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Formula for Counting Similar Rectangles

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ABSTRACT

The present paper is an attempt to illustrate the direct formula for calculating total number of similar rectangles present, when any rectangle is sub divided into $n\times n$ congruent rectangles. Subdivision of any rectangle into $n\times n$ congruent rectangles means sides of the rectangle divided into n equal parts and points are joined in a sense that it forms $n\times n = n^2$ number of congruent rectangles.

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Introduction

In this paper we consider a rectangle ABCD with length x and breadth y. Divide each side into n equal parts with lengths x/n, y/n. Rectangles with length x/n and breadth y/n are congruent to each other. There are $n\times n = n^2$ number of congruent rectangles. By the side ratio theorem rectangles with sides of length & breadth (x/n, y/n), (2x/n, 2y/n) (3x/n, 3y/n), ………………., ((n-1)x/n, (n-1)y/n), (x, y) are similar to each other. In the following investigation, we determine the number of similar rectangles when an arbitrary rectangle is subdivided into $n\times n = n^2$ numbers of congruent rectangles.

Congruency and similarity

Case-1 (2×2 subdivisions)

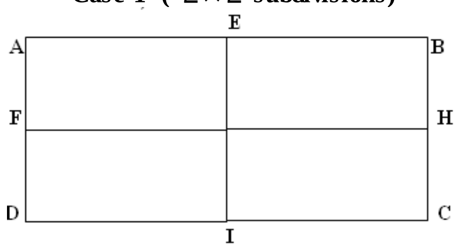

Fig 1.

In Fig – 1 we have consider rectangle ABCD with AB = CD = x, BC = AD = y. Let us subdivide it into $2\times 2 = 4$ congruent rectangles. Let E, F, H, I be mid points of AB, AD, BC, CD such that

AE = BE = CI = DI = $\frac{x}{2}$,

BH = HC = AF = DF = $\frac{y}{2}$.

Points are joined in a sense that it forms $2\times 2 = 4$ congruent rectangles.

Then

AE||FG||DI, EB||GH||IC,

AF||EG||BH, FD||GI||HC

So,

FG = GH = $\frac{x}{2}$, EG = GI = $\frac{y}{2}$.

Now rectangles AEGF, EBHG, GHCI, and FGID are congruent to each other with length x/2 and breadth y/2. So, by side ratio theorem rectangle AEGF, EBHG, GHCI, FGID, ABCD are similar to each other.

Case 2 (3×3 subdivisions)

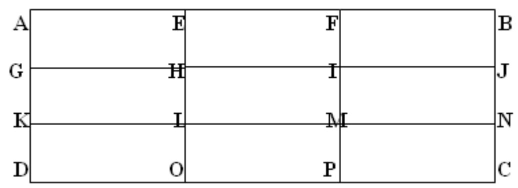

Fig 2.

In Fig - 2 we have consider rectangle ABCD with AB = CD = x, BC = AD = y. Let us subdivide it into $3\times 3 = 9$ congruent rectangle. Trisect segments AB, BC, CD, AD so that

AE = EF = FB = DO =OP = PC = $\frac{x}{2}$,

AG = GK = KD = BJ = JN = NC = $\frac{y}{2}$.

Points are joined in a sense that it forms $3\times 3 = 9$ congruent rectangles.

Then

AE||GH||KL||DO, EF||HI||LM||OP,

FB||IJ||MN||PC, AG||EH||FI||BJ,

GK||HL||IM||JN, KD||LO||MP||NC.

So,

AE = EF = FB = GH = HI = IJ = KL = LM = MN = DO = OP = PC = x/3,

AG = GK = KD = EH == HL = LO = FI = IM = MP = BJ = JN = NC = y/3.

Now, rectangles AEHG, EFIH, FBJI, GHIK, HIML, IJNM, KLOD, LMPO, and MNCP are congruent to each other with length x/3 and breadth y/3.

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Again rectangles AFMK, EBNL, GIPD, and HJCO are congruent to each other with length 2x/3 and breadth 2y/3. So, by side ratio theorem rectangle AEHG, EFIH, FBJI, GHIK, HIML, IJNM, KLOD, LMPO, MNCP, AFMK, EBNL, GIPD, HJCO , ABCD are similar to each other.

Case-3 Generalization ($n \times n = n^2$ Subdivisions)

Similarly, we can investigate for $n \times n = n^2$ subdivisions. To keep track of similar rectangles, it is helpful to represent rectangles of various dimensions graphically as suggested in Fig – 3 to Fig - 7

Rectangles of length x/n and breadth y/n represented by

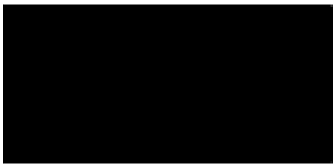


Fig 3. Rectangles of length 2x/n and breadth 2y/n represented by

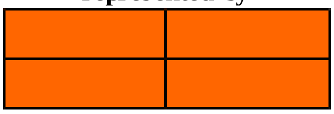


Fig 4. Rectangles of length 3x/n and breadth 3y/n represented by

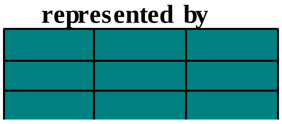


Fig 5. Rectangles of length 4x/n and breadth 4/n represented by

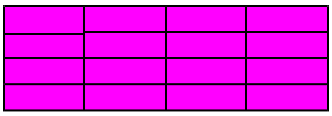


Fig 6. Rectangles of length 5x/n and breadth 5y/n represented by

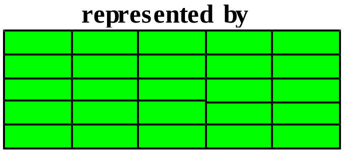


Fig 7.

We use above representations (Fig – 3 to Fig – 7) to count number of similar rectangles with $n \times n = n^2$ subdivisions for $1 \le n \le 5$, then look for pattern in our collected data in an effort to generalize our findings.

Description of Counting of Similar Rectangles

Now, we describe how to count similar rectangles when any rectangle is subdivided into $n \times n = n^2$ subdivisions for $1 \le n \le 5$ as follows:

(I) For (1 X 1) Rectangle

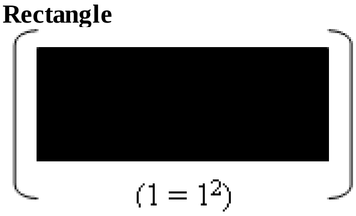


Fig 8. No. of Rectangle = 1=1^2

(2) For (2X 2) Rectangle

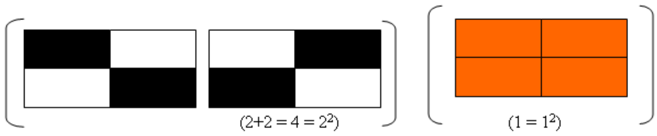


Fig 9. No. of Rectangles = 1+4 = 1^2+2^2

(3) For (3 X 3) Rectangle

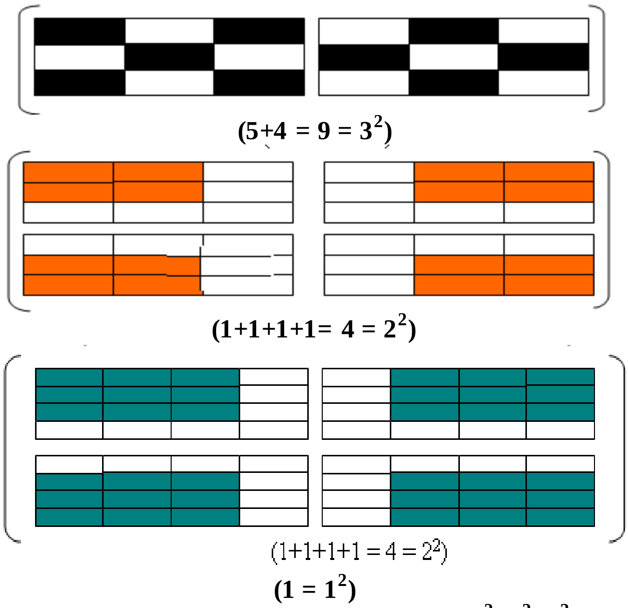


Fig 10. No. of Rectangles = 1+4+9 = 1^2+2^2+3^2

(4) For (4 X 4) Rectangle

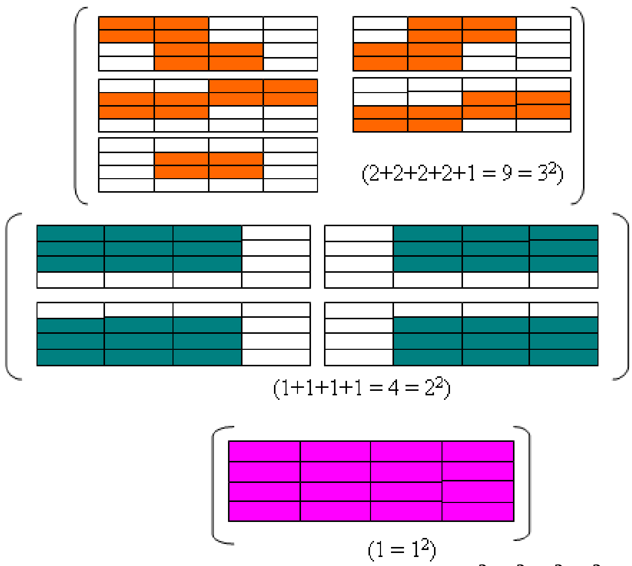
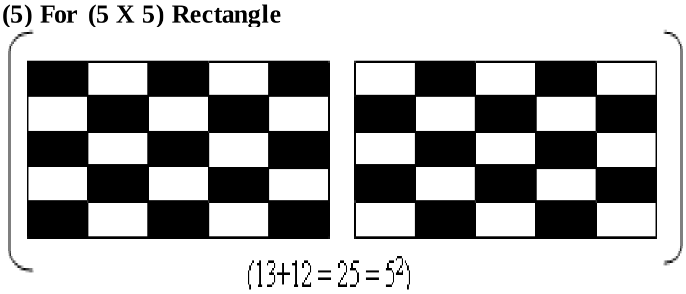
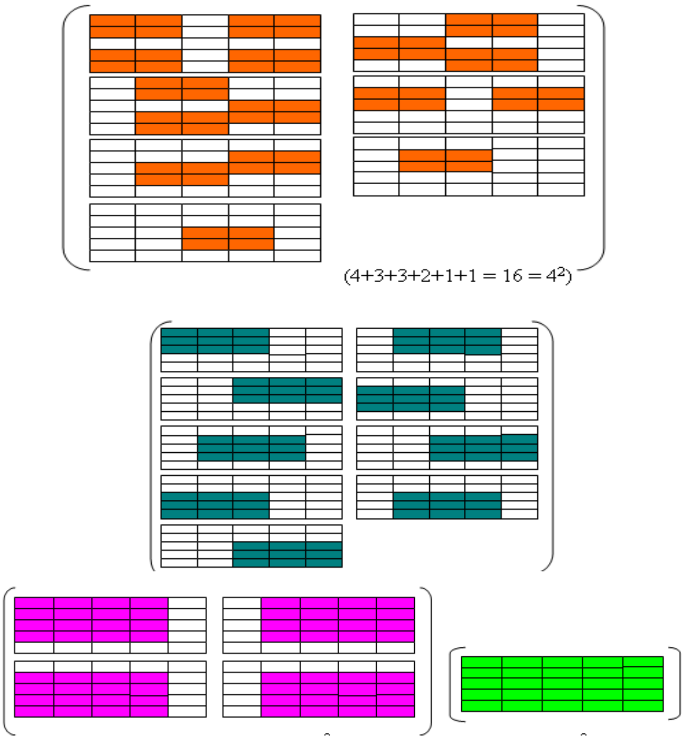


Fig 11. No. of Rectangles = 1+4+9+16 = 1^2+2^2+3^2+4^2

(5) For (5 X 5) Rectangle





$(1+1+1+1 = 4 = 2^2)$ **$(1 = 1^2)$**

Fig 12. No. of Rectangles = $1+4+9+16+25 = 1^2+2^2+3^2+4^2+5^2$

(6) For $n \times n = n^2$ Rectangles

By analyzing all the five cases we can write a possible expression for $n \times n = n^2$

Total Number of Similar Rectangles

$1^2 + 2^2 + 3^2 + \ldots\ldots\ldots\ldots + n^2 = n(n+1)(2n+1)/6$

Verification of general formula

Table – 1 contains the no. of similar rectangles present when any rectangle is subdivided into $n \times n = n^2$ congruent rectangles both by using general counting and the given formula.

$n \times n$	General Counting	By Formula	$n \times n$	General Counting	By Formula
1×1	1	1	2×2	5	5
3×3	14	14	4×4	30	30
5×5	55	55	6×6	91	91
7×7	140	140	8×8	204	204
9×9	285	285	10×10	385	385
11×11	506	506	12×12	650	650
13×13	819	819	14×14	1015	1015
15×15	1240	1240	16×16	1496	1496
17×17	1785	1785	18×18	2109	2109
19×19	2470	2470	20×20	2870	2870
21×21	3311	3311	22×22	3795	3795
23×23	4324	4324	24×24	4900	4900
25×25	5525	5525	26×26	6201	6201
27×27	6930	6930	28×28	7714	7714
29×29	8555	8555	30×30	9455	9455
31×31	10416	10416	32×32	11440	11440
33×33	12529	12529	34×34	13685	13685
35×35	14910	14910	36×36	16206	16206
37×37	17575	17575	38×38	19019	19019
39×39	20540	20540	40×40	22140	22140
41×41	23821	23821	42×42	25585	25585
43×43	27434	27434	44×44	29370	29370
45×45	31395	31395	46×46	33511	33511
47×47	35720	35720	48×48	38024	38024
49×49	40425	40425	50×50	42925	42925

Conclusion

In this paper we give a direct formula to find the total number of similar rectangles present when any rectangle is subdivided into $n \times n = n^2$ congruent rectangles. By this formula easily one can find the result without general counting.

References

1. Posamentier, A. (2002). "Advanced Euclidean Geometry", Emeryville, CA: Key College Press.
2. Abbott, P. (1948). "Teach Yourself Geometry", The English University Press, London.
3. Hilbert, D. (1971). "Foundations of Geometry", Open Court. Chicago.