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Hydromagnetic unsteady flow of visco-elastic rivlin-ericksen and walters fluid through a porous media

S Banerjee
Saroj Mohan Institute of Technology, Hooghly-712512, (W.B) India.

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ABSTRACT
The aim of this paper is to study the flow of visco-elastic fluid of Rivlin-Ericksen and Walters type through a porous media bounded by a rectilinear pipe of uniform cross section in the presence of a transverse magnetic field. The flow takes place under the action of transient and periodic pressure gradients. Using integral transform technique, the exact solution for velocities have been obtained in case of visco-elastic Rivlin-Ericksen fluid and Walters fluid. Finally various cases of permeability of the porous media have been discussed for both the fluids.

Introduction

The remarkable progress in the field of fluid dynamics of inviscid and viscous fluids will be found in the informative books of Lamb [1], Milne-Thomson [2], Batchelor [3], Landau and Lifshitz [4]. Various hydrodynamic problems and the corresponding development of the theories will be found in the monographs of Cowling [5], Ferraro-Plumpton [6], Cabannes [7] and Jeffrey [8]. In the area of non -Newtonian fluids the works of Bhatnagar [10] and the work of Joseph [11] are very worthy to mention. A survey monograph of non-Newtonian fluid flows of Kapur, Bhatt and Sacheti [9] may also be referred. The flow of visco-elastic fluid between two parallel plates under uniform, exponential and periodic pressure gradients has been investigated by Das [12] and Pal and Sengupta [13]. Roy, Sen and Lahiri [14] studied the problem of unsteady flow of Rivlin-Ericksen fluid through a rectangular duct under impulsive pressure gradient. Bagchi [15] has considered a similar problem through rectangular channel and through two parallel plates with transient pressure gradient. Basak and Sengupta [16] studied the unsteady flow of visco-elastic Maxwell fluid through a straight tube under uniform magnetic field. Das and Sengupta [17] studied the unsteady flow of conducting viscous fluid through a straight rectangular tube. Sengupta and Kundu [18] considered the hydromagnetic unsteady flow of generalised visco-elastic liquid through porous media. In this paper the authors have studied the hydromagnetic unsteady flow of visco-elastic Rivlin-Ericksen and Walters fluid through a porous media.

Basic theory and equation of motion

A conducting viscous incompressible fluid moving in a magnetic field is governed by the following set of equations:

1. Firstly, we consider Maxwell's electromagnetic equations which are

$$\vec{\nabla}.\vec{B} = 0$$

$$\vec{\nabla}.\vec{E} = \frac{\rho_e}{\varepsilon}$$

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\vec{\nabla} \times \vec{E} = \mu_e \vec{J}$$

Tele:
E-mail addresses: drsbanerjee98@rediffmail.com

Where σ is the excess electric charge density ε and μ_e are respectively electrical permitivity and magnetic permeability of the medium, $\vec{J}$ is the current density, $\vec{E}$ and $\vec{B}$ are electric and magnetic field intensity vectors respectively. When the frequency of the applied field is considered low, displacement current is neglected and since no charge separation takes place, ρ_e is also taken zero. So we can write $\vec{\nabla}.\vec{J}=0$.

2. Secondly, we consider the mechanical equations embodying the effect of the electromagnetic forces as well as other body forces. In view of this the Navier-Stokes equation is

$$\frac{dq}{dt}=-\frac{1}{\rho}\vec{\nabla}p+\nu\nabla^2\vec{q}+\vec{F}+\frac{1}{\rho}(\vec{J}\times\vec{B})$$

Where $\vec{q}$ is the velocity of the fluid, ρ is density, p is pressure, $\vec{F}$ is the body force vector (such as gravity force).

3. Thirdly, we consider the equation of continuity in cases of uniform field density, in the following form :

$$\vec{\nabla}.\vec{q}=0$$

Again a fluid moving with velocity $\vec{q}$ is subject to a total electric field $(\vec{E}+\vec{q}\times\vec{B})$, thus if

σ be the electrical conductivity, Ohm's law assumes the form:

$$\vec{J}=\sigma(\vec{E}+\vec{q}\times\vec{B})$$

The mechanical force of electromagnetic origin is perpendicular to the magnetic field, it has no direct influence on the motion parallel to the field. When the motion is perpendicular to the field, we can write,

$\vec{J}\times\vec{B}=\sigma(\vec{E}+\vec{q}\times\vec{B})\times\vec{B}$ [assuming there is no electric field, only magnetic field is present and as such $\vec{E}=0,\vec{B}=0$]

$$=-\sigma\vec{B}\times(\vec{q}\times\vec{B})$$

$$=-\sigma[(\vec{B}.\vec{B})\vec{q}-(\vec{B}.\vec{q})\vec{B}]$$

$=-\sigma B^2\vec{q}$ [assuming $\vec{q}$ is perpendicular. Now if B_0 be the uniform magnetic field and perpendicular to the flow field $\vec{q}$, then $\vec{J}\times\vec{B}=-\sigma B_0{}^2\vec{q}$

In view of this the Navier-Stokes equation becomes:

$$\frac{dq}{dt}=-\frac{1}{\rho}\vec{\nabla}p+\nu\nabla^2\vec{q}-\frac{\sigma B_0{}^2}{\rho}\vec{q}+\vec{F}$$

Rivlin-Ericksen Fluid

Mathematical Formulation

The visco-elastic fluid of Rivlin-Ericksen type is constituted by the rheological equations,

$$\tau_{ij}=-p\delta_{ij}+\tau'_{ij}$$

$$\tau'_{ij}=2\mu(1+\mu_1\frac{\partial}{\partial t})e_{ij}$$

$$e_{ij}=\frac{1}{2}(u_{i,j}+u_{j,i})$$

Where τ_{ij} is the stress tensor, τ'_{ij} is the deviatoric stress tensor, e_{iij} is the rate of strain tensor, μ_1 is the kinematical co-efficient of visco-elasticity, μ is the co-efficient of viscosity, p represents the hydrodynamic pressure, u_1 are the velocity components of the fluid and δ_{ij} is the kronecker delta. From above :

$$\tau'_{ij} = 2\mu(1+\mu_1 \frac{\partial}{\partial t})e_{ij} = 2\mu^* e_{ij} (say)$$

$$Where\ \mu^* = \mu(1+\mu_1 \frac{\partial}{\partial t}) \text{ and } \nu^* = \frac{\mu^*}{\rho} = \nu(1+\mu_1 \frac{\partial}{\partial t}) \text{ as } \nu = \frac{\mu}{\rho}$$

Fundamental Navier-Stokes equation of motion is :

$$\frac{dq}{dt} = -\frac{1}{\rho}\vec{\nabla}p + \nu^* \nabla^2 \vec{q} + \vec{F}$$

$$\text{i.e. } \frac{dq}{dt} = -\frac{1}{\rho}\vec{\nabla}p + \nu(1+\mu_1 \frac{\partial}{\partial t})\nabla^2 \vec{q} + \vec{F}$$

Where ρ is the density, $\vec{q}$ is the velocity and $\vec{F}$ is the body force of the fluid and $\nu = \frac{\mu}{\rho}$ is the kinematic co-efficient of viscosity.

Here we study the unsteady flow of visco-elastic fluid of Rivlin-Ericksen type through a porous medium bounded by a rectilinear tube under the action of transverse magnetic field B_0. We consider the z-axis parallel to the length of the tube whose boundary is given by $x = \pm a, y = \pm b.$ The fluid is initially at rest and the effect due to perturbation of the flow and due to induced magnetic field have been neglected.

The Navier-Stokes equation of motion of visco-elastic Rivlin-Ericksen fluid in view of the above assumptions becomes,

$$\frac{\partial W}{\partial t} = -\frac{1}{\rho}\frac{\partial p}{\partial z} + \nu(1+\mu_1 \frac{\partial}{\partial t})(\frac{\partial^2 W}{\partial x^2} + \frac{\partial^2 W}{\partial y^2}) - (\frac{\sigma B_0{}^2}{\rho} + \frac{\nu}{k})W \qquad ..(1)$$

Where W(x,y,t) is the velocity of the fluid in z-direction, $B_0(=\mu_e H_0)$ is the constant magnetic field perpendicular to the direction of the flow, σ is the electrical conductivity, μ_e is the magnetic permeability, H_0 is the intensity of the transverse magnetic field, k is the permeability of the porous media.

We now introduce the following non-dimensional quantities:

$$W' = \frac{Wa}{\nu}, \mathrm{p}' = \frac{pa^2}{\rho\nu^2}, \mathrm{t}' = \frac{\mathrm{t}\nu}{\mathrm{a}^2}, \mu_1' = \frac{\mu_1\nu}{a^2}, \mathrm{x}' = \frac{x}{a}, \mathrm{y}' = \frac{y}{a}, \mathrm{z}' = \frac{z}{a}$$

Then dropping the primes equation (1) takes the form :

$$\frac{\partial W}{\partial t} = -\frac{\partial p}{\partial z} + (1+\mu_1 \frac{\partial}{\partial t})(\frac{\partial^2 W}{\partial x^2} + \frac{\partial^2 W}{\partial y^2}) - (M^2 + \frac{a^2}{k})W \qquad ..(2)$$

where $M = \sqrt{\frac{\sigma B_0{}^2 a^2}{\rho\nu}}$ is the Hartmann number. Due to symmetric condition, the flow is considered in the region x≥0, y≥0, with no slip boundary condition. The fluid is at rest initially and the flow takes place under the action of time varying pressure gradient. The initial and boundary conditions are :

$$W(x,y,0) = 0 \qquad ..(3)$$

$$\left.\begin{array}{l} W(1,y,t) = 0; 0 \le y \le l, t \ge 0 \\ \frac{\partial W}{\partial x} = 0;\ \mathrm{x} = 0 \end{array}\right\} \qquad ..(4)$$

$$\left.\begin{array}{l} W(x,l,t) = 0; 0 \le x \le 1, t \ge 0 \\ \frac{\partial W}{\partial y} = 0;\ \mathrm{y} = 0 \end{array}\right\} \qquad ..(5)$$

$$\text{where } l = \frac{b}{a}$$

Solution of the problem

Flow of the fluid under the action of periodic pressure gradient

We consider the pressure gradient and the local velocity are periodic in time. So we assume:

$$\left.\begin{aligned} -\frac{\partial p}{\partial z} &= \mathrm{Re}(P_0 e^{i\omega t}) \\ W &= \mathrm{Re}(W_1(x,y)e^{i\omega t}) \end{aligned}\right\} \quad ..(6)$$

Where P_0 is real but W_1 is complex. Using (6) from (2) we have:

$$(1+i\omega\mu_1\frac{\partial}{\partial t})(\frac{\partial^2 W_1}{\partial x^2}+\frac{\partial^2 W_1}{\partial y^2})-(M^2+\frac{a^2}{k}+i\omega)W_1 = -\mathrm{P}_0 \quad ..(7)$$

Now, the following finite Fourier cosine transforms are used to find out the solution:

$$W_c(m,y,t)=\int_0^1 W_1(x,y,t)\cos(p_m x)dx \quad ..(8)$$

$$W_c(x,n,t)=\int_0^l W_1(x,y,t)\cos(p_n y)dy \quad ..(9)$$

$$\text{where} \left.\begin{aligned} \mathrm{p_m} &= (2m+1)\frac{\pi}{2} \\ \mathrm{p_n} &= (2n+1)\frac{\pi a}{2b} \end{aligned}\right\} \quad ..(10)$$

Multiplying equation (7) by $\cos(p_m x)\cos(p_n y)$ and integrating twice w.r.t x and y in the limit x=0 to x=1 and y=0 to $y=l$ and by the boundary conditions (4) and (5) we get :

$$W_c(m,n)=\frac{(-1)^{m+n}P_0}{p_m p_n(\xi+i\eta\omega)}$$

$$where\ \xi=\mathrm{p_m}^2+\mathrm{p_n}^2+\frac{a^2}{k}+M^2,\ \eta=\mu_1(\mathrm{p_m}^2+\mathrm{p_n}^2)+1$$

Now, we apply the following inversion formula for finite cosine transform as:

$$W_1(x,y)=\frac{4}{l}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}W_c(m,n)\cos(p_m x)\cos(p_n y)$$

Thus we get:

$$W_1(x,y)=\frac{4P_0}{l}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{(-1)^{m+n}\cos(p_m x)\cos(p_n y)}{p_m p_n(\xi+i\omega\eta)}$$

$$W(x,y,t)=\mathrm{W}_1(x,y)e^{i\omega t}$$

$$=\frac{4aP_0}{b}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{(-1)^{m+n}(\xi\cos\omega t+\omega\eta\sin\omega t)\cos(p_m x)\cos(p_n y)}{p_m p_n(\xi^2+\omega^2\eta^2)} \quad ..(11)$$

$$where\ \xi=\mathrm{p_m}^2+\mathrm{p_n}^2+\frac{a^2}{k}+M^2,\ \eta=\mu_1(\mathrm{p_m}^2+\mathrm{p_n}^2)+1$$

$$p_m = (2m+1)\frac{\pi}{2},\ p_n = (2n+1)\frac{\pi a}{2b}$$

Flow of the fluid under the action of transient pressure gradient

Here we consider the pressure gradient and the local velocity both are transient in time and we assume :

$$\left.\begin{array}{l} -\dfrac{\partial p}{\partial z} = P\,e^{-Nt} \\ W = W^*(x,y)e^{-Nt} \end{array}\right\} \quad ..(12)$$

Using (12) from (2) we get

$$(1 - N\mu_1 \frac{\partial}{\partial t})(\frac{\partial^2 W^*}{\partial x^2} + \frac{\partial^2 W^*}{\partial y^2}) - (M^2 + \frac{a^2}{k} - \mathrm{N})\mathrm{W}^* = -\mathrm{P}$$

Now we consider the following cosine transforms,

$$W_c^*(m,y,t) = \int_0^1 W^*(x,y,t)\cos(p_m x)dx \quad ..(13)$$

$$W_c^*(x,n,t) = \int_0^1 W^*(x,y,t)\cos(p_n y)dy \quad ..(14)$$

Thus proceeding as above:

$$W_c^*(m,n) = \frac{(-1)^{m+n}P}{p_m p_n \zeta}$$

$$where\,\zeta = (1 - N\mu_1)(\mathrm{p_m}^2 + \mathrm{p_n}^2) + (\frac{a^2}{k} + M^2 - N)$$

Now, we apply the following inversion formula for finite cosine transform as:

$$W^*(x,y) = \frac{4}{l}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty} W_c^*(m,n)\cos(p_m x)\cos(p_n y)$$

Thus we get: $$W^*(x,y) = \frac{4P}{l}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{(-1)^{m+n}\cos(p_m x)\cos(p_n y)}{p_m p_n \zeta}$$

$$W(x,y,t) = \mathrm{W}^*(x,y)e^{-N\mathrm{t}}$$

$$= \frac{4aPe^{-N\mathrm{t}}}{b}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{(-1)^{m+n}\cos(p_m x)\cos(p_n y)}{p_m p_n \zeta} \quad ..(15)$$

Walters Fluid

Mathematical Formulation

We are now discussing the unsteady flow of visco-elastic Walters fluid, through a porous media bounded by a rectilinear tube. The Navier-Stokes equation of motion of visco-elastic Walters fluid in view of the above assumption becomes:

$$\frac{\partial W}{\partial t} = -\frac{1}{\rho}\frac{\partial p}{\partial z} + \nu(1 - \mu_1 \frac{\partial}{\partial t})(\frac{\partial^2 W}{\partial x^2} + \frac{\partial^2 W}{\partial y^2}) - (\frac{\sigma \mathrm{B_0}^2}{\rho} + \frac{\nu}{k})W \quad ..(16)$$

Where W(x,y,t) is the velocity of the fluid in the z-direction.

Now introducing the non-dimensional quantities and dropping the primes as they were in the Rivlin-Ericksen fluid we get from (17),

$$\frac{\partial W}{\partial t} = -\frac{\partial p}{\partial z} + (1-\mu_1\frac{\partial}{\partial t})(\frac{\partial^2 W}{\partial x^2} + \frac{\partial^2 W}{\partial y^2}) - (M^2 + \frac{a^2}{k})W \qquad ..(17)$$

Solution of the problem

Flow of the fluid under the action of periodic pressure gradient

We consider the pressure gradient and the local velocity are periodic in time.

So we assume,

$$\left.\begin{aligned} -\frac{\partial p}{\partial z} &= \mathrm{Re}(P_0 e^{i\omega t}) \\ W &= \mathrm{Re}(W_1(x,y)e^{i\omega t}) \end{aligned}\right\} \qquad ..(18)$$

Now from equation (18) we have,

$$(1-i\omega\mu_1\frac{\partial}{\partial t})(\frac{\partial^2 W_1}{\partial x^2} + \frac{\partial^2 W_1}{\partial y^2}) - (M^2 + \frac{a^2}{k} + i\omega)W_1 = \text{-P}_0 \qquad ..(19)$$

Thus proceeding similar to Rivlin-Ericksen fluid we get,

$$\mathrm{W(x,\,y\,,t)} = \frac{4aP_0}{b}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{(-1)^{m+n}(\xi\cos\omega\,\mathrm{t} + \omega\,\eta\sin\omega\,\mathrm{t})\cos(p_m x)\cos(p_n y)}{p_m p_n(\xi^2 + \omega^2\,\eta_1{}^2)} \qquad ..(20)$$

Where $\eta_1 = 1-\mu_1(p_m{}^2 + p_n{}^2)$

Flow of the fluid under the action of transient pressure gradient

Here we assume,

$$\left.\begin{aligned} -\frac{\partial p}{\partial z} &= P\,e^{-Nt} \\ W &= W^*(x,y)e^{-Nt} \end{aligned}\right\}$$

Now from equation (18) we have

$$(1+N\mu_1\frac{\partial}{\partial t})(\frac{\partial^2 W^*}{\partial x^2} + \frac{\partial^2 W^*}{\partial y^2}) - (M^2 + \frac{a^2}{k} \text{ - N })\mathrm{W}^* = \text{-P}$$

Thus proceeding similar to Rivlin-Ericksen fluid we get,

$$\mathrm{W(x,\,y\,,t)} = \frac{4aPe^{-N\,\mathrm{t}}}{b}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{(-1)^{m+n}\cos(p_m x)\cos(p_n y)}{p_m p_n \zeta_1} \qquad ..(21)$$

$$where\,\zeta_1 = (1+N\mu_1)(\mathrm{p_m}{}^2 + \mathrm{p_n}{}^2) + (\frac{a^2}{k} + M^2 - N)$$

Various Cases

Rivlin-Ericksen Fluid

Pressure gradient is periodic

(i) When k is very small, i.e. the medium is highly porous then, velocity is given by,

$$\mathrm{W} = \frac{4P_0}{a^3 b}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{(-1)^{m+n}k[a^2\cos\omega\,\mathrm{t} + k\{\omega\,\eta\sin\omega\,\mathrm{t} + (p_m{}^2 + p_n{}^2 + M^2)\cos\omega\,\mathrm{t}\}]\cos p_m x\cos p_n y}{p_m p_n} \qquad ..(22)$$

(ii) When k→ ∞ i.e. no porosity is offered by the medium then velocity will be,

$$\mathrm{W} = \frac{4aP_0}{b}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{(-1)^{m+n}[\omega\,\eta\sin\omega\,\mathrm{t} + (p_m{}^2 + p_n{}^2 + M^2)\cos\omega\,\mathrm{t}]\cos p_m x\cos p_n y}{p_m p_n[(p_m{}^2 + p_n{}^2 + M^2)^2 + (\omega\,\eta)^2]} \qquad ..(23)$$

(iii) In case of steady flow i.e. $\omega = 0$ and for very small k we get,

$$\mathrm{W(x,\ y)} = \frac{4P_0}{a^3 b}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{(-1)^{m+n}k[a^2 + k(p_m{}^2 + p_n{}^2 + M^2)\cos\omega\,\mathrm{t}\}]\cos p_m x \cos p_n y}{p_m p_n} \qquad ..(24.1)$$

For k→∞ we have :

$$\mathrm{W} = \frac{4aP_0}{b}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{(-1)^{m+n}\cos p_m x \cos p_n y}{p_m p_n (p_m{}^2 + p_n{}^2 + M^2)} \qquad ..(24.2)$$

(iv) When x=0, y=0 and k is very small,

$$\mathrm{W} = \frac{4P_0}{a^3 b}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{(-1)^{m+n}k[a^2\cos\omega\,\mathrm{t} + k\{\omega\,\eta\sin\omega\,\mathrm{t} + (p_m{}^2 + p_n{}^2 + M^2)\cos\omega\,\mathrm{t}\}]}{p_m p_n} \qquad ..(25)$$

Pressure gradient is transient

(i) When k is very small, i.e. the medium is highly porous then, velocity is given by,

$$\mathrm{W} = \frac{4Pe^{-Nt}}{ab}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{(-1)^{m+n}k\cos p_m x \cos p_n y}{p_m p_n} \qquad ..(26)$$

(ii) When k→∞ i.e. no porosity is offered by the medium then velocity will be,

$$\mathrm{W} = \frac{4aPe^{-Nt}}{b}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{(-1)^{m+n}\cos p_m x \cos p_n y}{p_m p_n[(1-\mathrm{N}\mu_1)(p_m{}^2 + p_n{}^2 + M^2)]} \qquad ..(27)$$

(iii) In case of steady flow i.e. N=0 and for very small k we get,

$$\mathrm{W(x,\ y)} = \frac{4P}{ab}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{(-1)^{m+n}k\cos p_m x \cos p_n y}{p_m p_n} \qquad ..(28.1)$$

For k→∞ we have:

$$\mathrm{W(x,\ y)} = \frac{4aP}{b}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{(-1)^{m+n}\cos p_m x \cos p_n y}{p_m p_n (p_m{}^2 + p_n{}^2 + M^2)} \qquad ..(28.2)$$

(iv) When x=0, y=0 and k is very small,

$$\mathrm{W} = \frac{4Pe^{-Nt}}{ab}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{(-1)^{m+n}k}{p_m p_n} \qquad ..(29)$$

Walters Fluid

Pressure gradient is periodic

(i) When k is very small, i.e. the medium is highly porous then, velocity is given by,

$$\mathrm{W} = \frac{4P_0}{a^3 b}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{(-1)^{m+n}k[a^2\cos\omega\,\mathrm{t} + k\{\omega\,\eta_1\sin\omega\,\mathrm{t} + (p_m{}^2 + p_n{}^2 + M^2)\cos\omega\,\mathrm{t}\}]\cos p_m x \cos p_n y}{p_m p_n} \qquad ..(30)$$

(ii) When k→∞ i.e. no porosity is offered by the medium then velocity will be,

$$\mathrm{W} = \frac{4aP_0}{b}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{(-1)^{m+n}[\omega\,\eta_1\sin\omega\,\mathrm{t} + (p_m{}^2 + p_n{}^2 + M^2)\cos\omega\,\mathrm{t}]\cos p_m x \cos p_n y}{p_m p_n[(p_m{}^2 + p_n{}^2 + M^2)^2 + (\omega\,\eta_1)^2]} \qquad ..(31)$$

(iii) In case of steady flow and when k→0 and k→∞ the velocities will be same as equation (24.1) and (24.2) respectively.

(iv) When x=0, y=0 and k is very small,

$$\mathrm{W} = \frac{4P_0}{a^3 b}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{(-1)^{m+n}k[a^2\cos\omega\,\mathrm{t} + k\{\omega\,\eta_1\sin\omega\,\mathrm{t} + (p_m{}^2 + p_n{}^2 + M^2)\cos\omega\,\mathrm{t}\}]}{p_m p_n} \qquad ..(32)$$

Pressure gradient is transient

(i) When the medium is highly porous then the velocity will be same as equation (26).

(ii) When k→∞ i.e. when no porosity is offered by the medium then velocity becomes,

$$W = \frac{4aPe^{-Nt}}{b}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{(-1)^{m+n}\cos p_m x \cos p_n y}{p_m p_n[(1+N\mu_1)(p_m^{\ 2}+p_n^{\ 2}+M^2)]} \quad ..(33)$$

(iii) In case of steady flow and when k→0 and k→ ∞ the velocities will be same as equation (28.1) and (28.2) respectively.

(iv) When x=0, y=0 and k is very small, velocity will be same as equation (29).

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