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Non-linear Growth Functions for Modelling Tree Height - Diameter Relationships for *Gmelina arborea* (Roxb.) in Ibadan, Southwest Nigeria

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ABSTRACT

Tree height-diameter relationship can be used as an important input component in forest growth and yield models and description of stand dynamics. Five non-linear growth functions were fitted to tree height-diameter data of 21 years old *Gmelina arborea* plantation in Ibadan, South west Nigeria. The data consisted of three sets: 2004, 2008 and 2011 on total tree height and diameter at breast height (dbh). According to the model statistics, the five growth functions fitted the data equally well, but resulted in different asymptote estimates. Modified exponential fit was observed to give the best fit for the three data sets based on least square error, coefficient of determination and significance. The models are: Ht = $2.71e^{(-1.16/dbh)}$ (R^2 = 36.21%), Ht = $2.95e^{(-1.36/dbh)}$ (R^2 = 37.53%) and Ht = $3.07e^{(-1.47/dbh)}$ (R^2 = 34.74%) for 2004, 2008 and 2011 data sets. The predicted values follow the same nonlinear pattern and formed close to the line of best fit without much outlier. The result of this study revealed that the ability of dbh in determining height is not strong enough based on the model's goodness of fit and the model's ability for predictive purposes. Hence, more variables such as age, crown area and soil fertility were recommended to be incorporated in future prediction of the tree height in the study area. Also, the potential tree height-diameter equations should be evaluated and validated for their predictive capabilities across a range of tree diameters. This useful information can help forest researchers and managers to select and apply the appropriate models.

Introduction

Tropical rainforests are the most varied natural environments to be found on the earth, perhaps the most inspiring and popular. These forests, with their large trees and extraordinary flora and fauna constitute the planet's richest habitats, and one of our most precious natural resources (Longman and Jenik, 2000). Like all tropical rainforests, the Nigerian rainforest is characterized by high species diversity and morphological uniformity of the constituent species. The forest contains trees from a wide range of ages and size classes. Hence, its management requires more precise tools or technique in achieving sustainable forest management.

Measurements of tree heights and diameters are essential in forest assessment, modeling and management. Tree heights are used for estimating timber volume, site index and other important variables related to forest growth and yield, succession and carbon budget models (Peng, *et al* 2001). Considering that the diameter at breast height (dbh) can be more accurately obtained, and at lower cost than total tree height, only a sub-sample of heights is usually measured in the field. Height–diameter equations are then used to predict the heights of the remaining trees, thus reducing the cost of data acquisition. For these reasons, developing suitable height–diameter models may be considered one of the most important elements in forest design and monitoring.

The development of simple and accurate height–diameter models, based on easily obtainable tree and stand characteristics, is a common precursor to using inventory and sample plot data to calculate volume and other stand attributes. A number of height–diameter equations have been developed using only dbh as the predictor variable for estimating total height (e.g., Huang *et al*., 1992; Moore *et al*., 1996; Zhang, 1997; Peng, 1999; Fang and Bailey, 1998; Fekedulengn *et al*., 1999; Jayaraman and Zakrzewski, 2001; Robinson and Wykoff, 2004). However, the relation between the diameter of a tree and its height varies among stands (Calama and Montero, 2004) and depends on the growing environment and stand conditions (Sharma and Zhang, 2004). For a particular height, trees that grow in high density stands will have smaller diameters than those growing in less dense stands, because of greater competition among individuals (Lopez Sanchez *et al*., 2003; Calama and Montero, 2004). The height–diameter relationship is also not constant over time even within the same stand (Curtis, 1967). These factors indicate that additional predictor variables are required to develop generalized height–diameter models in order to avoid having to establish individual height–diameter relationships for every stand (Temesgen and Gadow, 2004).

The diversity observed in modeling approaches was easily dwarfed by the multiplicity in model forms developed. Numerous linear and nonlinear models have been applied in the quest to produce accurate height–diameter relationships. Candidate functions were often based on their mathematical features (e.g., demonstrating appropriate sigmoidal shape, possessing a sufficient number of parameters to achieve

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flexibility without compromising parsimony), and the possible biological interpretation of model parameters (Peng *et al*., 2001). Biologically reasonable models generally produced more accurate predictions beyond the range of data used in model fitting (Fekedulengn *et al*., 1999).

Gmelina arborea timber is a fast growing deciduous tree, occurring in most part of the country as a plantation species. It is reasonably strong for its weight. It is used in constructions, furniture, carriages, sports, musical instruments and artificial limbs. Once seasoned, it is a very steady timber and moderately resistant to decay and ranges from very resistant to moderately resistant to termites. Its timber is highly esteemed for door and window panels, joinery and furniture especially for drawers, wardrobes, cupboards, kitchen and camp furniture, and musical instruments because of its lightweight, stability and durability. It is also used for bentwood articles. In boat building it is used for decking and for oars (Anonymous, 2002). Hence, its choice as plantation species that are mostly sort for by loggers either legally or illegally. Therefore, the objective of this study is to develop and compare selected nonlinear models fitted for *G. arborea* tree height–diameter relationship in the study area for sustainable management.

Methodology

Study Area

The study was carried out on 21 years old *Gmilina arborea* plantation at Forestry Research Institute of Nigeria, Ibadan, Oyo state, Nigeria. It falls within the rainforest zone of Nigeria and lies between latitude 7^0 20'N and longitude 3^0 56'E with an altitude of 150m above sea level.

The study site is a permanent sample plot (PSP), under the management of Forestry Research Institute of Nigeria (FRIN). It has a total size of 0.85ha in which total enumeration on the diameter at breast height (dbh) and total height have been carried out. Three sets of data on the PSP were used for this study. These were the 2004, 2008 and 2011 data sets.

Basal Area Estimation

The Basal Area (BA) of individual trees was estimated using the formula in equation 1 (Husch *et al*, 2003)

$$BA = \frac{\pi}{4} D^2 \qquad (1)$$

Where BA = Basal area (m^2), D = dbh (cm).

Slenderness coefficient (SLC) Estimation

$$SLC = \frac{THT}{Dbh}$$

Where THT = Total height

Data analysis

The data collected were analyzed using a nonlinear regression model (Philip, 1994) of the form:

$$\phi_t = f(t_i, \beta) + \varepsilon_i \qquad i = 1, 2, \ldots\ldots, n, \qquad (2)$$

Where ϕ is the response variable (height), t is the independent variable (dbh), β is the vector of parameter β_j to be estimated i.e. $(\beta_1, \beta_2, \ldots\ldots\ldots\ldots, \beta_k)$, ε_i is a random error term, k is the number of unknown parameters and n is the number of the observation. The estimators of β_j's were obtained by minimizing the sum of squares error (SS_{err}) function as below

$$SS_{err} = \sum_{i=1}^{n} [\phi_i - f(t_i, \beta)]^2 \qquad (3)$$

Under the assumption that ε_i are normal and independent with mean zero and common variance σ^2. Since ϕ_i and t_i are fixed observations, the sum of square residual is a function of β. Least square estimates of β are values which when substituted into equation 3 made SS_{err} a minimum. These were obtained by differentiating equation 3 with respect to each parameter and setting the result to zero. This provides the k normal equations that must be solved for $\hat{\beta}$. These normal equations take the form

$$\sum_{i=1}^{n} \{\phi_i - f(t_i, \beta)\} \left[\frac{\partial f(t_i, \beta)}{\partial \beta_j} \right]_{\theta = \hat{\theta}} = 0 \qquad (4)$$

For j = 1,2,……..,k. when the model is nonlinear in the parameters so are the normal equations, consequently, for the nonlinear models considered above, it is impossible to obtain a closed form solution to the least squares estimate of the parameters by solving the k normal equations described in equation 4. Hence an iterative method was employed to minimize the SS_{err}. The NLIN (nonlinear regression) procedure in SAS (1992) was used to fit the models to the data and estimate the parameters. The quantity in the square bracket is the partial derivative of F(t,β) wrt θ with all the θ replaced by the corresponding θ. e.g. $\beta_1 = \beta_1$, $\beta_2 = \beta_2$ e.t.c.

Fitting of the Models

Five non-linear models were fitted using curve expert for the height–diameter relationship and ranked according to their best of fit using the correlation coefficient, least square error and significance at 5% level of probability.

The models are:

1. Modified Exponential (Khamis, and Ismail, 2004): Y = $a * e^{(b/x)}$
2. Exponential Association (Tsoularis, and Wallace, 2002): Y = $a(1 - \exp(-bx))$
3. Saturation Growth-Rate (Ismail *et al*, 2003): Y = $\frac{ax}{(b+x)}$
4. Modified Geometric Fit (Jaafar, 1999): Y = $a * x^{(b/x)}$
5. Hyperbolic Fit (Philip, 1994): Y = $a + b/x$

Where: Y = Height (m)
X = Dbh (cm)
a & b = model parameters

The model predictions functions for each year of assessment were compared using the root mean square errors and biases/residuals of stand diameter class. The absolute root mean square error (RMSE) was calculated as:

$$RMSE = \sqrt{\sum_{i=1}^{n} (Vi - \lambda i)^2 / n} \qquad (5)$$

Where n = Number of sample stands

Vi = Diameter/height class of growing stock in stand i

λi = The diameter/height of stand i estimated from the predicted distribution.

The bias of the predictions was calculated as

$$Bias = \sum_{i=1}^{n} (Vi - \lambda i) / n \qquad (6)$$

Result and discussion

The frequencies of the selected trees are 932, 928 and 917 for year 2004, 2008 and 2011 respectively (Table 1). An observation of downward trend in the number of stems per hectare in the plot between 2004 and 2011 was similar to the observation of Oguntala (1981). In his observation of tree population in a permanent sample plot in Gambari forest reserve, Nigeria, he reported that there was consistent decline in the number of trees per hectare for a period of 22 years. This decline in the number of stems per hectares may have been through natural mortality since exploitation and other forms of human disturbance have been strictly prohibited in the plot. Basal area per hectare throughout this period of assessment followed the same trend with the number of stems per hectare. This can be explained by the fact that basal area per hectare is computed from number of trees per hectare and the dbh of the standing trees. Meanwhile, the values of slenderness coefficient obtained for all year of assessment were moderate. The slenderness coefficient values obtained from the analysis of the data were classified into three categories as suggested by Navratil, *et al* (1994) as stated bellow:

SLC values > 99 = High slenderness coefficient
70 < SLC > 99 = Moderate slenderness coefficient
SLC < 70 = Low slenderness coefficient

Adjudging the result of this study with above classification, all the tree species have low slenderness coefficient (i.e SLC < 70). The value ranges from 36.17 to 142.86, 37.50 to 140 and 36 to 138.60 for 2004, 2008 and 2011 respectively. This implies that all the tree species are not prone to wind throw. Therefore, they all have good standing and vigor. In the same vein, the result of the basal area shows that the trees have high basal areas correlated to the tree enspacement. dbh and height are functions of basal area and volume (Husch *et al*, 2003). As a result of this, values obtained for the dbh and height can be use to project the corresponding tree volume.

Based on the result in table 2, it was observed that the modified exponential model is the best for the height-diameter relationship as 36.21%, 37.21% & 34.74% obtained as the coefficient of determination in the three cases considered. This revealed that 36.21%, 37.21% & 34.74% variation in height were explained by the diameter. Also, the curve was best captured by the modified exponential fit in the three cases considered (fig 1, 2 & 3). The predicted values follow the same nonlinear pattern and formed close to the line of best fit without much outlier. The modified exponential fit also yielded the minimum root mean square error which is an indication that the estimator is efficient compared with other growth curves used in the study. Gregoire *et al*. (1995) pointed out that the determination of fixed and random effects parameters in a model is a flexible decision subject to debate. Pinheiro and Bates (1998) suggest that all parameters in the model should first be considered mixed if convergence is possible. Fang and Bailey (2001) suggest that parameters with the high variability and less overlap in confidence intervals obtained by fitting at each individual plot separately should be considered mixed if the convergence is not achieved when considering all the parameters as mixed. Therefore, the result of this study revealed that the ability of dbh in determining height is not strong enough based on the model's goodness of fit and the model's ability for predictive purposes; hence, as a form of correcting this in the future, we shall include other independent variables like Age, soil fertility, basal area, crown area etc so as to check their effects in determining height.

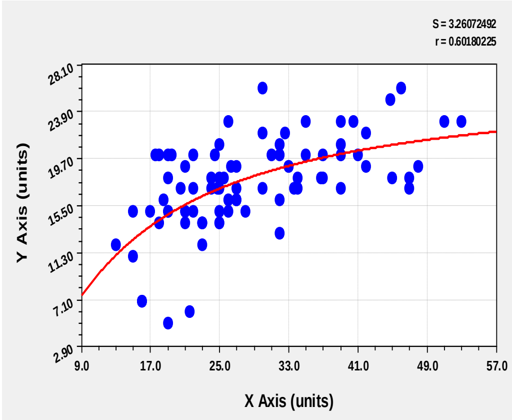

Fig 1

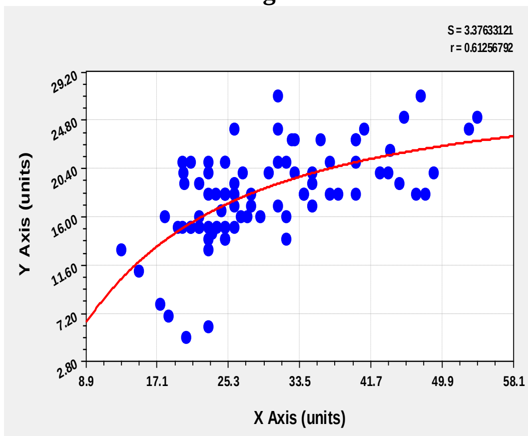

Fig 2

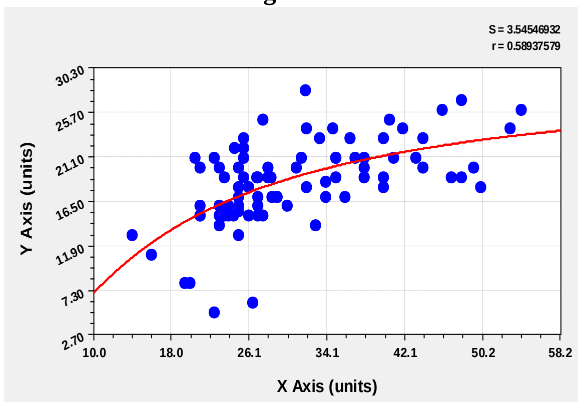

Fig 3

Fig 1, 2 & 3: The modified exponential fit for year 2004, 2008 and 2011 respectively

Conclusion

Non-linear growth functions have been commonly used for modelling tree height-diameter relationships. Traditionally the `best ' equation is evaluated and selected based on its statistical properties such as mean squared error, asymptotic *t*-statistics for the parameters, R^2 of the model, and residual analysis (Lianjun, 1997). The result of this study has evidently revealed that nonlinear model can be use to model the tree height-diameter relationship. Meanwhile, modified exponential fit was observed the best fit out of the five models considered. Though, the relationship was weak, this called for more variables such as age, crown area and soil fertility to be incorporated in future prediction of the tree height in the study area. More importantly, the potential tree height-diameter equations should be evaluated and validated for their predictive capabilities (e.g. accuracy, precision, and flexibility) across a range of tree diameters.

Table 1. Summary statistics of growth variables

Years of assessment	Variables	No of trees/ha	Mean	Maximum	Minimum	Standard deviation
2004	dbh (cm)	1,109	28.36	53.00	10.00	9.91
	Height (m)		17.46	26.00	5.00	4.06
	Basal Area (m^2/ha)		20.19	58.50	3.50	0.05
	SLC		65.91	142.86	36.17	19.70
2008	dbh (cm)	1,104	29.01	54.00	10.00	10.08
	Height (m)		18.06	27.00	5.00	4.25
	Basal Area (m^2/ha)		25.80	70.12	4.30	0.05
	SLC		66.40	140.00	37.50	19.33
2011	dbh (cm)	1,084	30.02	54.20	11.00	10.03
	Height (m)		18.54	28.00	5.00	4.36
	Basal Area (m^2/ha)		26.70	72.62	4.81	0.05
	SLC		68.55	138.16	36.00	19.17

Table 2. Summary of the fitted models ranked according to their suitability

Year of assessment	Name of the model	R^2 (%)	a	b	RMSE
2004	Modified exponential	36.21	2.71	-1.16	3.2607
	Exponential association	36.17	2.30	5.34	3.2619
	Saturation growth-rate	36.01	3.07	2.05	3.2661
2008	Modified exponential	37.53	2.95	-1.36	3.3763
	Exponential association	37.42	2.50	4.58	3.3792
	Modified geometric	37.19	3.58	-5.78	3.3854
2011	Modified exponential	34.74	3.07	-1.47	3.5455
	Exponential association	34.59	2.60	4.25	3.5496
	Hyperbolic	34.39	2.76	-2.61	3.5549

This useful information can help forest researchers and managers to select and apply the appropriate models. Model validation can be conducted using independent validation data sets or Monte Carlo cross-validation methods.

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