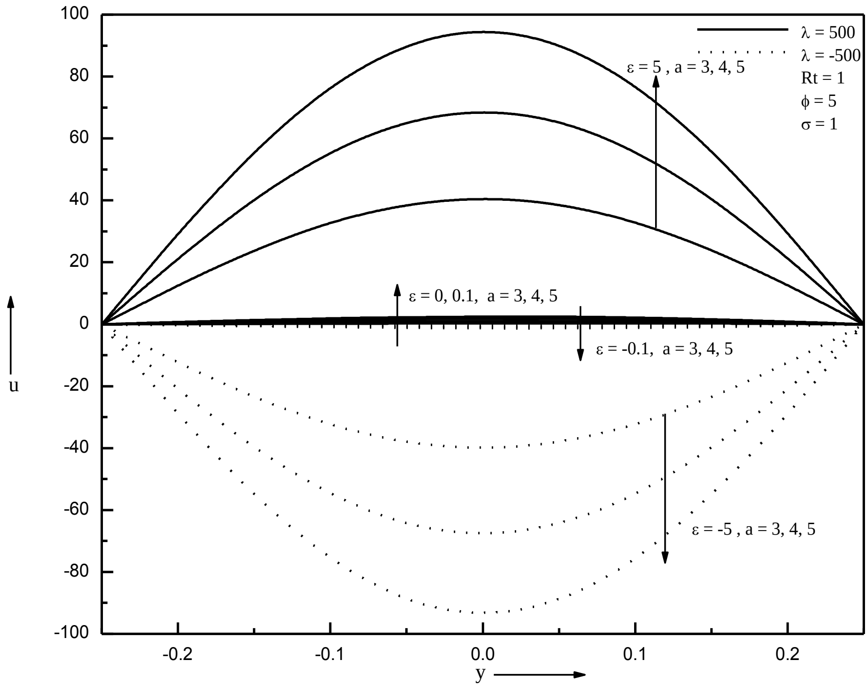

Fig. 5 Plots of u versus y in the case of asymmetric heating for different values of couple stress parameter a and ε

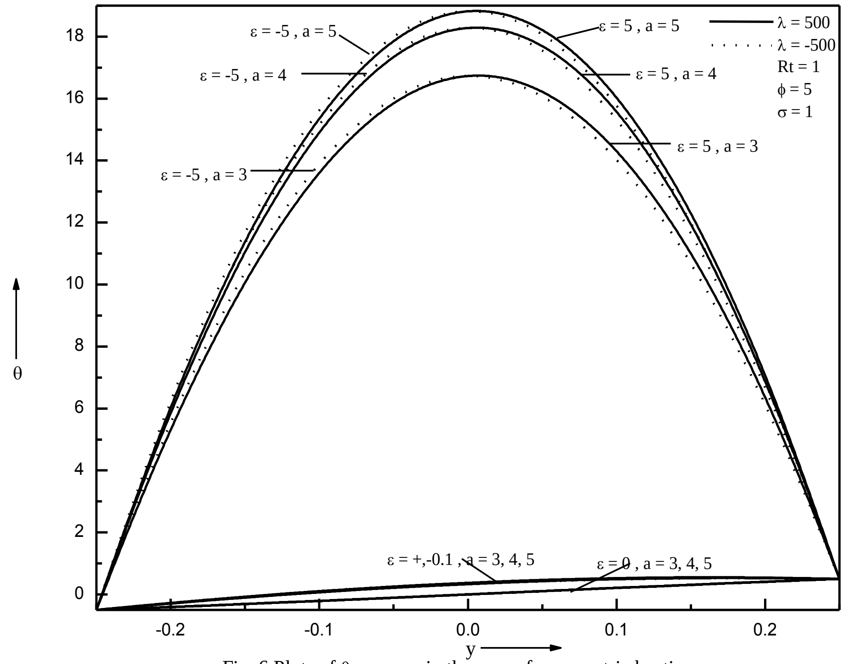

Fig. 6 Plots of θ versus y in the case of asymmetric heating for different values of couple stress parameter a and ε

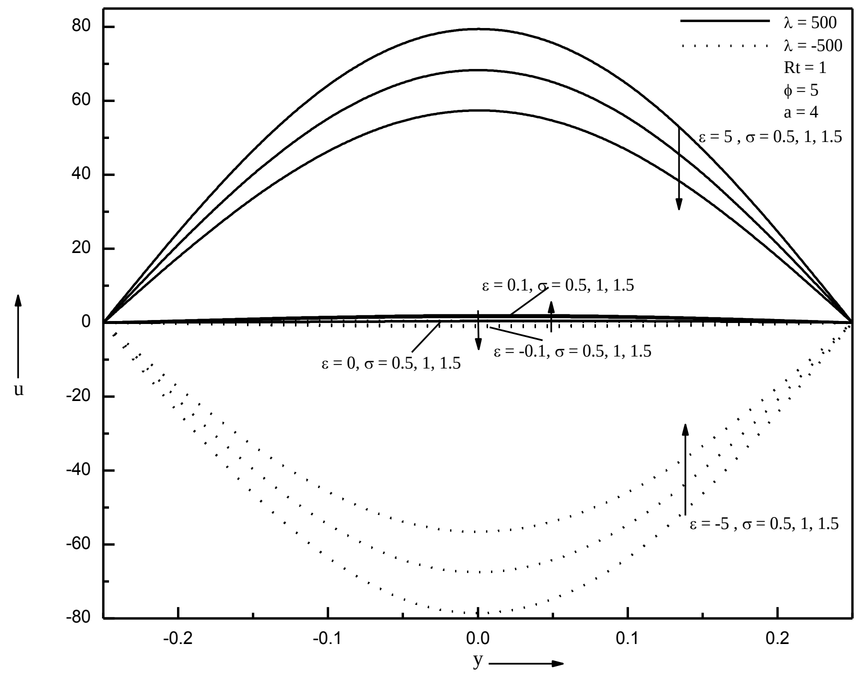

Fig. 7 Plots of u versus y in the case of asymmetric heating for different values of porous parameter σ and ε

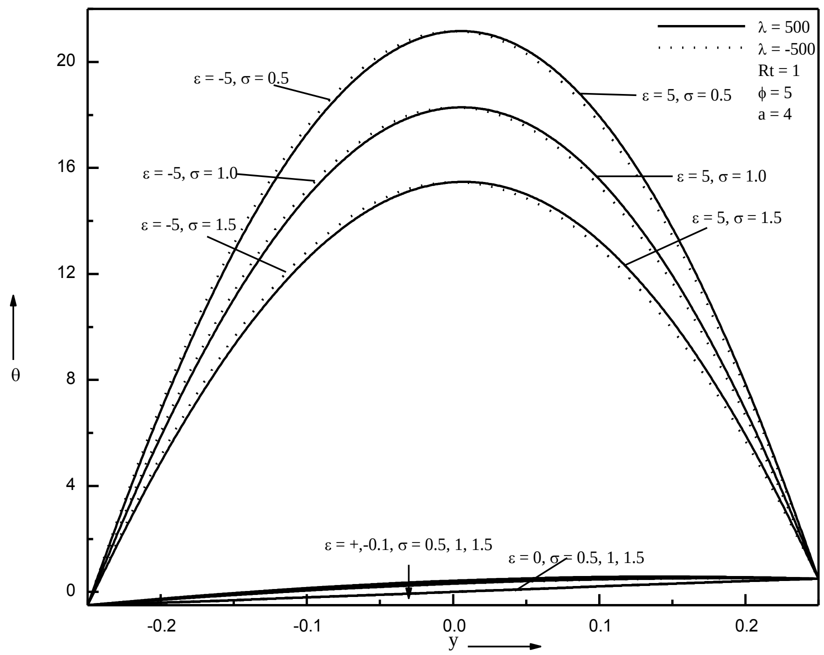

Fig.8 Plots of u versus y in the case of asymmetric heating for different values of porous parameter σ and ε

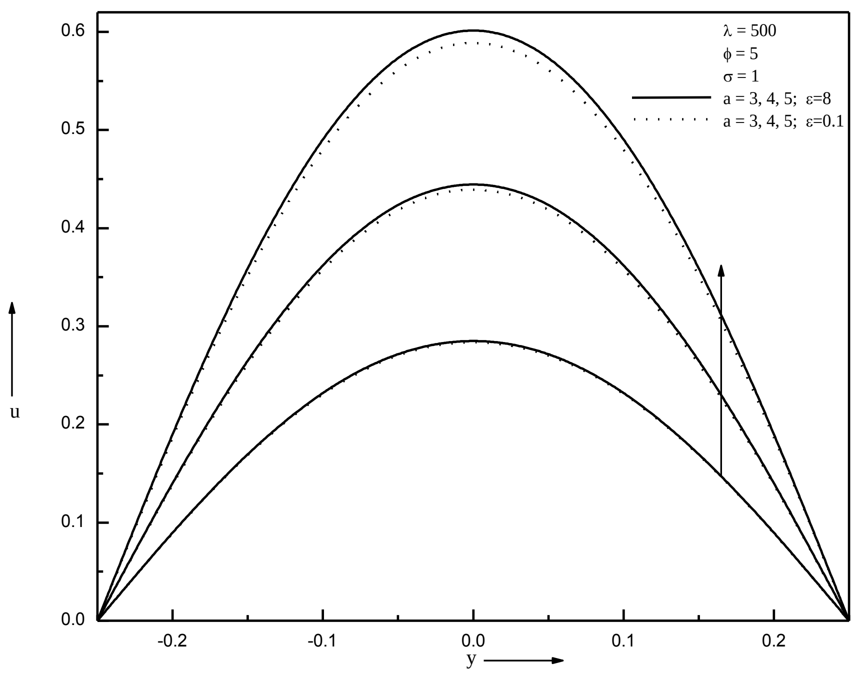

Fig. 9 Plots of u ersus y in the case of symmtric heating for different values of couple stress parameter a and ε

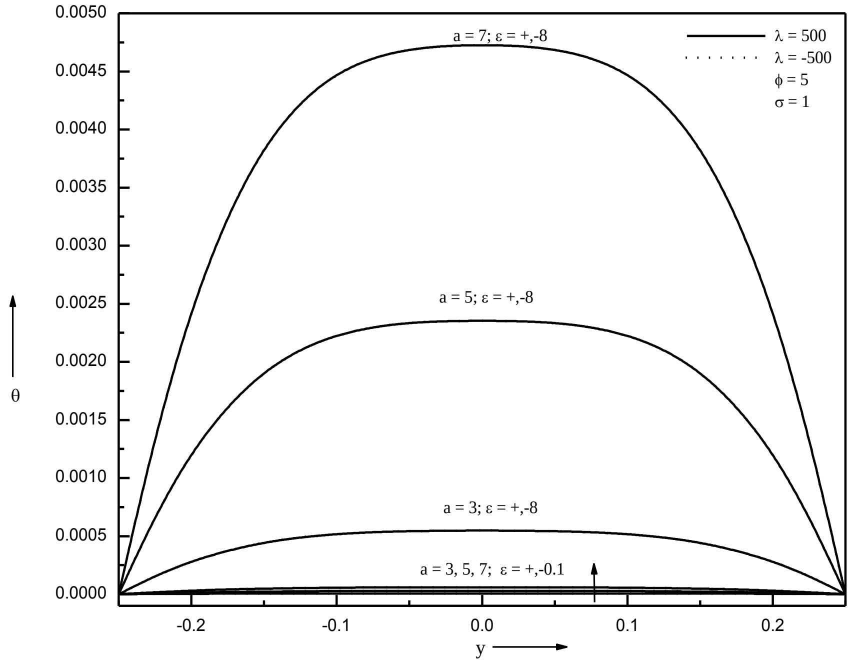

Fig.10 Plots of θ versus y in the case of symmetric heating for different values of couple stress parameter a and ε

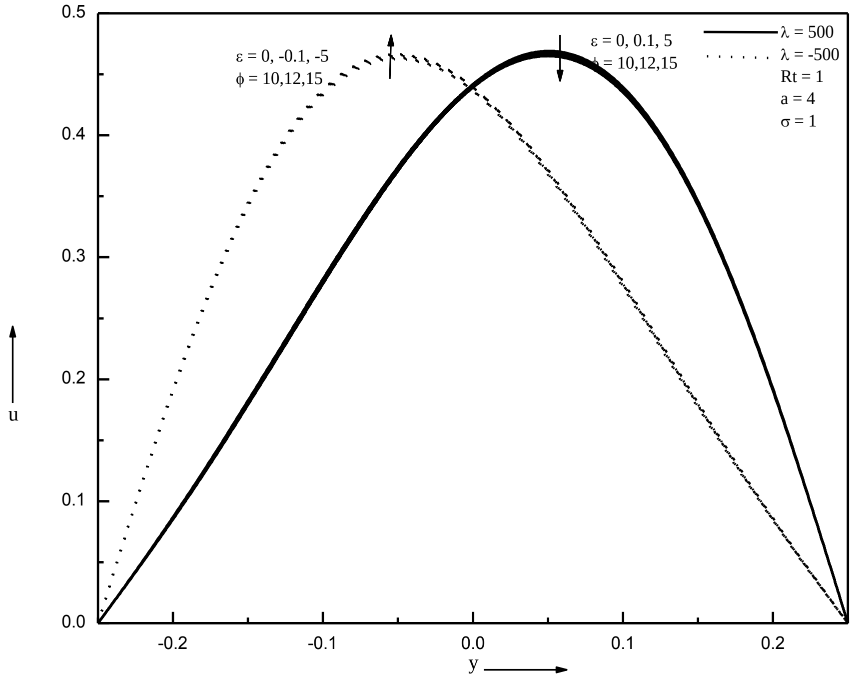

Fig.11 Plots of u versus y in the case of asymmetric heating for different values of heat absorption coefficient ϕ and ε

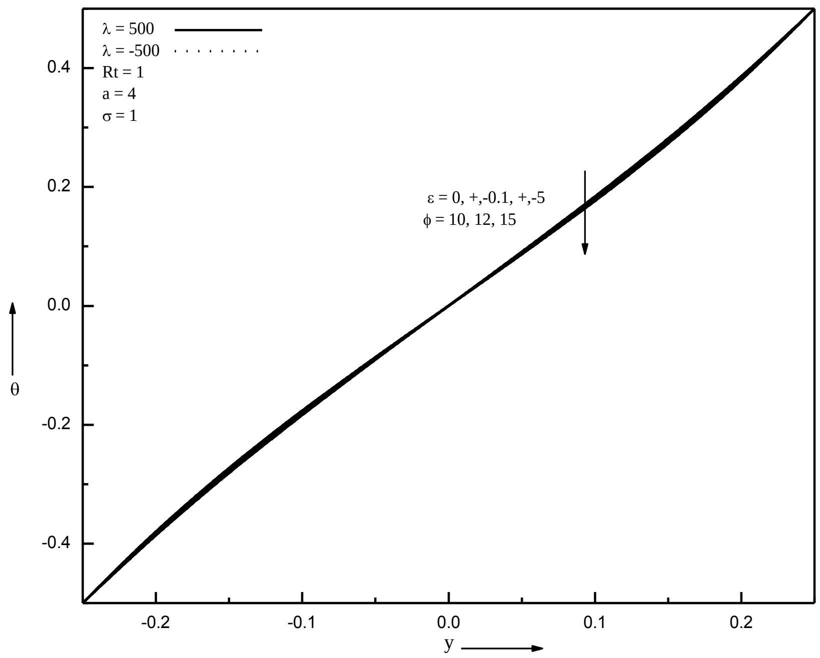

Fig.12 Plots of θ versus y in the case of asymmetric heating for different values of heat absorption cofficient ϕ and ε

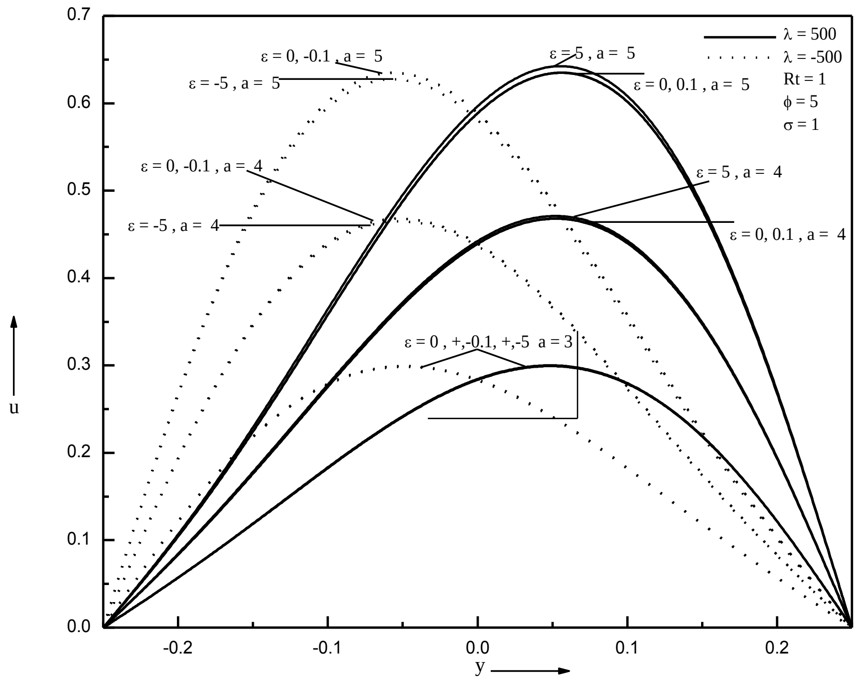

Fig. 13 Plots of u versus y in the case of asymmetric heating for different values of couple stress parameter a and ε

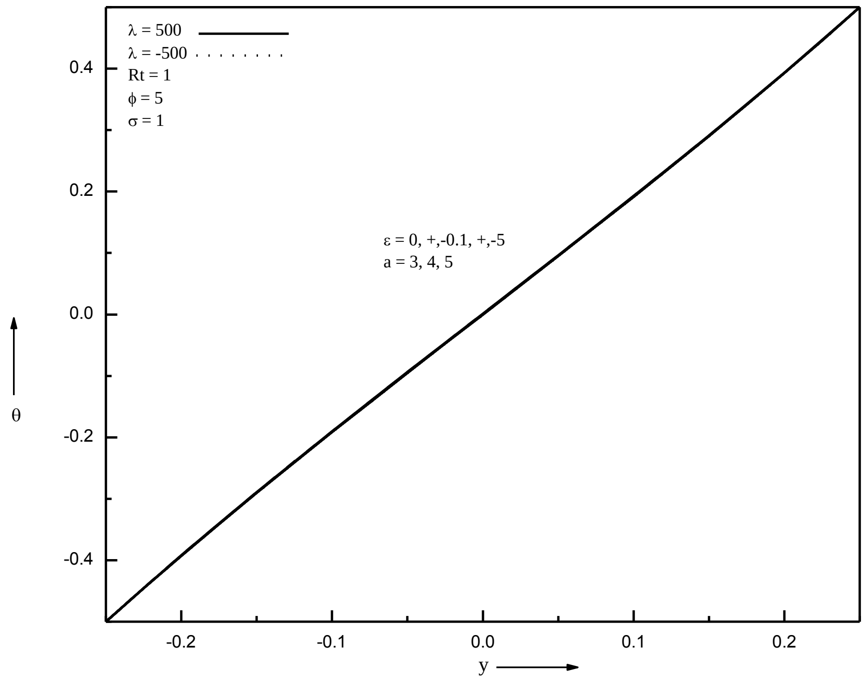

Fig. 14 Plots of θ versus y in the case of asymmetric heating for different values of couple stress parameter a and ε

Conclusions

Mixed convection of a permeable couple stress fluid flow in a vertical channel with heat generation or absorption was analyzed with symmetric and asymmetric wall temperatures. The plate exchanged heat with an external fluid. The governing equations are solved analytically for small values of the product of mixed convection parameter and Brinkman number. For an upward flow, the velocity and temperature at each position are an increasing function of ε, whereas for downward flow velocity is a decreasing function of ε and temperature is an increasing function of ε. Increase in the values of porous parameter results in the reduction of flow for both equal and unequal wall temperature. Flow reversal near the wall was obtained for asymmetric wall temperatures and was found to increase in the presence of a porous matrix. As couple stress parameter increases velocity and temperature increases and it is notified that the maximum velocity occurs at both left and right walls.

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